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Lattice-Gas Quantum Computation

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Abstract

We present a quantum lattice gas model for a quantum computer operating with continual local decoherence; entanglement of the wavefunction occurs locally over small spatial regions between nearby qubits for only a short time period. The quantum lattice gas is a noiseless method that directly models the lattice-gas particle dynamics at the mesoscopic scale. The system behaves like a viscous Navier-Stokes fluid. Numerical simulations indicate the viscosity of the quantum lattice-gas fluid is lower than its classical lattice-gas counterpart's.

1 Introduction

Many body quantum mechanical systems are notoriously difficult to simulate on classical computers. The computational difficulty of simulating a system of N quantum spins on a lattice in an external uniform magnetic field (spin up and down are the discrete spin states) is in the exponential complexity class. This is because the number of dimensions of the Hilbert space is 2^N. Yet it is possible to impose simplifications to make the problem tractable. We present a quantum spin system where the spins move between lattice sites and where multiple spins interact on site. The wavefunction is coherent locally within a lattice cell for a short time period. This system, even with a large number of spins, can be simulated on a classical computer and has the useful attribute of mimicking the mesoscopic behavior of an artificial many body system of classical particles moving and colliding on a discrete spacetime lattice. We call the spin system a *quantum lattice gas*. Quantum lattice gas models have been previously studied

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by Succi [13], Meyer [10], and Boghosian and Taylor [1] for the relavisitic Dirac equation and the nonrelativisitic Schrödinger equation.

Lattice gases model kinetic processes for a large number of particles in a fine-grained parallel manner; at the macroscopic scale the lattice-gas system behaves like a viscous Navier-Stokes fluid [6, 5]. The drawback is they are quite noisy and have very high viscosity To overcome these difficulties, we have considered integer lattice gases [2]. The integer lattice gas is less noisy and has a lower viscosity, but its viscosity is still too high even in the infinite integer limit. Directly modeling the lattice gas at its mesoscopic scale with a lattice Boltzmann equation is another approach in use today [9]. It is a noiseless lattice-gas method that significantly reduces the viscosity of the fluid as well as corrects some anomalies [3]. It is a first order finite difference method that retains the lattice-gas metaphor for particle streaming but it violates the principle of detailed balance for collisions; the BGK approximation of the collision operator is used and is valid only when the particle distribution is everywhere near local equilibrium [11]. If the particle distribution is far from equilibrium at even a single site of the lattice, the model is subject to collisional instabilities at that site and eventually becomes unstable everywhere.

We present a lattice gas approach that also directly models the system at the mesoscopic scale yet retains detailed balance in its collisions. Hence, it is an unconditionally stable numerical method. The model is a generalized classical lattice gas where digital bits are replaced with quantum bits. This model is a quantum lattice gas where each spin of the quantum spin system represents a qubit in a superposition of the states $\mid 0\rangle$ and $\mid 1\rangle$. There is an amplitude, α, of it being in the "zero" state and another amplitude, β, of finding it in the "one" state, $\mid q\rangle = \alpha \mid 0\rangle + \beta \mid 1\rangle$. To normalize the qubit's state, the two probabilities add to unity: $\langle q \mid q\rangle = \mid \alpha \mid^2 + \mid \beta \mid^2 = 1$.

In recent years, qubits have been physically embodied in several experiments, for example using the internal states of a barium ion localized in a linear trap [4], or the nuclear spin states of an atom in a chloroform molecule subject to a uniform external magnetic field [7]. To date, only quantum computers with two qubits have been constructed. Only a small number of qubits is possible because it is extremely difficult to isolate them from the surrounding environment. Interference effects between qubits are essential to the quantum algorithms (for example factoring [12] or search [8]) and any uncontrolled contact with the external world causes the qubits to decohere and this destroys the computation. Therefore, it is useful to find quantum computational algorithms that require only short term coherence between only a few qubits.

The main theme of this paper is the following: A parallel network of quantum computers can simulate mesoscopic fluid dynamics where each quantum computer comprises a few qubits situated at a lattice node and where these qubits are entangled for only a short time τ. Such an network is a *fine-grained quantum computer*. Its dynamics is determined by a unitary evolution operator $e^{-i\hat{H}\tau/\hbar} = \hat{S}\hat{C}$, the product of a streaming operator $\hat{S}$ and a collision operator

$\hat{C}$. The spin system's wavefunction $\mid \Psi\rangle$ advances forward in unit time steps[1]

$$\mid \Psi(t+\tau)\rangle = \hat{S}\hat{C} \mid \Psi(t)\rangle. \tag{1}$$

Equation (1) is a general way to represent the dynamics of a lattice-gas quantum computer. The particular choice of evolution operator in (1) specifies the "algorithm" for the quantum computer.[2]

2 Quantum Lattice Gases

Consider a fine-grained quantum computer with the following properties: There is a parallel network of V quantum computers, each containing B qubits, where $V \gg B$. So the total number of qubits is $N = BV$. The full Hilbert space with 2^N dimensions is not used in the quantum computation, only a small fraction of size $V2^B$ is used. That is, there are V independent quantum manifolds, each 2^B dimensional. The full wavefunction $\mid \Psi(\vec{x}_1, \ldots, \vec{x}_V; t)\rangle$ can be written as a tensor product state over V number of *on-site kets*, denoted $\mid \psi\rangle$,

$$\mid \Psi(\vec{x}_1, \ldots, \vec{x}_V; t)\rangle = \mid \psi(\vec{x}_1, t)\rangle \otimes \cdots \otimes \mid \psi(\vec{x}_V, t)\rangle, \tag{2}$$

where each on-site ket is formed as a tensor product over the B qubits at the site

$$\mid \psi(\vec{x}, t)\rangle \equiv \mid q_1(\vec{x}, t)\rangle \otimes \mid q_2(\vec{x}, t)\rangle \otimes \cdots \otimes \mid q_B(\vec{x}, t)\rangle. \tag{3}$$

Therefore, $\mid \psi\rangle$ resides in a Hilbert manifold of size 2^B.

A property of the quantum lattice gas is that its collision operator $\hat{C}$ in (1) is block diagonal with V blocks each of size $2^B \times 2^B$ so it can be written as a tensor product over V number of *on-site unitary collision matrices*, denoted $\hat{U}$. That is, $\hat{C} = \bigotimes_{x=1}^{V} \hat{U}$. Each $\hat{U}$ acts on an on-site ket

$$\mid \psi'(\vec{x}, t)\rangle = \hat{U} \mid \psi(\vec{x}, t)\rangle. \tag{4}$$

The prime on the L.H.S. of (4) indicates that the ket is an *outgoing* collisional state. An *equivalence class* is the set of all on-site configurations with equal mass and momentum. $\hat{U}$ itself is also block diagonal, blocked over the equivalence classes of the on-site configurations. Therefore, quantum entanglement occurs only between members of an equivalence class.

Denote the lattice directions by the unit vectors $\hat{e}_a$, and $a = 1, 2, \ldots, B$. The quantum lattice gas dynamics is separated into two processes: streaming between neighboring sites[3]

$$\mid q_a(\vec{x} + \varepsilon\ell\hat{e}_a, t + \varepsilon^2\tau)\rangle = \mid q'_a(\vec{x}, t)\rangle, \tag{5}$$

[1] Of coarse time advances continuously in quantum mechanical spin systems and the wavefunction exists at say time $\tau/2$, but we consider the state of the spin system at unit multiples of the time period τ.

[2] We believe it is important to find useful quantum computational algorithms for physical modeling because they serve as a guide to constructing a fine-grained quantum computer.

[3] $\delta x = \varepsilon\ell$ and $\delta t = \varepsilon^2\tau$. Diffusive ordering, $\delta t \sim \delta x^2 \sim \varepsilon^2$, holds for lattice-gas systems.

and on-site collisions

$$\langle q'_a(\vec{x},t) \mid \hat{n} \mid q'_a(\vec{x},t)\rangle = \langle \psi(\vec{x},t) \mid \hat{U}^\dagger \hat{n}_a \hat{U} \mid \psi(\vec{x},t)\rangle, \tag{6}$$

where $\hat{n} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ is the *single qubit number operator* and $\hat{n}_a$ is the *multiple qubit number operator* which is a $2^B \times 2^B$ size matrix. The collision process defined by (6) involves the collapse or decoherence of the local on-site ket $\mid \psi' \rangle$, a kind of measurement. This decoherence is necessary to allow the qubits to stream independently to their nearest neighbors according to the streaming equation (5). In this way, translation of qubits between neighboring sites occurs in direct analogy with the streaming procedure in a classical lattice gas.

3 Mesoscopic Dynamics

The mesosopic particle distribution function $f_a(\vec{x},t)$ is defined in terms of the qubits at the site or terms of the on-site ket as[4]

$$f_a(\vec{x},t) \equiv \langle q_a(\vec{x},t) \mid \hat{n} \mid q_a(\vec{x},t)\rangle = \langle \psi(\vec{x},t) \mid \hat{n}_a \mid \psi(\vec{x},t)\rangle. \tag{7}$$

That is, the amplitude for the "one" state is defined as the square root of f_a

$$\mid q_a(\vec{x},t)\rangle = \sqrt{1 - f_a(\vec{x},t)} \mid 0\rangle + \sqrt{f_a(\vec{x},t)} \mid 1\rangle. \tag{8}$$

The quantum lattice-gas transport equations (5) and (6) can be written more conventionally as a discretized lattice Boltzmann equation

$$f_a(\vec{x} + \varepsilon \ell \hat{e}_a, t + \varepsilon^2 \tau) = f_a(\vec{x},t) + \Omega_a(\vec{x},t), \tag{9}$$

where the collision term is

$$\Omega_a(\vec{x},t) = \langle \psi(\vec{x},t) \mid \hat{U}^\dagger \hat{n}_a \hat{U} - \hat{n}_a \mid \psi(\vec{x},t)\rangle. \tag{10}$$

4 Mass and Momentum Conservation

The mass density, ρ, and momentum density, $\rho\vec{v}$, for the quantum lattice gas are defined as

$$\rho = \sum_{a=1}^{B} m_a f_a = \sum_{a=1}^{B} m_a \langle q_a \mid \hat{n} \mid q_a\rangle = \langle \psi \mid \hat{Q}_\circ \mid \psi\rangle \tag{11}$$

$$\rho v_i = \sum_{a=1}^{B} m_a c \hat{e}_a f_a = \sum_{a=1}^{B} m_a c \hat{e}_a \langle q_a \mid \hat{n} \mid q_a\rangle = \langle \psi \mid \hat{Q}_i \mid \psi\rangle, \tag{12}$$

[4] In a classical lattice gas, $f_a \equiv \langle n_a \rangle$ is defined as an ensemble average over the discrete occupation variable, $n_a = 0$ or 1.

where the mass and momentum operators are defined as[5]

$$\hat{Q}_\circ \equiv \sum_{a=0}^{B-1} m_a \hat{n}_a \qquad \text{and} \qquad \hat{Q}_i \equiv \sum_{a=0}^{B-1} m_a c \hat{e}_{ai} \hat{n}_a. \tag{13}$$

For conservation, we require that the matrix elements (11) and (12) remain constant after each time step

$$\langle \psi' \mid \hat{Q}_\alpha \mid \psi' \rangle = \langle \psi \mid \hat{Q}_\alpha \mid \psi \rangle, \tag{14}$$

where $\hat{Q}_\alpha \equiv (\hat{Q}_\circ, \hat{Q}_i)$ and $\mid \psi' \rangle = \hat{U} \mid \psi \rangle$. This occurs only when the matrices $\hat{Q}_\alpha$ commute with $\hat{U}$[6]

$$[\hat{U}, \hat{Q}_\alpha] = 0. \tag{15}$$

Let $\hat{g}$ denote the generator $\hat{U} = \exp(i\varepsilon\hat{g})$, where ε is an "Euler angle". Consider a "rotation" through an infinitesimal angle ε so that $\hat{U}$ can be expanded to first order as $\hat{U} = \mathbf{1} + i\varepsilon\hat{g}$. The unitary condition, $\hat{U}^\dagger\hat{U} = \mathbf{1}$, implies that the generator is hermitian

$$\hat{g} - \hat{g}^\dagger = 0 + \mathcal{O}(\varepsilon^2). \tag{16}$$

From (15), we see that mass and momentum conservation is ensured provided that

$$\hat{Q}_\alpha\hat{g} - \hat{g}^\dagger\hat{Q}_\alpha = 0 + \mathcal{O}(\varepsilon^2). \tag{17}$$

The solution of the set of linear equations (16) and (17) give the Lie algebra for the unitary group. Therefore, the mass density (11) and the momentum density (12) are conserved when each equivalence class block of the collision operator is an element of the unitary group $U(n)$, where n is the size of the associated equivalence class.

5 A Simple Example Model

Let us consider a simple lattice gas as a concrete example. The lattice gas is one dimensional and has three bits per site, a rest particle with mass two and speed ± 1 particles with mass one. The mass and momentum at a lattice site is

$$m = 2n_0 + n_1 + n_2 \qquad \text{and} \qquad p_x = n_1 - n_2. \tag{18}$$

There are two local configurations both with $m = 2$ and $p_x = 0$: (1) $(n_0, n_1, n_2) = (1, 0, 0)$ and (2) $(n_0, n_1, n_2) = (0, 1, 1)$. These two configurations are members of the only equivalence class for this system. The classical lattice-gas microscopic transport equations are

$$\begin{aligned} n_0(x, t+\tau) &= n_0(x,t) + \Omega(x,t) \\ n_{1,2}(x \mp \ell, t+\tau) &= n_{1,2}(x,t) - \Omega(x,t), \end{aligned} \tag{19}$$

[5]Let $b_{\mu a}$ denote the a^{th}-bit of the μ^{th} ket. Alternatively, we can define the mass and momentum operators in terms of the bit occupations as follows: $(Q_\circ)_{\mu\nu} = \delta_{\mu\nu}\sum_{a=0}^{B-1} b_{\mu a} m_a$ and $(Q_i)_{\mu\nu} = \delta_{\mu\nu}\sum_{a=0}^{B-1} b_{\mu a} m_a c e_{ai}$.

[6]Note that $[\hat{U}, \hat{n}_a] \neq 0$

where the single collision is specified by the function $\Omega = n_1\ n_2\ (1-n_0) - n_0\ (1-n_1)(1-n_2)$. The associated quantum lattice gas has three qubits per site, $\mid\ q_a\rangle = \alpha_a \mid 0\rangle + \beta_a \mid 1\rangle$ for $a = 0, 1, 2$. The zeroth qubit represents a rest particle of mass two and the first and second qubits represent moving particles of speeds ± 1, translating in the right and left going directions, respectively. The mass and momentum densities for the quantum lattice-gas system are

$$\begin{aligned} \rho &= 2\langle q_0 \mid \hat{n} \mid q_0\rangle + \langle q_1 \mid \hat{n} \mid q_1\rangle + \langle q_2 \mid \hat{n} \mid q_2\rangle & (20)\\ \rho v_x &= \langle q_1 \mid \hat{n} \mid q_1\rangle - \langle q_2 \mid \hat{n} \mid q_2\rangle. & (21) \end{aligned}$$

The $m = 2$, $p_x = 0$ equivalence class is spanned by the states $\mid 100\rangle$ and $\mid 011\rangle$. Collisional entanglement occurs only between these two states $\xi \mid 100\rangle + \chi \mid 011\rangle$, where ξ and χ are c-numbers. The on-site ket, $\mid \psi\rangle = \mid q_0\rangle \otimes \mid q_1\rangle \otimes \mid q_2\rangle$, is

$$\begin{aligned} \mid \psi\rangle = \ & \beta_0\beta_1\beta_2 \mid 111\rangle + \beta_0\beta_1\alpha_2 \mid 110\rangle + \beta_0\alpha_1\beta_2 \mid 101\rangle + \beta_0\alpha_1\alpha_2 \mid 100\rangle + \\ & \alpha_0\beta_1\beta_2 \mid 011\rangle + \alpha_0\beta_1\alpha_2 \mid 010\rangle + \alpha_0\alpha_1\beta_2 \mid 001\rangle + \alpha_0\alpha_1\alpha_2 \mid 000\rangle. \end{aligned} \quad (22)$$

The outgoing state $\mid \psi'\rangle = \hat{U} \mid \psi\rangle$ is

$$\begin{pmatrix} \beta_0\beta_1\beta_2 \\ \beta_0\beta_1\alpha_2 \\ \beta_0\alpha_1\beta_2 \\ a\beta_0\alpha_1\alpha_2 + b\alpha_0\beta_1\beta_2 \\ c\alpha_0\beta_1\beta_2 + d\alpha_0\beta_1\beta_2 \\ \alpha_0\beta_1\alpha_2 \\ \alpha_0\alpha_1\beta_2 \\ \alpha_0\alpha_1\alpha_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a & b & 0 & 0 & 0 \\ 0 & 0 & 0 & c & d & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \beta_0\beta_1\beta_2 \\ \beta_0\beta_1\alpha_2 \\ \beta_0\alpha_1\beta_2 \\ \beta_0\alpha_1\alpha_2 \\ \alpha_0\beta_1\beta_2 \\ \alpha_0\beta_1\alpha_2 \\ \alpha_0\alpha_1\beta_2 \\ \alpha_0\alpha_1\alpha_2 \end{pmatrix}, \quad (23)$$

where the local collision operator is the 8×8 matrix with one 2×2 block, which is a member of the U(2) unitary group satisfying

$$\mid a \mid^2 + \mid b \mid^2 = \mid c \mid^2 + \mid d \mid^2 = 1 \qquad \text{and} \qquad ac^* + bd^* = a^*c + b^*d = 0. \quad (24)$$

The outgoing configuration of qubits $\mid q'_a\rangle$ is computed using the collision equation (6)

$$\langle q'_a \mid \hat{n} \mid q'_a\rangle = \langle \psi' \mid \hat{n}_a \mid \psi'\rangle \ \text{ for all } \ a = 0, 1, 2. \quad (25)$$

The $\hat{n}_a$'s are 8×8 diagonal matrices. Their diagonal components are $(n_0)_{ii} = (11110000)$, $(n_1)_{ii} = (11001100)$, and $(n_2)_{ii} = (10101010)$. Using these number operators, (25) becomes

$$\begin{aligned} \mid \beta'_0 \mid^2 &= \mid \beta_0\beta_1\beta_2 \mid^2 + \mid \beta_0\beta_1\alpha_2 \mid^2 + \mid \beta_0\alpha_1\beta_2 \mid^2 + \mid a\beta_0\alpha_1\alpha_2 + b\alpha_0\beta_1\beta_2 \mid^2 & (26)\\ \mid \beta'_1 \mid^2 &= \mid \beta_0\beta_1\beta_2 \mid^2 + \mid \beta_0\beta_1\alpha_2 \mid^2 + \mid c\beta_0\alpha_1\alpha_2 + d\alpha_0\beta_1\beta_2 \mid^2 + \mid \alpha_0\beta_1\alpha_2 \mid^2 \\ \mid \beta'_2 \mid^2 &= \mid \beta_0\beta_1\beta_2 \mid^2 + \mid \beta_0\alpha_1\beta_2 \mid^2 + \mid c\beta_0\alpha_1\alpha_2 + d\alpha_0\beta_1\beta_2 \mid^2 + \mid \alpha_0\alpha_1\beta_2 \mid^2 . \end{aligned}$$

The outgoing qubit states are then set according to (8): $\mid\ q'_a\rangle = \sqrt{1 - \mid \beta'_a \mid^2}\ \mid\ 0\rangle + \sqrt{\mid \beta'_a \mid^2}\ \mid\ \ 1\rangle$. Using (26), we find that mass and momentum conservation (14) holds provided the constraints $\mid\ a\ \mid^2\ +\ \mid\ c\ \mid^2 = \mid\ b\ \mid^2\ +\ \mid\ d\ \mid^2 = 1$ and

$ab^* + cd^* = a^*b + c^*d = 0$ are satisfied by the components of the $\hat{U}$. These constraints differ from (24) but are also satisfied for unitary $\hat{U}$.

At equilibrium, the collision term (10) vanishes when $\hat{U} \mid \psi\rangle = \mid \psi\rangle$; this implies

$$\mid \beta_0\alpha_1\alpha_2 \mid^2 = \mid a\beta_0\alpha_1\alpha_2 + b\alpha_0\beta_1\beta_2 \mid^2 \qquad \text{and} \qquad \mid \alpha_0\beta_1\beta_2 \mid^2 = \mid c\beta_0\alpha_1\alpha_2 + d\alpha_0\beta_1\beta_2 \mid^2 . \tag{27}$$

Using the unitarity of $\hat{U}$, (27) reduces to the following single equation

$$\frac{\mid \beta_0 \mid^2}{1 - \mid \beta_0 \mid^2} = \left(\frac{\mid \beta_1 \mid^2}{1 - \mid \beta_1 \mid^2}\right)\left(\frac{\mid \beta_2 \mid^2}{1 - \mid \beta_2 \mid^2}\right). \tag{28}$$

Since $\mid \beta_a \mid^2 = \langle q_a \mid \hat{n} \mid q_a \rangle$, we find that f_a^{eq} must be the function $\left[\exp(\frac{\mathcal{E}_a}{k_{\text{B}}T}) + 1\right]^{-1}$, where $\mathcal{E}_a$ is a scalar linear combination of the invariant quantities of the system, the mass density and flow velocity: $f_a^{\text{eq}} = f_a^{\text{eq}}(\rho, \vec{v})$.

It is possible to perform a Chapman-Enskog analysis of the quantum lattice gas. Just as is done for the classical lattice gas, expand $f_a^{\text{eq}}(\rho, \vec{v})$ about $\vec{v} = 0$ as a Taylor series in the macroscopic conserved quantities, the mass density and flow velocity. A lattice Boltzmann equation is determined by Taylor expanding (9) about $(\vec{x}, t)$ to order ε^2. Then, the low Mach number expansion of f_a^{eq} can be inserted into the lattice Boltzmann equation. Taking the moments of the lattice Boltzmann equation gives a set of partial differential equations governing the quantum lattice-gas system at the macroscopic scale. For our simple quantum lattice gas, there is a mass continuity equation and a one dimensional Navier-Stokes equation.

6 Simulation Results

We coded two one-dimensional lattice-gas systems: a classical lattice gas with three bits per site and its generalized quantum counterpart with three qubits per site. The primary difference in the numerical outcomes is the classical lattice gas requires ensemble averaging or coarse-grain averaging whereas the quantum lattice gas does not; it simulates the mesoscopic scale directly, consistent with (7).

Sound waves were set up and tested in both systems, depicted in Figures 1. A mass density perturbation of $\delta d = 0.04$ and $\lambda = 127\ell$ on a background density of $d = 0.4$ causes a standing sound wave in both systems. The mass density field for the classical case is extremely noisy because of the discreteness of the occupation variable, $n = 0, 1$. Ensemble averaging is performed to recover the mesoscopic scale (see Figure 1a). In contrast, the mass density field of the quantum lattice is continuous because the matrix element $\langle q \mid \hat{n} \mid q \rangle$ is a real numbered mesoscopic quantity (see Figure 1b). The average of the absolute value peak amplitudes were recorded for 2000 time steps and are plotted in Figure 1c. Another observed difference is the decay rates caused by viscous damping of the form $\exp(k^2\zeta t)$, where the wave number is $k \equiv 2\pi/\lambda$, and ζ is

the bulk viscosity (compare the outcomes plotted in Figure 1c, the data from the quantum lattice gas is clipped for clarity). The sound wave in the quantum lattice gas decays at a much slower rate because at every site and at every time step a collision can occur; there is always some non-zero amplitude for occupation of the collisional states $\mid 011\rangle$ and $\mid 100\rangle$. Hence, the effective mean-free path between collisions is lower for the quantum lattice gas because its collision frequency is greater.

7 Conclusion

On a classical computer, it is possible to simulate a fine-grained quantum computer comprising a large number of qubits with short term local coherence. A small portion of the full Hilbert space is used in the quantum computation. One dimensional simulations were carried out to test the validity of the quantum lattice gas algorithm.

The following points summarize what we presently know about quantum lattice gases:

(1) The quantum lattice gas behaves like a viscous Navier-Stokes fluid. The unitary local update procedure can be done in parallel, where only nearest neighborhood qubits interact. Each block of $\hat{U}$ associated with an equivalence class of size n can be any member of the unitary group $U(n)$, causing local entanglement. We partially collapse the wavefunction locally at each site of the lattice at every time step while keeping mass, momentum, and probability fixed. This continual local collapse of the wavefunction is a built-in decoherence mechanism. Imposing local decoherence allows for particle streaming by exchanging qubits without causing any global entanglement of the full wavefunction.

(2) A quantum lattice-gas algorithm can be thought of as a numerical lattice Boltzmann scheme that simulates the mesoscopic dynamics of an ensemble of classical lattice gases. A quantum lattice-gas algorithm simulates continuous mass and momentum fields, whereas a classical lattice gas requires ensemble averaging or coarse-grain averaging which are computationally expensive. The method is noiseless to within floating-point round off error in representing the c-numbers. The mesoscopic fields are continuous since they are stored in an ordered array of qubits, each in superposition of the $\mid 0\rangle$ and $\mid 1\rangle$ states.

(3) Because of quantum mechanical superposition of states, a quantum lattice gas can have lower viscosity than its classical counterpart.

The quantum lattice gas treatment presented here applies to systems in any number of spatial dimension. It is straightforward to apply the method to simulate two or three dimensional fluid flows; this will be presented in another paper. Lattice-gas quantum computation could also be applied to simulate more complex hydrodynamic systems and other physical systems.

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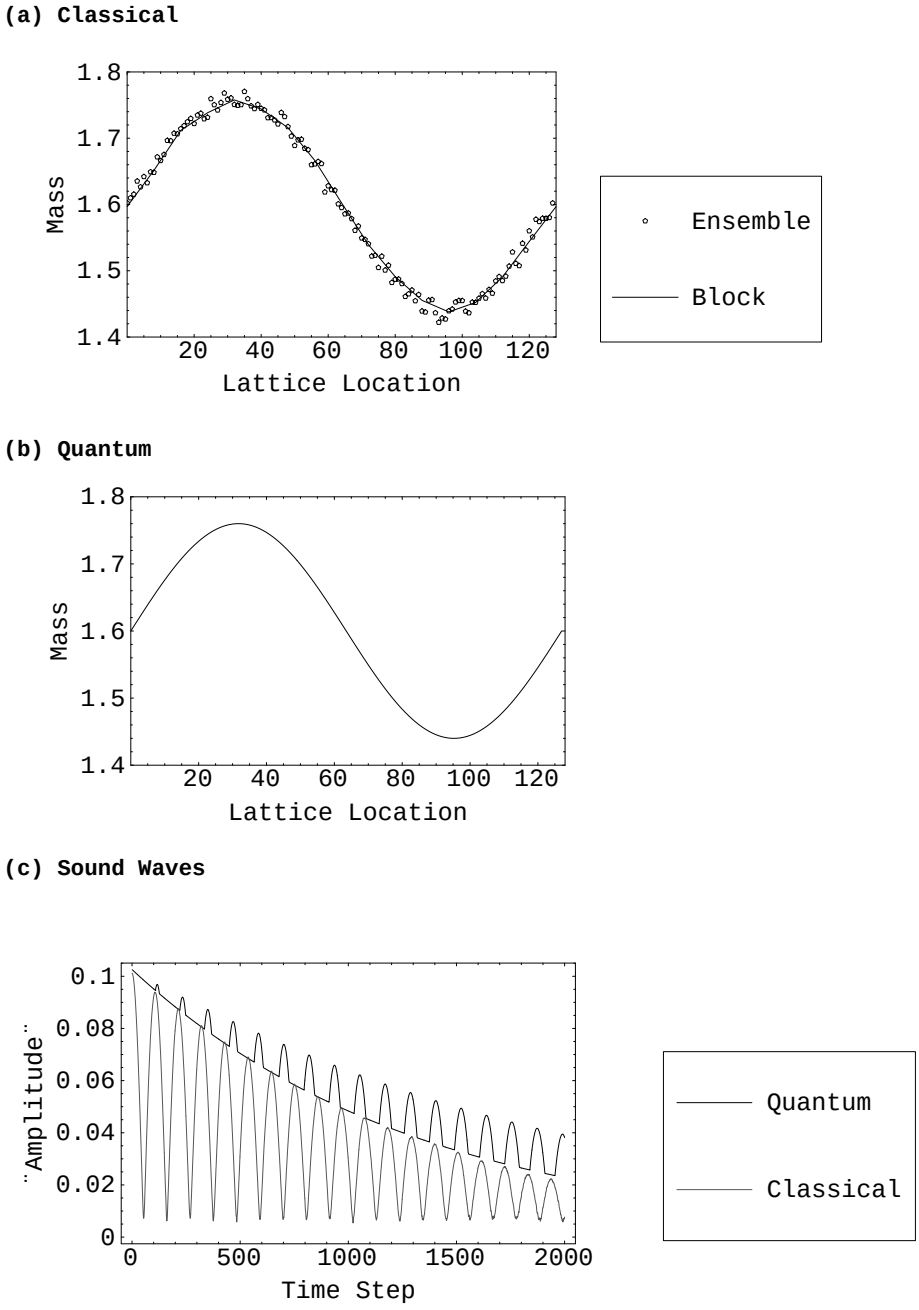

Figure 1: Sound waves for a 1D classical lattice gas with 3 bits per site and a quantum lattice gas with 3 qubits per site. Lattice size=127, background density d=0.4, with perturbation $\delta d = 0.04$ and $\lambda = 127$. Plotted is the: (a) initial state of the classical lattice-gas system with an ensemble size of 20,000; (b) initial state of the quantum lattice-gas system; and (c) time variation of the absolute value of the amplitude of both sound waves. Using superposition of states, the mass density of the quantum lattice-gas system is continuous, allowing for a smooth initial profile with neither coarse-graining nor ensemble averaging. The clipped data in (c) for the quantum system shows a slower decay rate, indicating lower bulk viscosity.