A Comment on the Experimental Determination of $|V_{ts}/V_{td}|^2$.

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We propose a method to extract the ratio $|V_{ts}/V_{td}|^2$ from a measurement of $\Delta\Gamma/\Gamma$ for the B_s meson. This method is experimentally more sensitive than the conventional method for large values of $|V_{ts}|$ but depends on the accuracy of parton level calculations.

The measurement of the mixing parameter $x_s = \Delta m/\Gamma$ for the B_s meson is one of the goals of high energy collider experiments and experiments planned for the facilities of the future [1], [2]. A measurement of x_s combined with a determination of x_d the corresponding quantity for the B_d meson allows the determination of the ratio of the KM matrix elements $|V_{td}|^2/|V_{ts}|^2$ from the ratio [3]

$$\frac{x_s}{x_d} = \frac{(m_{B_s}\eta_{QCD}^{B_s} B_{B_d} f_{B_s}^2)}{(m_{B_d}\eta_{QCD}^{B_d} B_{B_s} f_{B_d}^2)}\frac{\tau_s}{\tau_d}|\frac{V_{ts}^2}{V_{td}^2}| \tag{1}$$

The factor which multiplies the ratio of KM matrix elements is estimated to be unity up to $SU(3)$ breaking effects and has been estimated to be of order (1.3) [4]. Since time integrated measurements of B_s mixing are insensitive to x_s when mixing is maximal, one must make time dependent measurements in order to extract this parameter. A severe experimental difficulty is the rapid oscillation rate of the B_s meson, as recent experimental limits indicate that $x_s > 8.5$ [4] and theoretical fits to the Standard Model parameters suggest that x_s lies in the range $10-40$.

It should be noted that there is another parameter of the B_s meson which can also be measured, this is $\Delta\Gamma/\Gamma$, the difference between the widths of the two B_s eigenstates. For $|V_{ts}| \sim 0.043$ this could lead to a value of $\Delta\Gamma/\Gamma$ of order $10-20\%$ which is measurable at high energy experiments or asymmetric B factories. In parton calculations [3]

$$\Delta\Gamma = \frac{-G_F^2 f_B^2 m_B m_b^2 \lambda_t^2}{4\pi}\,[1+\frac{4}{3}\frac{\lambda_c}{\lambda_t}\frac{m_c^2}{m_b^2}+O(m_c^4/m_b^4)] \tag{2}$$

Comparing to the dispersive term, this gives

$$\frac{\Delta\Gamma_{B_s}}{\Delta m_{B_s}} \approx \frac{-3}{2}\pi\,\frac{m_b^2}{m_t^2}\times\frac{\eta_{QCD}^{\Delta\Gamma(B_s)})}{\eta_{QCD}^{\Delta M(B_s)}} \tag{3}$$

where m_b, m_t are the masses of the b and t quark respectively and terms of order m_c^2/m_b^2, m_b^2/m_t^2 are neglected [3]. The last factor in the above expression, the ratio of QCD corrections for $\Delta\Gamma$ and ΔM, is expected to be of order unity. All of the above factors in $\Delta\Gamma$ have a common mass dependence of m_b^2 in the leading term. From equations (1) and (3), the ratio x_s/x_d is given by,

$$\frac{\Delta\Gamma_{B_s}}{\Delta m_{B_d}} = \frac{-3}{2}\pi\,\frac{m_b^2}{m_t^2}\,\frac{(m_{B_s}\eta_{QCD}^{\Delta\Gamma(B_s)}\;Bf_{B_s}^2)}{(m_{B_d}\eta_{QCD}^{\Delta M(B_d)}\;Bf_{B_d}^2)}\frac{|V_{ts}|^2}{|V_{td}|^2} \tag{4}$$

We have assumed that the lifetimes of the B_d and B_s mesons will have been measured to sufficient precision to extract this ratio. The above expression assumes unitarity since the leading term which enters in $\Delta\Gamma$ is

$$\lambda_u^2 + \lambda_c^2 + 2\lambda_u\lambda_c \tag{5}$$

which is expressed as

$$(\lambda_c + \lambda_u)^2 = (\lambda_t)^2 \tag{6}$$

via unitarity of the KM matrix. In fact, all determinations of $|V_{ti}|$ which depend on virtual t quarks necessarily rely on the assumed unitarity of the 3×3 KM matrix. The only way to obtain values of $|V_{ti}|$ free from this assumption is through direct on-shell measurements of t decays.

Several authors have pointed out that the quantity $\Delta\Gamma(B_s)$ may be large since there are intermediate final states such as $\bar{B}_s \to D_s^{(*)+} D_s^{(\bar{*})-}$ accesible to both B_s and $\bar{B}_s$ which have appreciable branching states [5]. The calculation of Aleksan, Le Yaouanc, Oliver, Pene, and Raynal [5] shows that the parton model estimate and the calculation using exclusive final states agree to within an accuracy of 30%. Given the large experimental uncertainties already present in the determination of the ratio $|V_{ts}/V_{td}|^2$, this is not yet a serious limitation. In the future, it will also be possible to verify that the assumptions in the parton model estimate and in the calculation of Aleksan *et al.* are appropriate by detailed measurements of B_s decay rates. For example, one may check that the W-exchange is small by searching for $\bar{B}_s \to D^{(*)0}\bar{D}^{(*)0}$ and $\bar{B}_s \to D^+\pi^-$ [6]. Verification of the expected pattern of branching ratios predicted in ref. [5] should also be possible. The required measurements would be possible at a lengthy dedicated run on the $\Upsilon(5S)$ resonance at a B Factory.

In order to measure $\Delta\Gamma/\Gamma$, one must determine the lifetimes of two samples of events [7]. One possiblity is to use the large samples of $\bar{B}_s \to \psi\phi$ events and $\bar{B}_s \to D_s^{(*)+}\ell^-\nu$ events. The first sample may be dominated by events in a single CP eigenstate as is the case for $B_{d,u} \to \psi K^*$. This can be verified experimentally by measuring the polarization in this decay. The latter sample of semileptonic decays will be an incoherent mixture of both CP eigenstates. The measured lifetime difference will be $\Delta\Gamma/\Gamma^2$, which can then be used to constrain $|V_{ts}|^2/|V_{td}|^2$. Another possibility is to obtain $\Delta\Gamma$ by fitting the lifetime distribution of a sample of $\bar{B}_s \to D_s^{(*)+}\ell^-\nu$ events to the sum of two exponential distributions and allowing for the oscillatory term.

The sensitivity of the two methods can be roughly compared as follows. The ALEPH lower limit on x_s (8.5) corresponds to the lower limit $\Delta\Gamma/\Gamma > 0.033$ (3.3%). A measurement of a 7% lifetime difference corresponds to a central value of $x_s = 15$ for a time dependent oscillation study. For large values of V_{ts}, the method using $\Delta\Gamma$ eventually becomes more sensitive. Good control of systematic effects from the boost correction in $\bar{B}_s \to D_s^+\ell^-\nu$ and the lifetime of the background sample are required. In published lifetime analyses, these effects have been estimated at the 3-7% level [4]. However, systematic uncertainties in high statistics measurements of charm meson lifetimes have been sucessfully reduced to the 1% level [4]. Similar improvements with higher statistics can be anticipated for B hadron lifetime measurements.

The disadvantage of this method of constraining the KM matrix (using $|\Delta\Gamma_{B_s}|$ and $|\Delta m_{B_d}|$) is the greater reliance on parton calculations. In addition to the methods described here which involve $B_s - \bar{B}_s$ mixing, constraints on $|V_{ts}|^2/|V_{td}|^2$ can be obtained by comparing the branching fractions of $B \to K^*\gamma$ and $B \to \rho\gamma$ decays. As the precision of these measurements of rare decays improves, a careful treatment of possible long distance contributions and $SU(3)$ breaking effects will be required [10], [11], [12].

Finally, it is expected that the contribution of $\Delta\Gamma$ to $B_d - \bar{B}_d$ mixing is small since there are no final states that are accessible to both B_d and $\bar{B}_d$ mesons that also have appreciable branching ratios. This plausible assumption should eventually be experimentally verified by comparing lifetime measurements of B_d in CP eigenstates such as $\bar{B}_d \to \psi K^0$ and $\bar{B}_d \to D^{*+}\ell^-\nu$. The present experimental precision allows for a substantial $\Delta\Gamma$ contribution to the observed time integrated B_d mixing rate.

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given by $\pm\tanh(\Delta\Gamma\tau)$ where the $\pm$ sign refers to the CP eigenvalue. This will allow the resolution of the four fold ambiguity and determine which is more massive of the short or long lived eigenstates. This may be feasible at B factories.

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