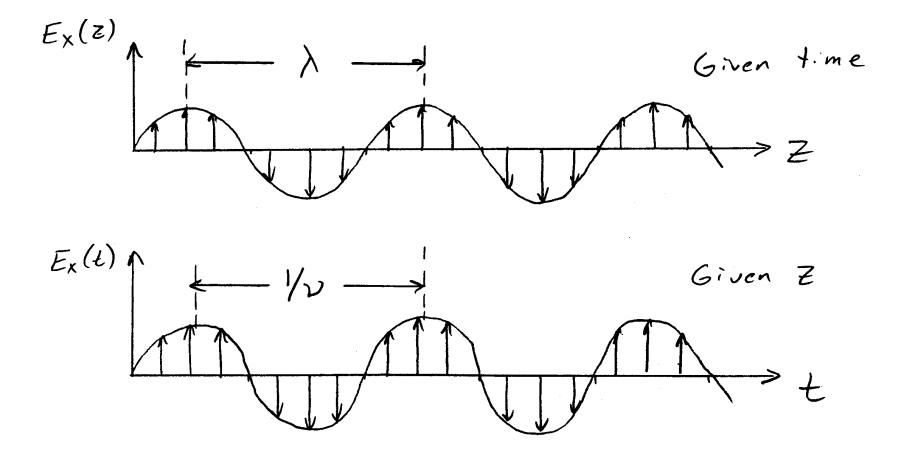
Polarization

Optics, Eugene Hecht, Chpt. 8

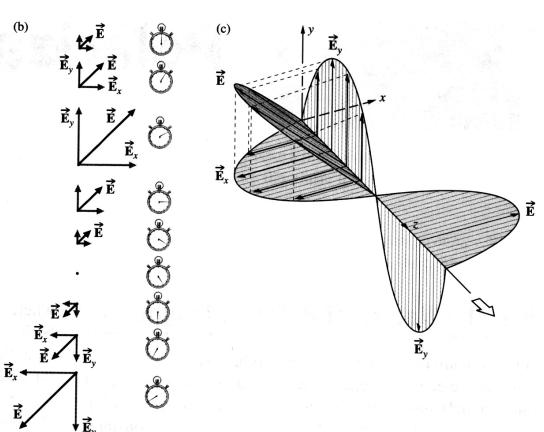
Electromagnetic Properties of Light



$$E_{x}(z,t) = \operatorname{Re}\left\{E_{0x}\exp[i(kz-\omega t)]\right\}$$
$$E_{y}(z,t) = \operatorname{Re}\left\{E_{0y}\exp[i(kz-\omega t)]\right\}$$

The complex amplitude, E_0 is the same for each component.

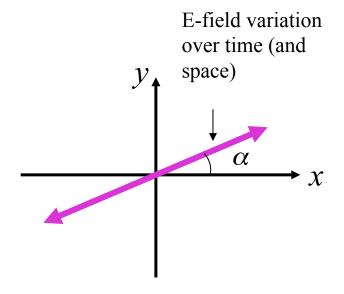
So the components are always in phase.



Arbitrary Angle Linear Polarization

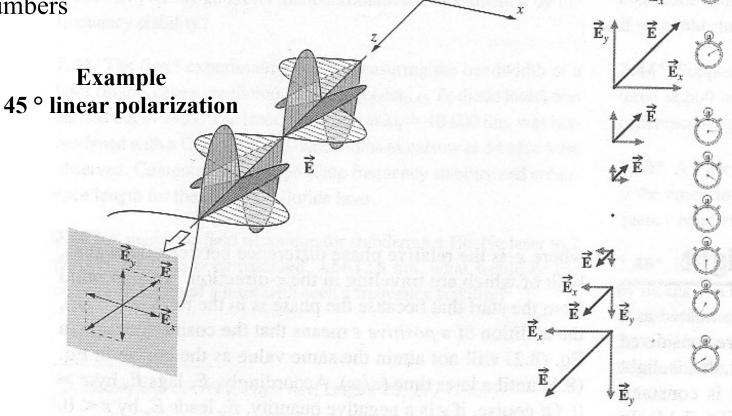
$$E_{x}(z,t) = \operatorname{Re}\left\{E_{0x}\cos(\alpha)\exp[i(kz-\omega t)]\right\}$$
$$E_{y}(z,t) = \operatorname{Re}\left\{E_{0y}\sin(\alpha)\exp[i(kz-\omega t)]\right\}$$

Here, the *y*-component is in phase with the *x*-component, but has different magnitude.



Linear polarization

- E-field magnitude oscillates
- Direction fixed
- Arbitrary polarization angle
 - superposition of x and y polarized waves
 - real numbers



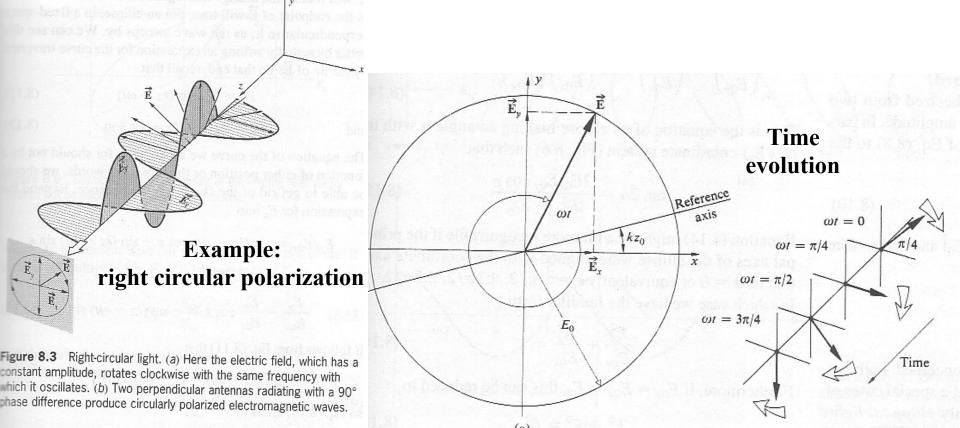
Time

evolution

(b)

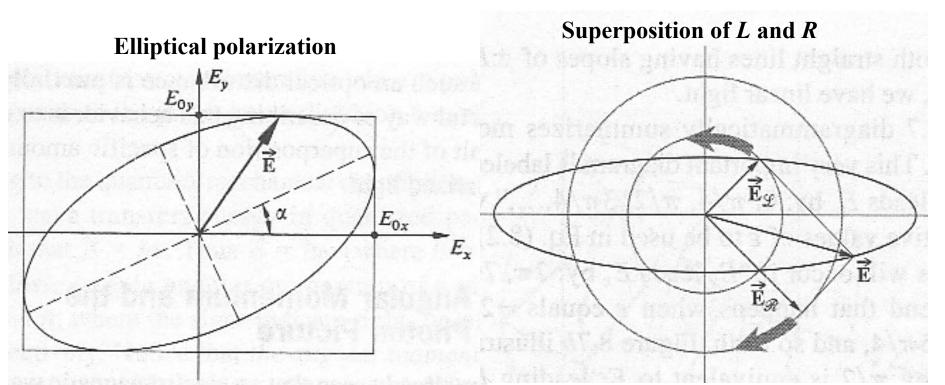
Circular polarization

- E-field magnitude constant
- Direction rotates
- Complex superposition of *x* and *y* polarizations
 - -x and y in quadrature



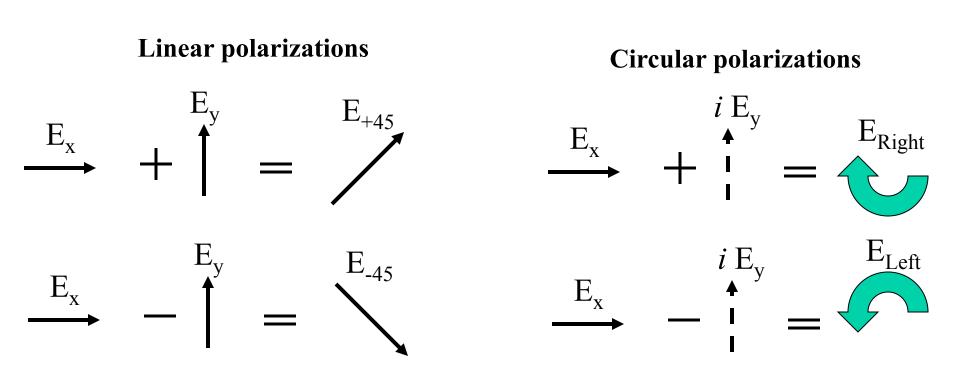
Elliptical polarization

- General case
 - polarization partly linear and partly circular
- E-field sweeps out ellipse
 - both magnitude and direction change with time
- Superposition of *L* and *R* states



Polarization summary

- Decompose into x and y polarizations
- Linear -- real superposition
- Circular -- quadrature superposition



Producing linear polarization

- Induce loss for one polarization direction
- Examples: wire grid, polaroid filter

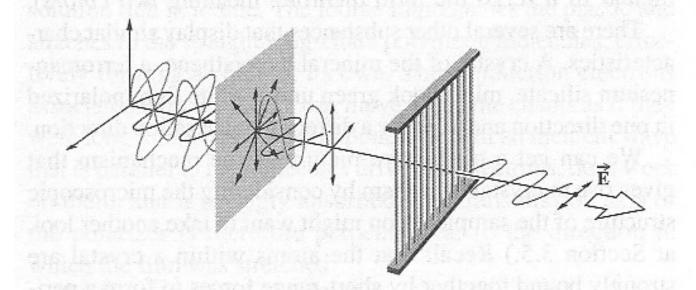


Figure 8.12 A wire-grid polarizer. The grid eliminates the vertical component (i.e., the one parallel to the wires) of the *E*-field and passes the horizontal component.

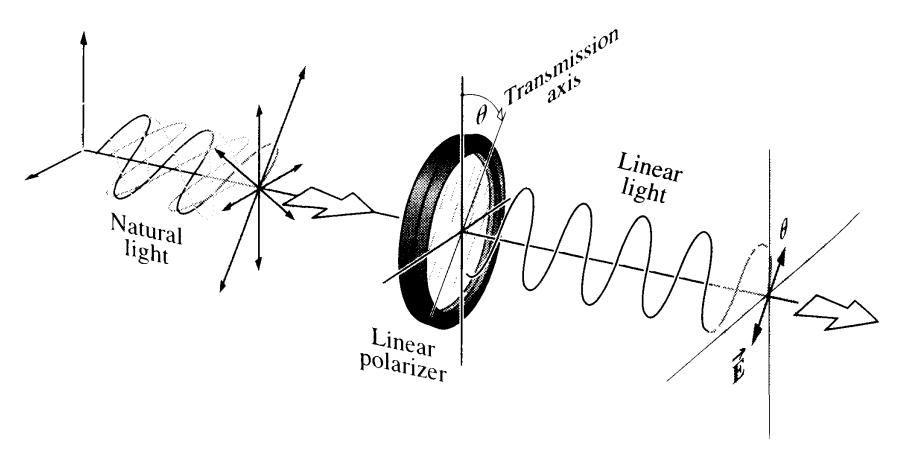


Figure 8.10 Natural light incident on a linear polarizer tilted at an angle θ with respect to the vertical.

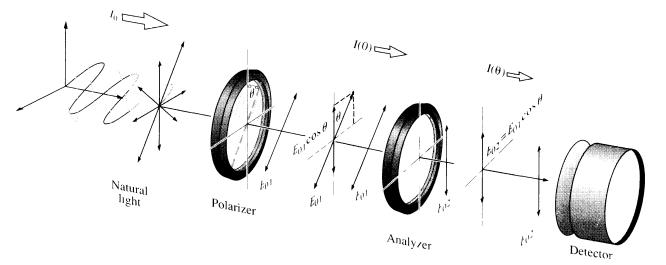


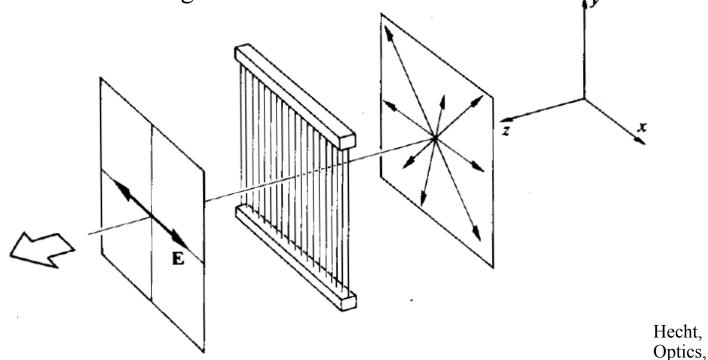
Figure 8.11 A linear polarizer and analyzer—Malus's Law. Natural light of irradiance I_0 is incident on a linear polarizer tilted at an angle θ with respect to the vertical. The irradiance leaving the first linear polarizer is $I_1 = I(0)$. The irradiance leaving the second linear polarizer (which makes an angle θ with the first) is $I(\theta)$.

$$I(\theta) = I(0) \cos^2 \theta \qquad (8.24)$$

This is known as **Malus's Law**, having first been published in 1809 by Étienne Malus, military engineer and captain in the army of Napoleon.

Dichroic Materials

Wire grid polarizer absorbs light with the polarization along the length of the wires because this polarization generates a current along the length of the wires. Resistance in the wires causes energy dissipation, and the phase of the EM wave radiated by the electrons is 180° out of phase with incoming field.

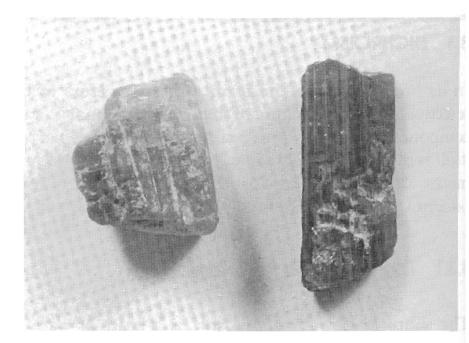


Chapter 8

Figure 8.12 A wire-grid polarizer.

Dichroic Materials

• Tourmaline is a naturally occurring crystal that has strong absorption for light polarized perpendicular to the optic axis and much weaker absorption for light polarized parallel to the optic axis. Polaroid films are formed from sheets of long hydrocarbon molecules that are aligned mechanically as the sheet is stretched. The molecules are then impregnated with iodine so that the molecules are axially conducting.



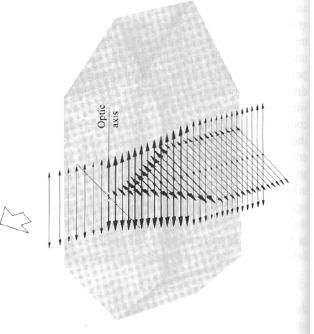


Figure 8.13 A diochroic crystal. The naturally occurring ridges evident in the photograph of the tourmaline crystals correspond to the optic axis. (Photo by E.H.)

Hecht, Optics, Chapter 8

Polarization by Scattering (Rayleigh scattering)

Rayleigh scattering from molecules is usually strongly polarized. For atoms and molecules that are spherically symmetric, like argon and methane, the polarization is nearly complete. For molecules like nitrogen, the scattering is slightly depolarized. Rayleigh scattering intensities are proportional to the fourth power of optical frequency – that's why the sky is blue.

> Hecht, Optics, Chapter 8

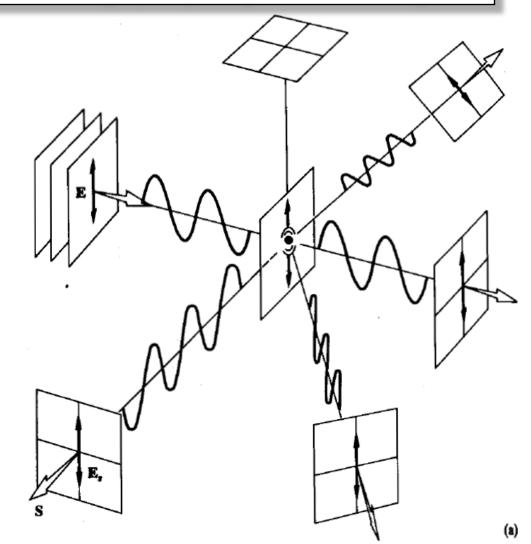
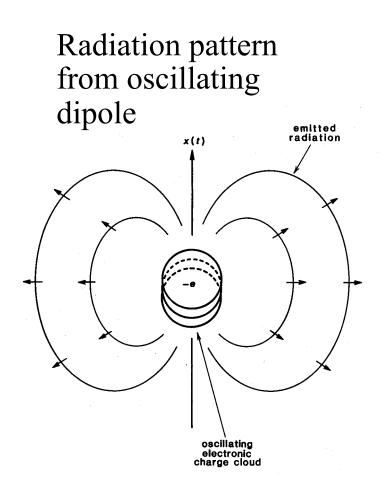


Figure 8.36 Scattering of polarized light by a molecule.

Polarization by Scattering (Dipoles/Rayleigh Scattering)

• The electrons in the atom oscillate in a direction parallel to the electric field vector. The radiation pattern of a classical oscillating dipole has a $\sin^2\theta$ dependence, where θ is the angle between the direction of the incident electric field and the scattering direction.



Siegman, Lasers, 1986

Polarization in Rayleight Scattering

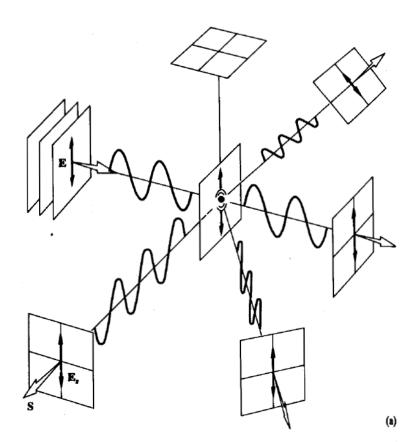
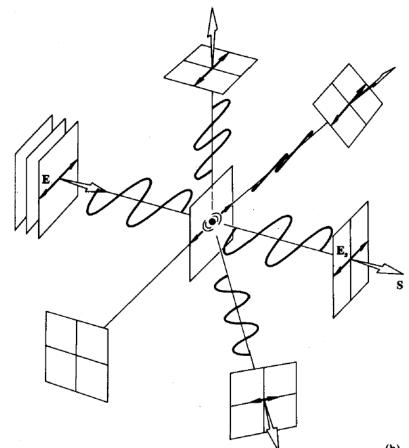


Figure 8.36 Scattering of polarized light by a molecule.



(b)

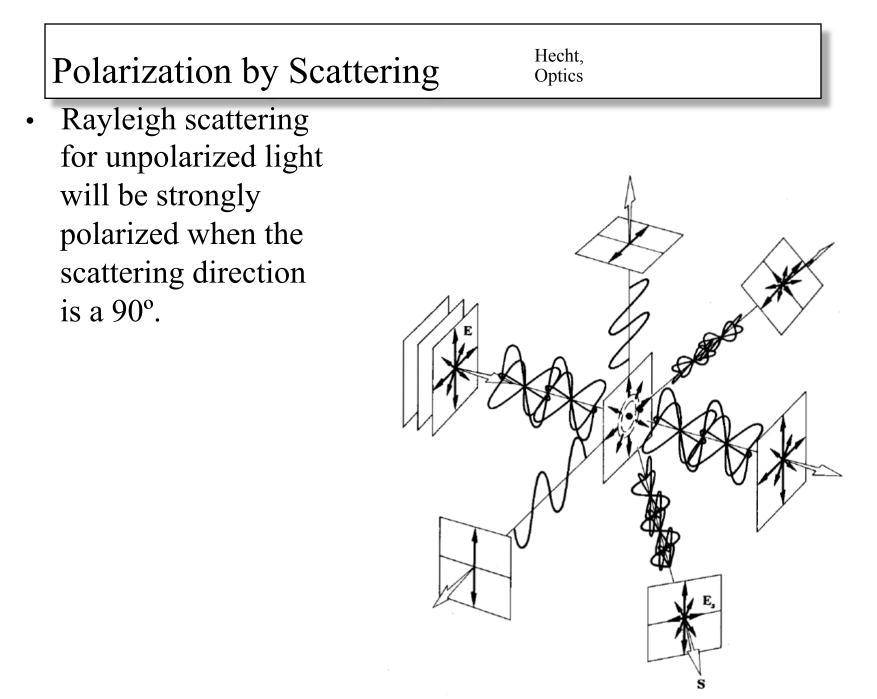
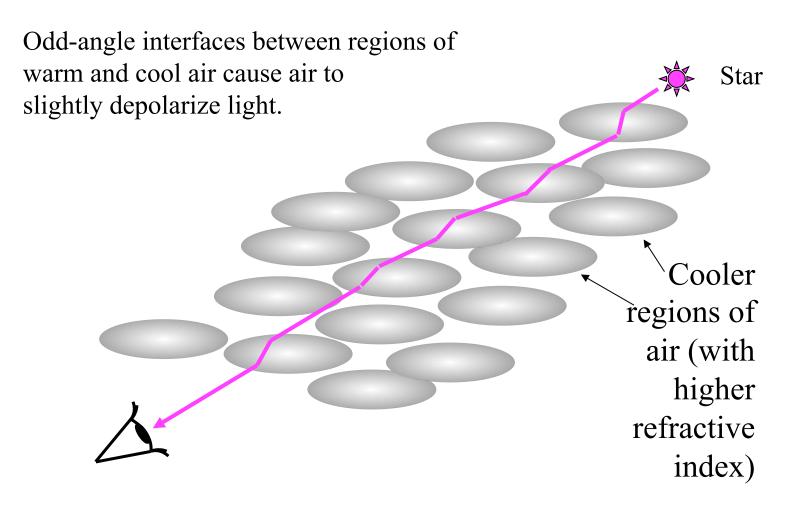


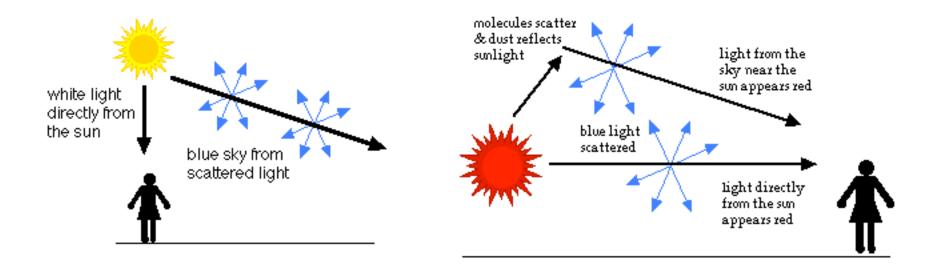
Figure 8.37 Scattering of unpolarized light by a molecule.

The atmosphere depolarizes light slightly.



Droplets of water (i.e., clouds) completely depolarize light.

Why is the sky blue ? Why are sunsets red ?



$$rac{d\sigma}{d\Omega} = \left(rac{\omega}{\omega_0}
ight)^4 r_e^2 \sin^2 heta,$$

Note frequency dependence !

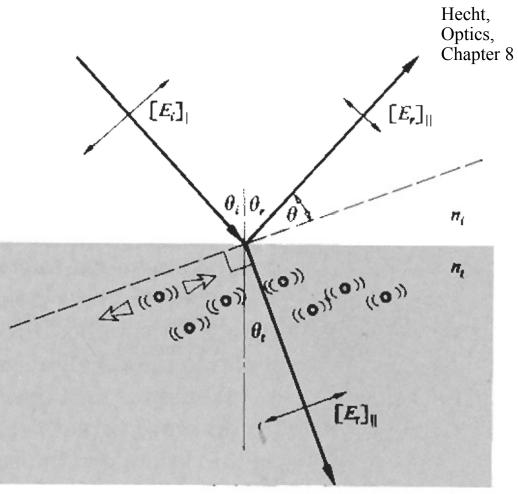
Polarization by Reflection from a Dielectric Surface

• What we have just learned about the polarization of scattered light can be used to understand the absence of reflected light for a TM wave incident at Brewster's angle. At Brewster's angle,

 $n_i \sin \theta_i = n_t \sin \theta_t$ $\theta_t = 90^\circ - \theta_i$

$$n_{i} \sin \theta_{i} = n_{t} \sin (90^{\circ} - \theta_{i})$$
$$= n_{t} \cos \theta_{i}$$

$$\theta_i = \theta_p = \tan^{-1}\left(\frac{n_t}{n_i}\right)$$



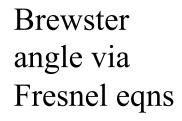
$$R_{\parallel} = r_{\parallel}^2 = [E_{0r}/E_{0i}]_{\parallel}^2$$
 and $R_{\perp} = r_{\perp}^2 = [E_{0r}/E_{0i}]_{\perp}^2$

Squaring the appropriate Fresnel Equations yields

$$R_{\parallel} = \frac{\tan^2 \left(\theta_i - \theta_t\right)}{\tan^2 \left(\theta_i + \theta_t\right)}$$
(8.26)
$$\sin^2 \left(\theta_i - \theta_t\right)$$

and

$$R_{\perp} = \frac{\sin^2 \left(\theta_i - \theta_t\right)}{\sin^2 \left(\theta_i + \theta_t\right)} \tag{8.27}$$



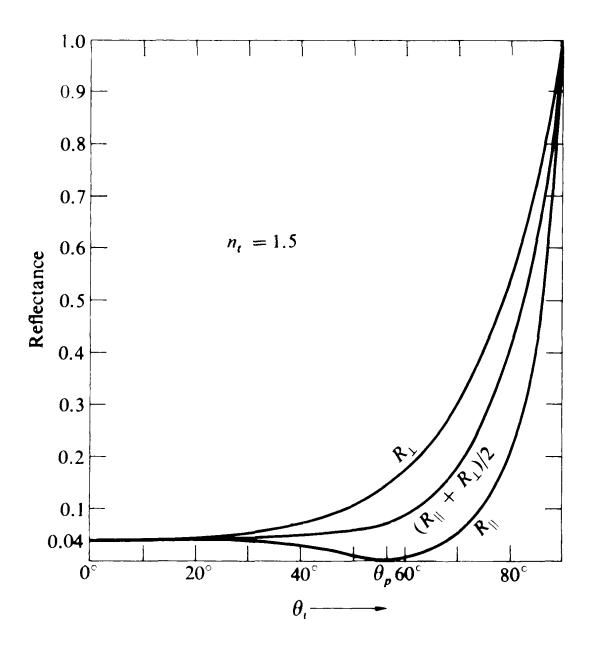
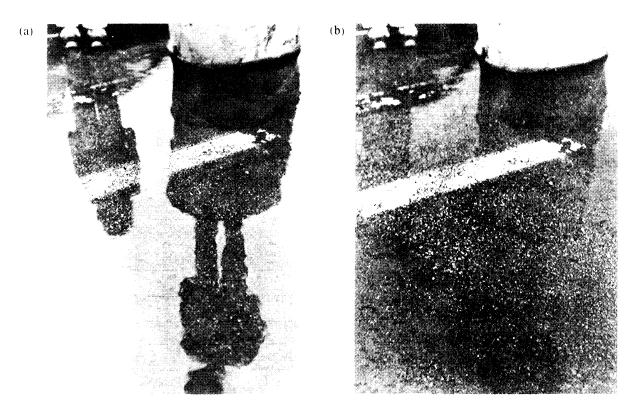


Figure 8.35 Reflectance versus incident angle.

Practical application of Brewster's angle

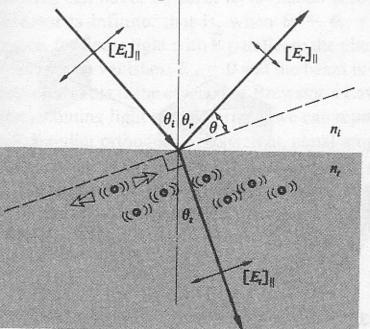


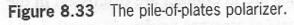
Light reflecting off a puddle is partially polarized. (*a*) When viewed through a Polaroid filter whose transmission axis is parallel to the ground, the glare is passed and visible. (*b*) When the Polaroid's transmission axis is perpendicular to the water's surface, most of the glare vanishes. (Photo courtesy Martin Seymour.)

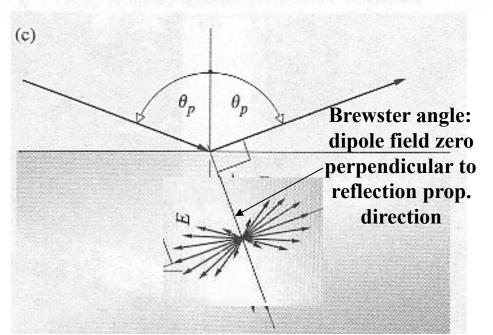
Producing linear polarization -- 3

- Brewster's angle
- Only one polarization reflected
- Reflected light polarized
- Works with most surfaces
- Good way to calibrate polarizers

Refracted beam creates dipoles in medium

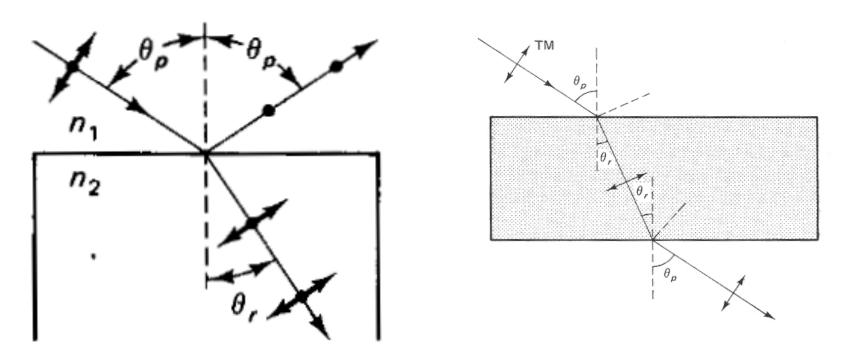






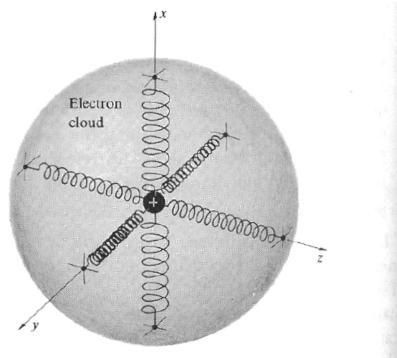
Polarization by Reflection from a Dielectric Surface

• For unpolarized light incident at Brewster's angle, the reflected beam will be pure TE but weak and the transmitted beam will be a mixture of TE and TM light. This situation can be improved dramatically by using a pile-of-plates polarizer



Birefringent Materials

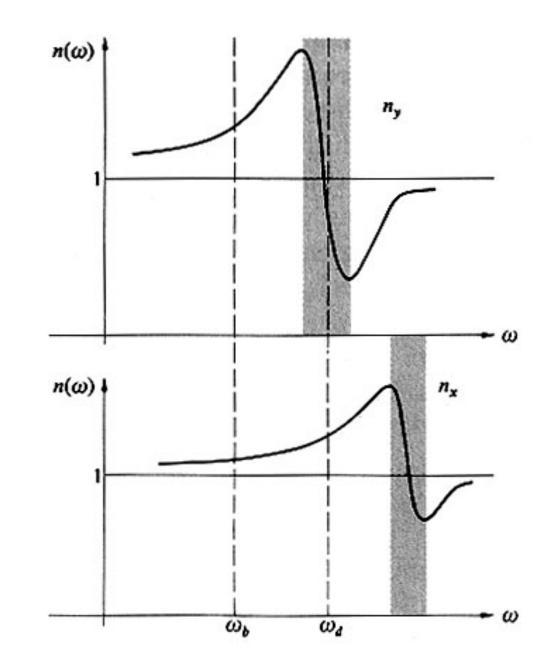
• Birefringent solids have a crystal structure such that the binding of the electron clouds about the nuclei in the lattice is anisotropic. Thus, the induced polarization in the medium will be anisotropic and the refractive index will depend on the orientation of the electric field vector with respect to the crystal lattice.



Hecht, Optics, Chapter 8

Figure 8.15 Mechanical model depicting a negatively charged shell bound to a positive nucleus by pairs of springs having different stiffness.

Indices of Refraction/Birefringence



The x- and y-polarizations can see different refractive index curves.

Birefringent Materials (Indices of Refraction)

• Amorphous solids and cubic crystals have isotropic refractive indices. On the other hand, solid crystals with asymmetric structure such as calcite will exhibit different refractive indices depending on the orientation of the electric field vector of the light field with respect to the crystal structure.

Isotropic	Sodium chloride		1.544	
(cubic)	Diamond		2.417	
	Fluorite		1.392	
Uniaxial	Positive:	n_{\parallel}	n_{\perp}	
(trigonal,	Ice	1.313	1.309	
tetragonal,	Quartz (SiO ₂)	1.5534	1.5443	
hexagonal)	Zircon (ZrSiO ₄)	1.968	1.923	
	Rutile (TiO ₂)	2.903	2.616	
	Negative:			
	Calcite (CaCO ₃)	1.4864	1.6584	
	Tourmaline	1.638	1.669	
	Sodium Nitrate	1.3369	1.5854	
	Beryl	1.590	1.598	
	$(Be_3Al_2(SiO_3)_6)$			
Biaxial		n_1	n_2	n_3
(triclinic,	Gypsum (CaSO ₄ ($2H_2O$))	1.520	1.523	1.530
monoclinic,	Feldspar	1.522	1.526	1.530
orthorhombic)	Mica	1.552	1.582	1.588
	Topaz	1.619	1.620	1.627

TABLE 15-1 REFRACTIVE INDICES FOR SEVERAL MATERIALS MEASURED AT SODIUM

 WAVELENGTH OF 589.3 nm

Birefringent Materials: Calcite

• Calcite is a natural, strongly birefringent material. It is commonly used for polarizers and exhibits the remarkable phenomenon of E_{\perp} double refraction.¹

fraction.^{$$\perp$$}

 $n_{\rm p} = 1.4864$ $n_{\perp} = 1.6584$

for $\lambda = 589.3 \ nm$

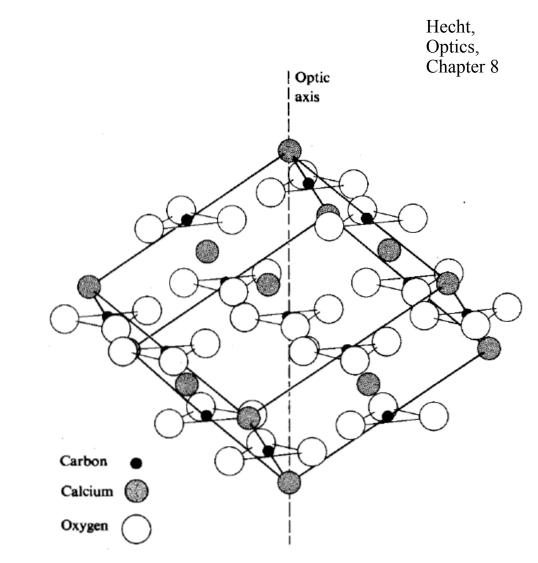
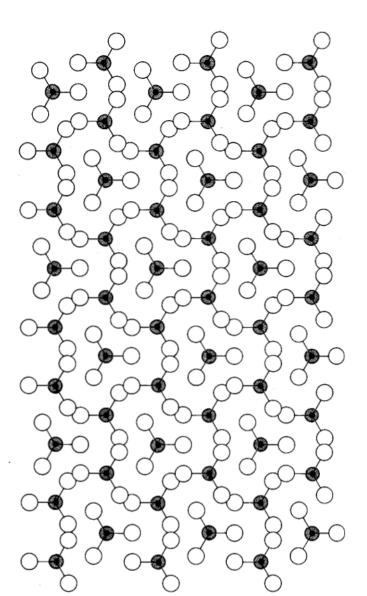


Figure 8.17 Arrangement of atoms in calcite.

Birefringent Materials: Calcite

 Looking up through the crystal along the direction of the optical axis we see a structure with threefold rotational symmetry.

Hecht, Optics, Chapter 8



Double Refraction in Birefringent Materials

The phenomenon of double refraction was observed in calcite and in the early 1800's was a key factor in the development of the theory of polarization of light.

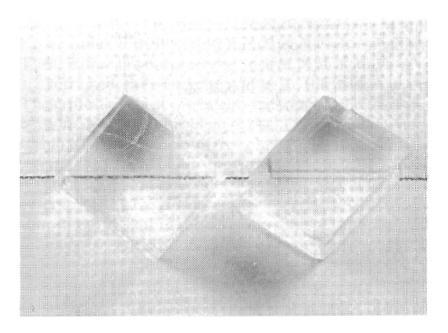


Figure 8.27 Images in sodium chloride and calcite single crystals. (Photo by E.H.)

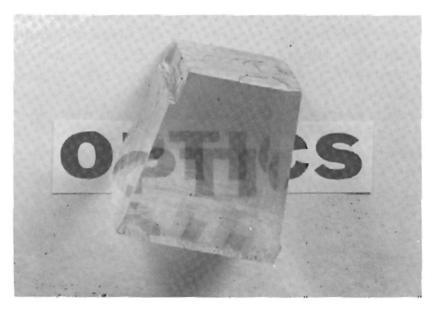
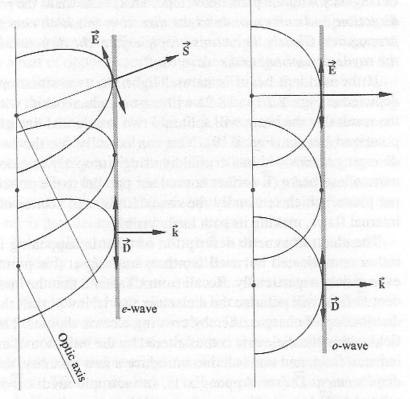


Figure 8.20 Double image formed by a calcite crystal (not cleavage form). (Photo by E.H.)

Hecht, Optics, Chapter 8

Producing linear polarization -- 2

- Separation in birefringent crystal -- ex: calcite
- Ordinary wave -- behaves as expected
- Extra-ordinary wave -- behaves differently
 - example -- E-field not perpendicular to propagation direction
- Input light converted to two polarized beams



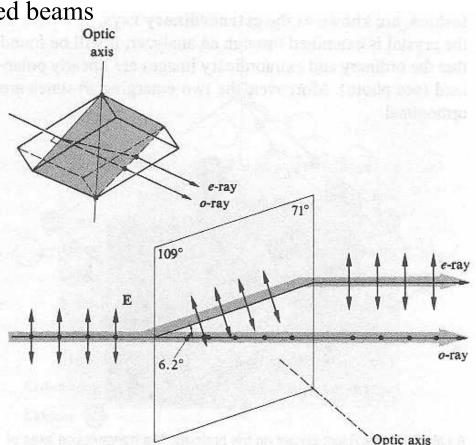
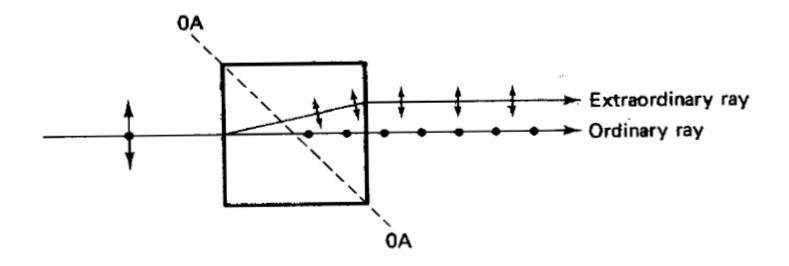


Figure 8.23 Orientations of the E-, D-, S-, and k-vectors.

Double Refraction in Birefringent Materials



Due to Snell's Law, light of different polarizations will bend by different amounts at an interface.

Double Refraction in Birefringent Materials

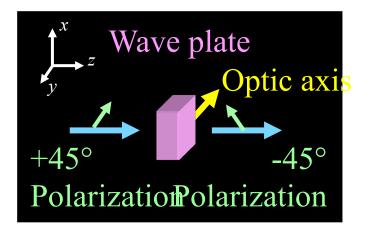
ons within any given The dichroic the previous sect special s e the crys here that attice at Saw metrically t completely the bin voi id be ar the electron We w 5163 ke use of the model fo q the electrica. rectrics. OF Fig. 3.14(b) we rep. the isotropic ing the simple mechanicar model of a spheri

Hecht, Optics, Chapter 8 **Figure 8.21** A calcite crystal (blunt corner on the bottom). The transmission axes of the two polarizers are parallel to their short edges. Where the image is doubled the lower, undeflected one is the ordinary image. Take a long look, there's a lot in this one. (Photo by E.H.)

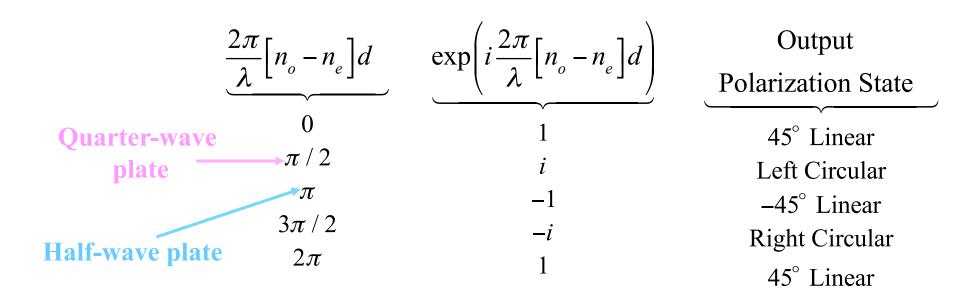
Wave Plates

When a beam propagates through a birefringent medium, one polarization sees more phase delay than the other.

This changes the relative phase of the *x* and *y* fields, and hence changing the polarization.



Wave Plates



A quarter-wave plate creates circular polarization, and a half-wave plate rotates linear polarization by 90°.

We can add an additional $2m\pi$ without changing the polarization, so the polarization cycles through this evolution as *d* increases further.

Thickness of wave plates

When a wave plate has less than 2π relative phase delay, we say it's a zeroorder wave plate. Unfortunately, they tend to be very thin.

Solve for *d* to find the thickness of a zero-order quarter-wave plate:

$$\frac{2\pi}{\lambda} |n_o - n_e| d = \frac{\pi}{2}$$

$$d = \frac{\lambda}{4|n_o - n_e|}$$

Using green light at 500 nm and quartz, whose refractive indices are $n_e - n_o = 1.5534 - 1.5443 = 0.0091$, we find:

$$d = 13.7 \,\mu m$$

This is so thin that it is very fragile and very difficult to manufacture.

A multi-order wave plate has more than 2π relative phase delay.

We can design a twentieth-order quarter-wave plate with $20\frac{1}{4}$ waves of relative phase delay, instead of just $\frac{1}{4}$:

$$\frac{2\pi}{\lambda} |n_o - n_e| d = 40\pi + \frac{\pi}{2} \qquad \longrightarrow_d \longleftarrow$$
$$d = \frac{41\lambda}{4|n_o - n_e|} = 41 d_{zero-order}$$
$$d = 561 \,\mu\text{m}$$

This is thicker, but it's now 41 times more wavelength dependent! It's also temperature dependent due to n's dependence on temperature.

Photoelasticity and Stress-Induced Birefringence

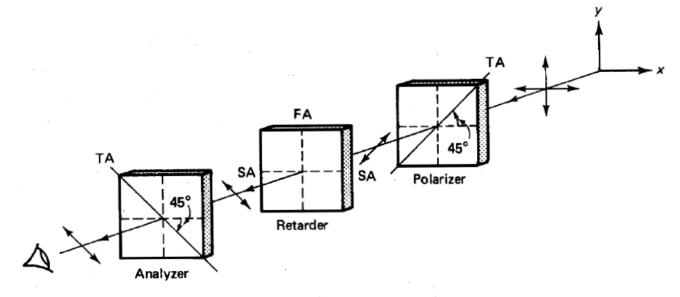
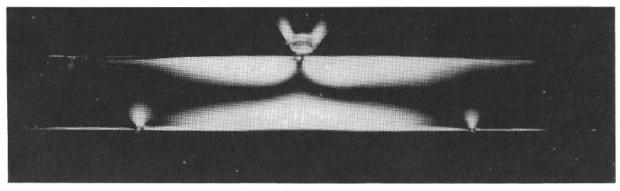
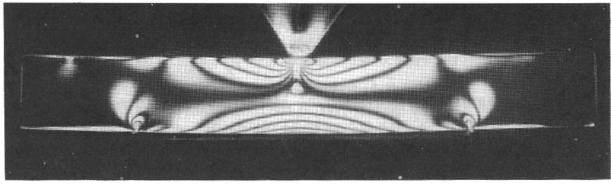


Figure 15-20 Light transmitted by cross polarizers when a birefringent material acting as a half-wave plate is placed between them.

Photoelasticity and Stress-Induced Birefringence



(a)



(b)

Figure 15-21 Photoelastic stress patterns for a beam resting on two supports and (a) lightly loaded at the center, (b) heavily loaded at the center. (From M. Cagnet, M. Francon, and J. C. Thrierr, *Atlas of Optical Phenomenon*, Plate 40, Berlin: Springer-Verlag, 1962.)

Waveplates

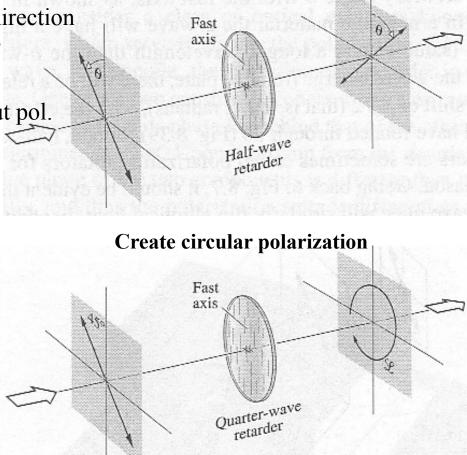
• Polarization converters

e-wave

- One linear polarization direction propagates faster
- Half wave plate -- phase delay 180°
 - rotate linear polarization up to 90°
 - fast axis at 45° to input polarization direction
- Quarter wave plate -- phase delay 90°
 - convert linear to circular polarization
 - R or L_{optic} fast axis +45 or -45 to input pol.

Retardation of

one polarization



Rotate linear pol. by angle 2θ

Other circular polarizers

- Use phase shift for total internal reflection
 45° over broad range of angles
- Two reflections give 90°
- Converts linear to circular polarization

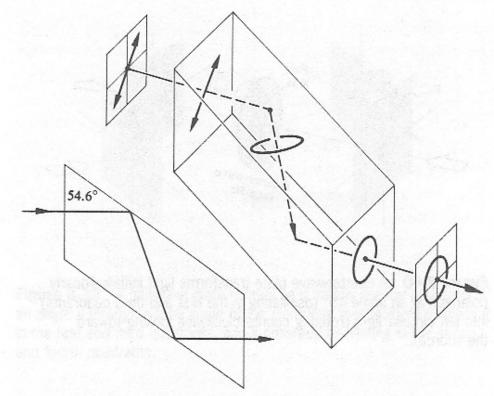
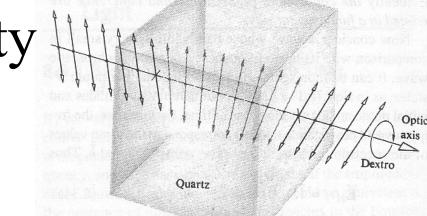
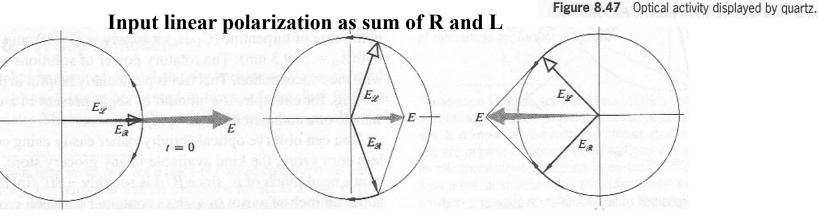


Figure 8.41 The Fresnel rhomb.

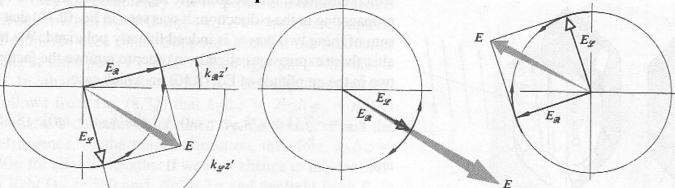
Optical activity \

- Rotate linear polarization
- Express linear as sum of R and L
- Different propagation speeds
- Phase delays give rotation



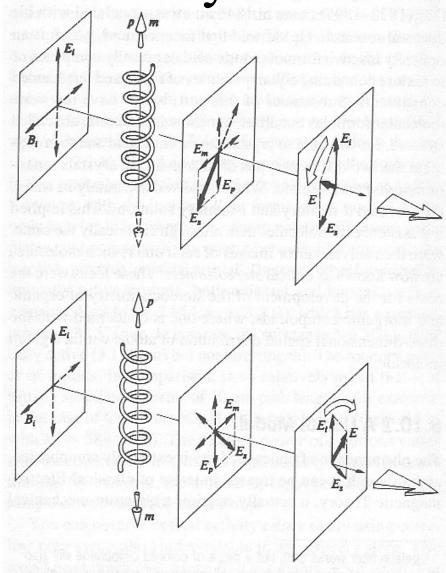


Rotated linear polarization as sum of R and L



Uses of optical activity

- Organic molecule ID
 - right and left handed molecules
 - Example: helical molecule
- Biological molecule ID
 - almost always pure right or left
 - not mixture



Liquid crystals ("Killer App")

- Electric field changes average orientation of molecules
 - Delay depends on polarization direction
- Phase modulator or variable waveplate
- Intensity modulator needs polarizers
- Used for displays -- ex: computer monitors

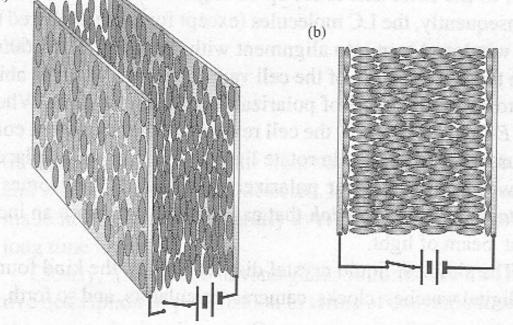


Figure 8.59 (a) A nematic liquid crystal between two transparent electrodes. The long molecules align parallel to a set of microgrooves on the inside faces of the two electrodes. (b) When a voltage is applied, the molecules rotate into alignment with the field.

Mathematical description of polarization

Stokes vectors:

- Elements give:
 - 1/2 total intensity/ I_0 \leftarrow Unpolarized state
 - horizontal lin
 - $+45^{\circ}$ linear
 - right circular

ear
$$\begin{pmatrix} I_0 \\ I_0 - I_H \\ I_0 - I_{+45} \\ I_0 - I_R \end{pmatrix}$$

only I_0 is non-zero

Jones vectors:

- Elements give
 - $\begin{pmatrix} E_x \\ E_y \end{pmatrix}$
 - E-field x-componentE-field y-component
- Only applicable to polarized light

Stokes and Jones vectors

• Special cases of pure polarization

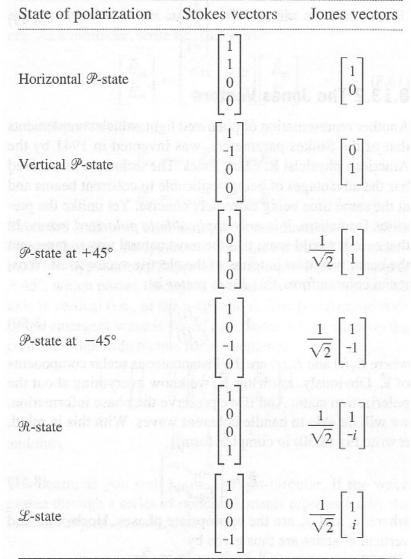
Action of optical elements

• matrix

Example -- vertical polarizer

- input H state -- zero output
- input V state -- no effect

TABLE 8.5Stokes and Jones Vectors for SomePolarization States



Jones and Mueller matrices

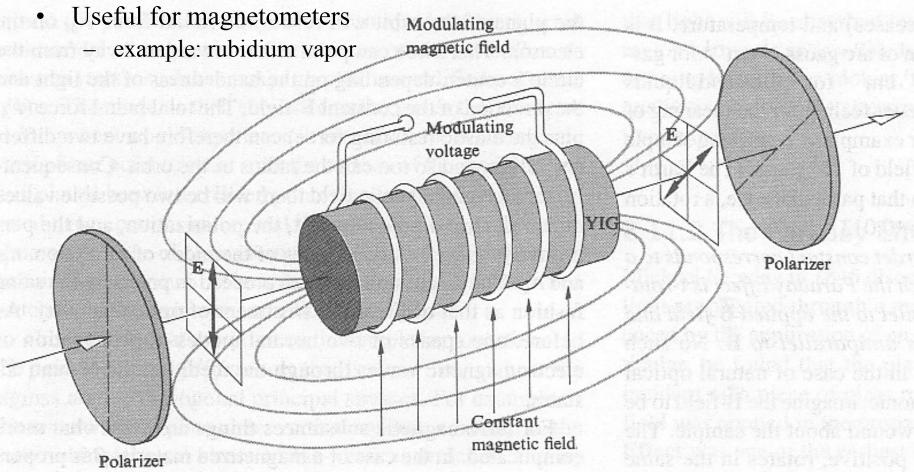
- Stokes parameters used in:
 - Target ID -- spectral-polarmetric
 - Quantum computing
 - V,H and +45,-45 entangled
 - can only measure in one basis
 - measurement destroys info in other basis

TABLE 8.6 Jones and Mueller matrices.					Magkit	$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$
Linear optical ele	ement	Jones matrix	Mueller matrix	Quarter-wave plate,	$e^{i\pi/4} \begin{bmatrix} 1 & 0 \end{bmatrix}$	0 1 0 0
Horizontal linear polarizer	⇔	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$	$\frac{1}{2} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0$	fast axis vertical	[0 - <i>i</i>]	$\begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$
Vertical linear		$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$ $\frac{1}{2} \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$	Quarter-wave plate, fast axis horizontal	$e^{i\pi/4} \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$
polarizer Linear polarizer at +45°	•	$\frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$ $\frac{1}{2} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$	Homogeneous circular polarizer right Q	$\frac{1}{2} \begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$	$\frac{1}{2} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$
Linear polarizer at -45°	Ń	$\frac{1}{2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$ $\frac{1}{2} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$	Homogeneous circular polarizer left 📿	$\frac{1}{2} \begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$	$\frac{1}{2} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$

Backup/Extra Slides

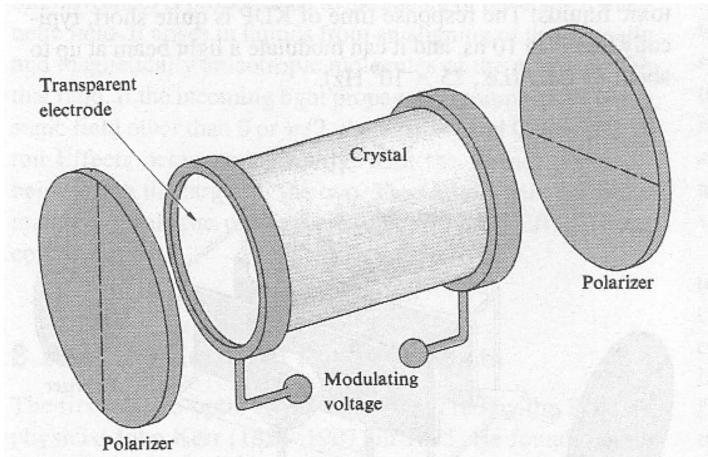
Faraday effect (481L experiment)

- Magnetic field induces polarization rotation
- Orients electron spins in medium
- Angular momenta of electrons and photons interact
- R and L have different propagation delays



Pockels effect

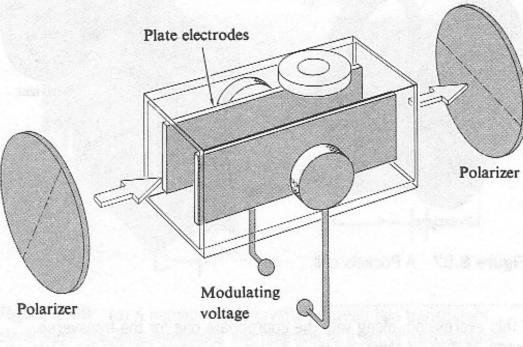
- Similar to Kerr effect
 - Apply electric field along propagation direction
 - Crystal with no center of symmetry -- also piezoelectric
- Delay linear in applied field -- Kerr effect quadratic



Kerr effect

Electro optic effect

- Polarization in direction of applied field changes propagation speed Input linear polarization
- In direction of applied field -- phase modulator
- Perpendicular to applied field -- nothing
- 45° to applied field -- variable waveplate
 - output polarizer gives intensity modulator TABLE 8.4 Electro-optic Constants (Room Temperature, $\lambda_0 = 546.1$ nm)



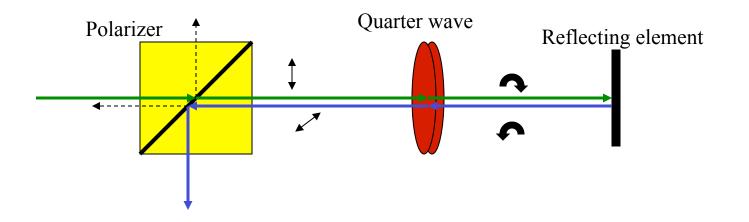
Material	r_{63} (units of 10 ⁻		<i>n</i> _o (approx.)	$V_{\lambda/2}$ (in kV)
ADP (NH ₄ H ₂ PO ₄)	8.5	- 440	1.52	9.2
KDP (KH ₂ PO ₄)	10.6		1.51	7.6
KDA (KH ₂ AsO ₄)	~13.0		1.57	~6.2
KD*P (KD ₂ PO ₄)	~23.3		1.52	~3.4

TABLE 8.3 Kerr Constants for Some Selected Liquids (20°C, $\lambda_0 = 589.3$ nm)

	Substance	$\frac{K \text{ (in units of}}{10^{-7} \text{ cm statvolt}^{-2})}$ 0.6	
Benzene	C ₆ H ₆		
Carbon disulfide	CS_2	3.2	
Chloroform	CHCl ₃	-3.5	
Water	H ₂ O	4.7	
Nitrotoluene	C ₅ H ₇ NO ₂	123	
Nitrobenzene	C ₆ H ₅ NO ₂	220	

Isolators -- 1

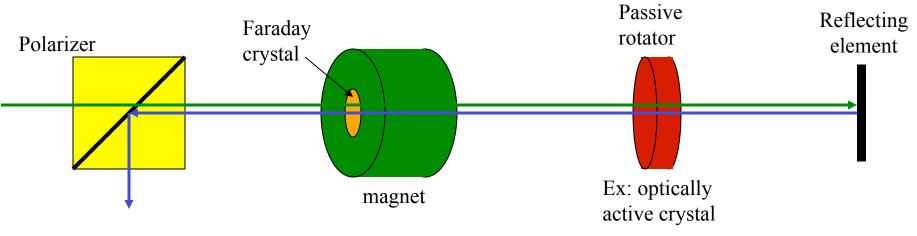
- Polarizer and quarter waveplate
- Double pass through quarter wave plate
 - same as half wave plate
 - rotate polarization by up to 45°
- Polarizer blocks reflected light



Isolators -- 2

- Faraday effect non-reciprocal
 - Opposite for different propagation directions
- Put passive polarization rotator and Faraday rotator in series
 - One direction -- no effect
 - Opposite direction -- rotate polarization 90°
- Polarizer blocks reflections
- Used for fiber optics, laser diodes

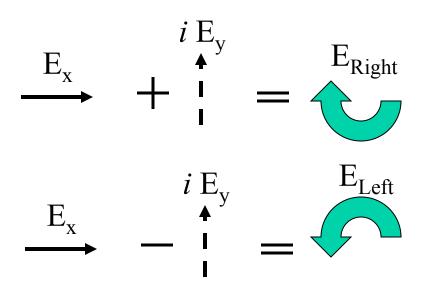
- performance good -- 10 - 30 dB



Angular momentum of light -- Spin

- Circular polarized light has angular momentum
- Like spin
- Can induce electron spin flips
 - important for spectroscopy etc.

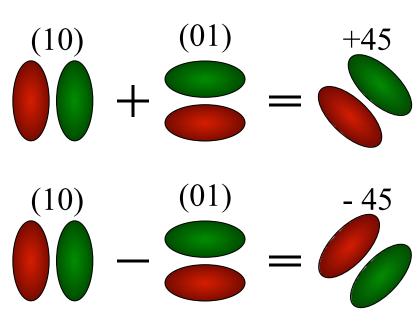
Circular polarizations



Light angular momentum

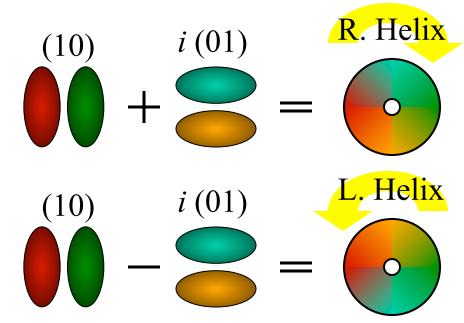
- Transverse laser modes
- Addition rules
 - similar to polarization
- Quadrature case
 - wavefront is helix
- "Orbital" angular momentum

Linear addition



Transverse laser modes(10) mode \square </t

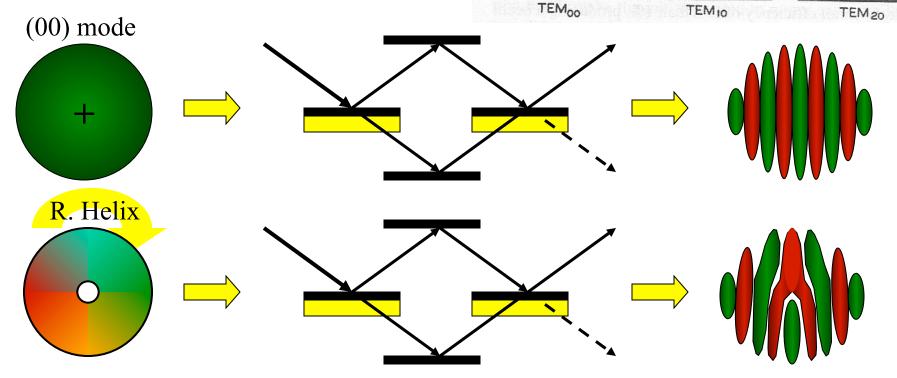
Quadrature addition



Properties of helical beams

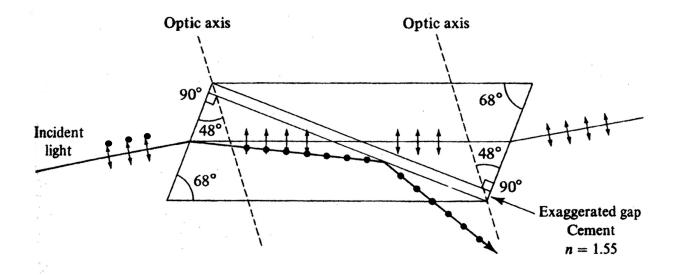
- Interference fringes have discontinuity
- Can have higher order
- Number of extra fringes
 - order number





The Nicol Polarizer uses two identical prisms of calcite and TIR off a layer of optical cement.

Combine two prisms of calcite (with parallel optical axes), glued together with Canada balsam cement (n = 1.55).



Snell's Law separates the beams at the entrance. The perpendicular polarization then goes from high index (1.66) to low (1.55) and undergoes total internal reflection, while the parallel polarization is transmitted near Brewster's angle.