

## 2 Motion

### 2.1 Speed and Velocity

The *speed* of a moving object is equal to the ratio of the distance traveled to the time that elapsed during the trip. For example, suppose you travel from Haleiwa to Hawaii Kai (~50 km away) in 1 hour. Then your (average) speed is

$$\text{speed} = v = \frac{\text{distance}}{\text{time}} = \frac{50\text{km}}{1\text{hour}} = 50\text{km/hr} \quad (\sim 30\text{mi/hr}).$$

Now your speed is not always exactly 50 km/hr. Sometimes you may be stopped for a traffic light, in which case you are not moving and your speed is zero. Some times you may be passing another car in which case your speed will probably be higher. The *instantaneous* speed is the ratio of the distance traveled to the elapsed time for very short periods of the elapsed time. The speedometer in your car (and a police radar gun!) measures instantaneous speed.

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Example:

What is the speed of the Earth in its orbit around the Sun?  
(The Earth's orbit around the Sun is almost a circle of radius  $r = 1.5 \times 10^{11}\text{m}$ .)

Answer:

$$v = \frac{\text{distance}}{\text{time}}$$

The distance is  $2\pi r$ , the circumference of the earth's orbit, and the time for one orbit is 1 year = 365 days  $\times$  24 hours/day = 8760 hours.

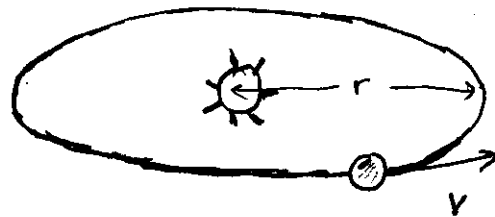
$$v = \frac{2 \times 3.14 \times 1.5 \times 10^{11}\text{m}}{8760\text{hr}} = \frac{9.4 \times 10^{11}\text{m}}{8.76 \times 10^3\text{hr}} = 1.1 \times 10^8\text{m/hr}$$

$$= 1.1 \times 10^5\text{km/hr} = 110,000\text{km/hr} \quad (\sim 70,000\text{mi/hr!!!})$$

Pretty fast!!!

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This is not the only characteristic of motion that is important: motion also has a direction. There is a big difference



The Earth's speed in its orbit around the Sun is 110,000km/hr.

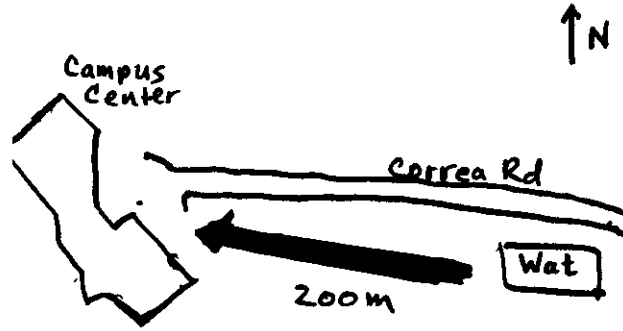
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between a car that is moving at 80 km/hr moving toward you and one moving 80 km/hr away from you. The term *velocity* (in contrast to the *speed*) is used to signify *both* the speed *and* the direction of a moving object. Physical quantities that denote both a magnitude and a direction are called *vectors*. We denote them by an arrow that points in the direction of the vector quantity; the length of the arrow is proportional to the magnitude of the vector quantity. Thus, while we use the symbol  $v$  to denote speed, the symbol  $\vec{v}$  is used to denote the velocity; it stands for both the speed and the direction. For example, consider the Earth in its orbit around the Sun. The magnitude the velocity, i.e. the “speed,” doesn’t change (by very much) during the year, yet the direction changes dramatically.



Although the Earth's speed stays more-or-less the same throughout the year, its velocity vector keeps changing.

Another example of a vector quantity is the difference in position between two places. Suppose that while you are standing in front of Watanabe Hall, someone asks you where the Campus Center is. If all you told him was “about 200 m away,” you would be correct, but not very helpful. A more useful answer would be “about 200m west of here;” that would tell the person everything he needed to know (if he had a compass, otherwise you need to designate the direction some other way—such as by pointing). This is why navigators and flight controllers always talk about “vectors” of airplanes: flight controllers need to know *both* the speed and direction of an airplane if they are to be able to do their job.



A complete set of directions includes the distance and the direction.

The word speed is reserved for the magnitude of the velocity. It does not imply any particular direction. Such a quantity is called a scalar. In physics, scalar quantities are those thing that can be specified by a single number. Other examples of scalars are: the temperature in this room; the number of students in the University; your SAT score; etc.

## 2.2 Acceleration

Changes in velocities are called *accelerations*. Velocities can change either by changing in magnitude (change in speed), or by changing in direction (by turning). The average acceleration is defined as

$$acceleration = a = \frac{change\ in\ velocity}{elapsed\ time}$$

For example, suppose a car dealer trying to sell you a car tells you that "this baby can go from 0 to 60 in 5 seconds." What he means is that the car can change its velocity from zero (0 km/hr) to 60 mi/hr (~100 km/hr) in a time interval of 5 seconds. The average acceleration is then

$$a = \frac{\text{final velocity} - \text{initial velocity}}{\text{elapsed time}} = \frac{100\text{km/hr}}{5\text{sec}} = 20 \frac{\text{km}}{\text{hr} \cdot \text{sec}}$$

Although our automobile (and moped) speedometers give us speeds in mi/hr or km/hr, it is neater for us only to use seconds as our unit of time, and meters as our unit of distance. Using the fact that  $1\text{hr} = 60\text{min} \times 60\text{secs/min} = 3600\text{sec}$  and  $1\text{km} = 1,000\text{m}$ , we get

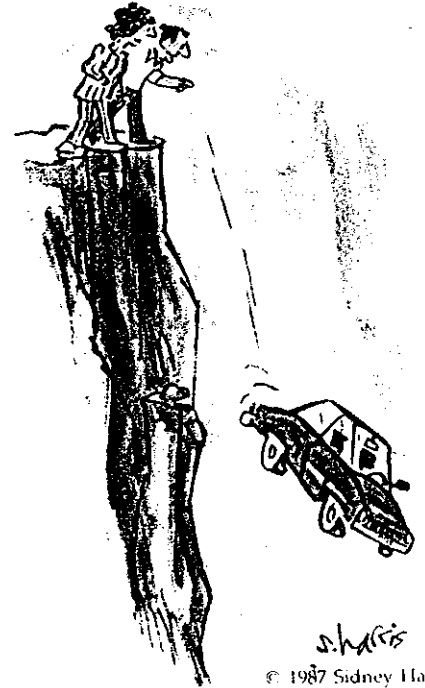
$$20 \frac{\text{km}}{\text{hr} \cdot \text{sec}} = \frac{20,000\text{m}}{3600\text{sec} \cdot \text{sec}} = 5.6 \frac{\text{m}}{\text{sec}^2}$$

Acceleration is also occurring when the direction of the velocity changes. During the year, as the Earth goes around the Sun, its speed doesn't change by very much, but its direction keeps changing. Thus, although the speed is more-or-less constant, this is accelerated motion. When a car goes around a curve, we feel as though we are pulled toward the outside of the curve. Now remember that the law of inertia says that we should want to "persevere in a state of motion in a straight line." But the car that we are sitting in is turning. Our "inertia" tries to keep us going in a straight line, which is toward the outside of the curve.

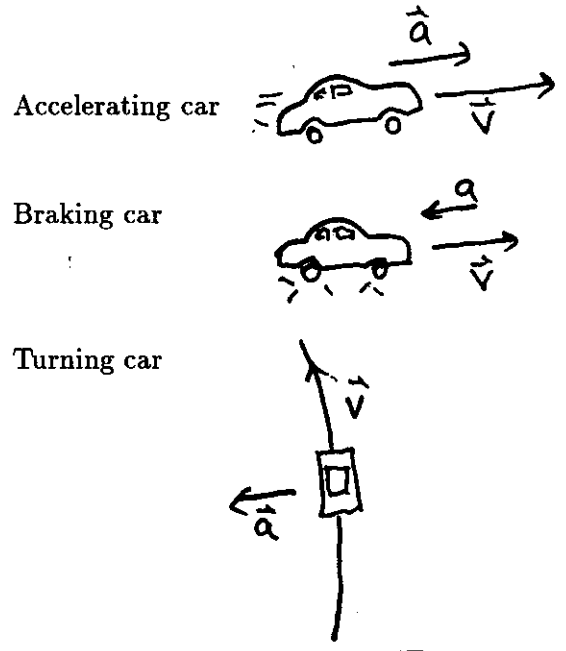
Acceleration is a vector quantity, it has a direction as well as a magnitude. When a car is accelerating forward, the acceleration vector points forward, parallel to the velocity vector. When a car is slowing down, the acceleration vector points backward, opposite to the direction of the velocity vector. When something is moving in a circle, the acceleration points toward the center of the circle, perpendicular to the direction of the velocity vector.

### 2.2.1 Acceleration due to Gravity

The most commonly felt acceleration is that due to gravity. If I hold a ball up in the air and then let go, the law of inertia doesn't hold at all, the ball falls and continues to gain



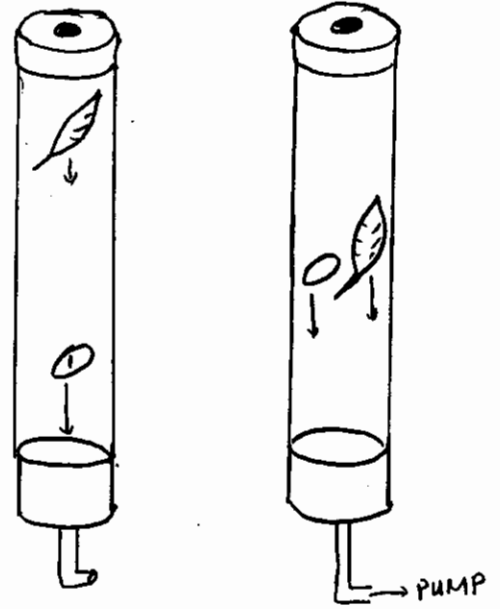
"It goes from zero to 60 in about 3 seconds."



velocity until it hits the floor. The acceleration is always pointed downward. Since the velocity changes, the law of inertia says that there must be “forces impressed thereon.” In this case this is the force of gravity, which we will study in more detail later. An interesting feature of the accelerations caused by gravity was first noted by Galileo:

*In the absence of air resistance, all bodies fall with the same constant acceleration.*

This may seem strange to you because, after all, if you drop a penny and a feather, the penny certainly falls faster than the feather. Galileo realized that this difference was due to the resistance to the motion caused by the air. If a penny and a feather are dropped in a vacuum, they both fall with the same speeds and acceleration.



In the absence of air, the feather and the penny fall at the same speed.



The acceleration due to gravity at the surface of the Earth is  $g = 9.8m/s^2$ . In this class we will approximate this with  $g = 10m/s^2$ , which is very close to the real value and much more convenient to work with. Thus, if you let go of something, it starts moving downward with a continuously increasing speed. After 1 second, its velocity is  $10 m/s$ ; after 2 seconds

20 m/s, etc. The distance it travels each second increases accordingly: in the first second it falls a distance of 5 m; after 2 seconds it has fallen 20 m, etc. These results are summarized in the equations for the distance fallen,  $d$ , and the downward velocity,  $v$ ,

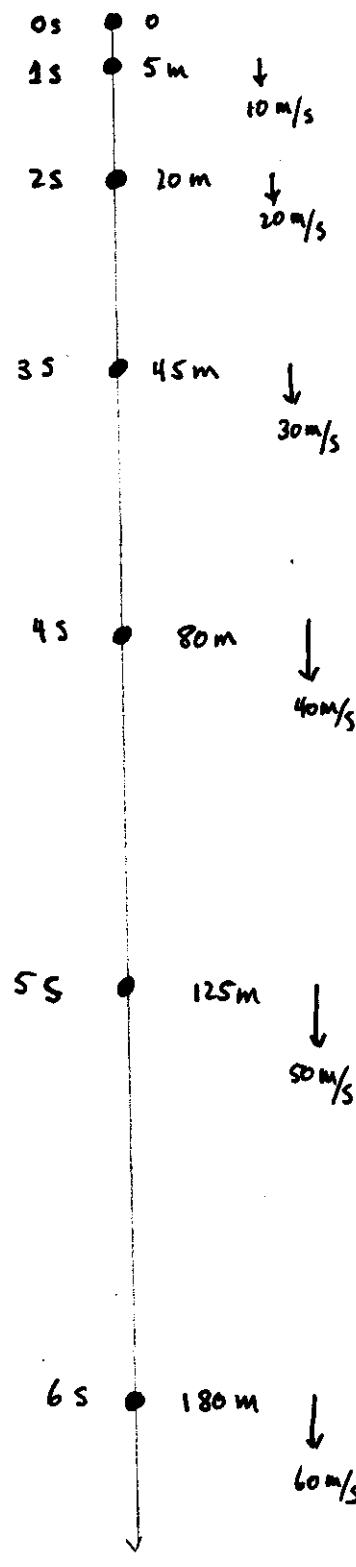
$$d = \frac{1}{2}gt^2 \quad \text{and} \quad v = gt.$$

These results are summarized in the following table, where we follow the motion of the object for 7 seconds.

time ( $t$ )	acceleration ( $a$ )	velocity ( $= at$ )	distance ( $= \frac{1}{2}at^2$ )
0 s	-10 m/s <sup>2</sup>	0	0
1 s	-10 m/s <sup>2</sup>	-10 m/s	-5 m
2 s	-10 m/s <sup>2</sup>	-20 m/s	-20 m
3 s	-10 m/s <sup>2</sup>	-30 m/s	-45 m
4 s	-10 m/s <sup>2</sup>	-40 m/s	-80 m
5 s	-10 m/s <sup>2</sup>	-50 m/s	-125 m
6 s	-10 m/s <sup>2</sup>	-60 m/s	-180 m
7 s	-10 m/s <sup>2</sup>	-70 m/s	-245 m

If we start out by throwing the ball upward with an initial upward velocity  $v_0 = 30\text{m/s}$ , the acceleration of gravity first slows down by  $10\text{m/s}$  every second until its upward velocity is zero, and then it accelerates it downward. This is illustrated in the figure at the left. Numerically we can reproduce what happens by realizing that the ball falls below the path it would take if there were no acceleration due to gravity by the same amounts that the ball dropped from rest falls. Without acceleration, the ball would continue upward at a rate of  $v_0t$ .

time ( $t$ )	acceleration ( $a$ )	velocity ( $= v_0 + at$ )	distance ( $= v_0t + \frac{1}{2}at^2$ )
0 s	-10 m/s <sup>2</sup>	30 m/s	0
1 s	-10 m/s <sup>2</sup>	20 m/s	25 m
2 s	-10 m/s <sup>2</sup>	10 m/s	40 m
3 s	-10 m/s <sup>2</sup>	0 m/s	45 m
4 s	-10 m/s <sup>2</sup>	-10 m/s	40 m
5 s	-10 m/s <sup>2</sup>	-20 m/s	25 m
6 s	-10 m/s <sup>2</sup>	-30 m/s	0 m
7 s	-10 m/s <sup>2</sup>	-40 m/s	-35 m



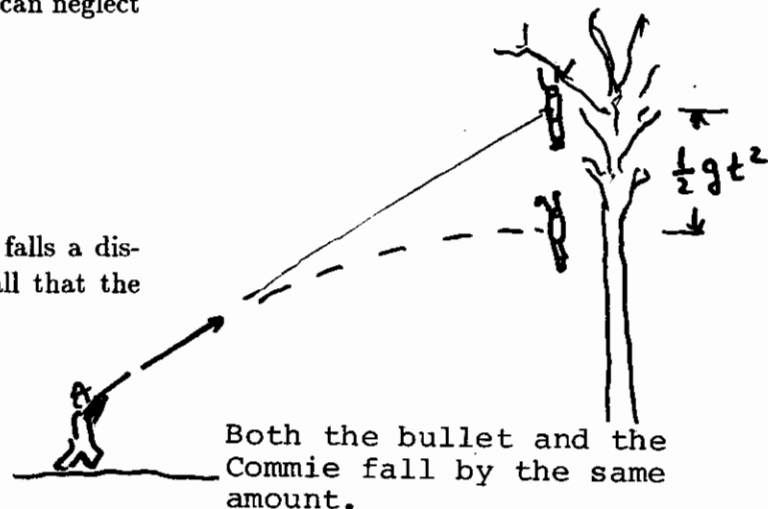
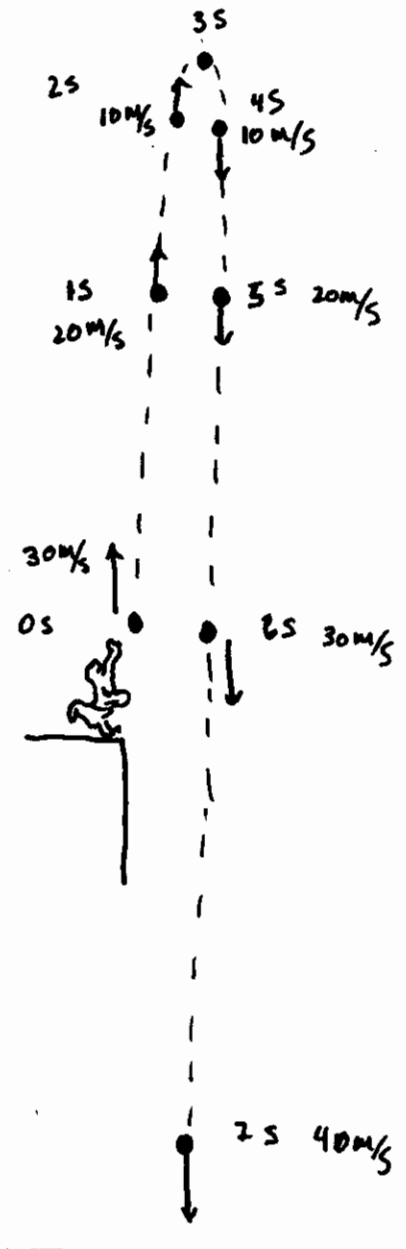
## 2.2.2 Projectile Motion

Remember, acceleration, like velocity, is a vector. Since the acceleration of gravity is straight downward, it only effects the vertical part of the velocity—the horizontal part of the motion remains unchanged. Suppose you throw the object sideways with an initial horizontal velocity what happens? The horizontal motion continues with the same speed, gravity only changes the vertical part of the motion. After the ball travels for 1 second, the acceleration due to gravity causes the it to fall below the horizontal by 5 meters, the same distance it would fall if released from rest. After 2 seconds it falls 20 meters, etc. Each second the object will travel the same horizontal distance but a different vertical distance. Similarly, if it is thrown upward at some angle, it will fall vertically below the straight-line path it would take if there were no acceleration by the same 5 m in the first second, 20 m after 2 seconds, etc.

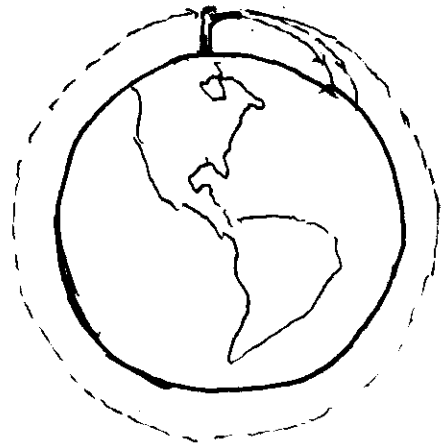
To illustrate this, let us consider the case where a Freedom Fighter comes across a Commie who is hiding in a tree. Immediately, the Freedom Fighter points his gun at the Commie and fires. As soon as he sees the flash from the rifle barrel, the Commie, in a desperate attempt to avoid the bullet, lets go and falls out of the tree. What happens? In fact, if the Commie lets go at exactly the same time that the gun is shot, will get hit. Why? Because not only does the Commie fall down, the bullet falls down also—both by the same distance. In the absence of gravity, the bullet would go in a straight-line path directly at the target. However, gravity effects the bullet just as it effects the Commie; the bullet doesn't go in a straight line but falls below a straight line due to the influence of gravity. In any time interval, the distance that the bullet falls below the straight-line trajectory is exactly equal to the distance that the Commie falls, so he buys the farm—as Galileo noted, “*all bodies fall with the same constant acceleration*” (at least to the extent that we can neglect air resistance).

## 2.2.3 Orbital Motion

We saw that if we throw an object sideways, it falls a distance of 5 m during the first second. Now recall that the



Earth is round. In fact, the Earth's curvature is such that it falls off 5 m every 8000 m of horizontal distance (on average). Thus, if we throw a ball sideways with a horizontal velocity of 8000 m/s, after 1 second, although the ball falls 5 m, it remains the same distance above the Earth's surface. (This assumes that the ball is thrown at a place where the Earth is smooth—no mountains or tall buildings—and we neglect air resistance.) In fact, if we could throw a ball sideways this fast (and there were no tall buildings or air in the way), the ball would continue to travel all around the Earth without ever hitting the ground. It would eventually come around the Earth and hit you in the back of the head! This is the principle behind orbital motion. In low Earth orbit, there is no atmosphere nor buildings or mountains in the way. Rocket engines are powerful enough to provide the necessary 8000 m/s ( $\sim 18,000\text{mph}$ ) sideways speed, and satellites continue free-falling all the way around the Earth.



A projectile with sufficient horizontal velocity will fall all the way around the Earth.

The time for a single orbit can be determined by solving the velocity-distance-time formula for the time:

$$\text{speed} = \frac{\text{distance}}{\text{time}} \Rightarrow \text{time} = \frac{\text{distance}}{\text{speed}},$$

inserting the circumference of the Earth ( $40 \times 10^6\text{m}$ ) for the distance, and 8000m/s for the velocity, giving

$$\text{time} = \frac{40 \times 10^6\text{m}}{8 \times 10^3\text{m/s}} = 5000\text{s} \simeq 83\text{minutes}.$$

You may recall that this is about the time it takes astronauts to make one full orbit.

### 2.3 Circular Motion

Accelerations that are perpendicular to the motion of an object produce curved trajectories—all motions that have curved trajectories are accelerated motions, even if the speed doesn't change. Suppose I have a ball on the end of a string and I swirl it around in a circle with a constant speed. The ball is not moving in a straight line, therefore there must be acceleration. In this case, i.e., if the speed stays constant, the acceleration points toward the center of the circle. Such an acceleration is called a *centripetal acceleration*.

### 2.3.1 Centripetal Acceleration

When an object moves in a circle of radius  $r$  at constant speed  $v$ , it is accelerating with a *centripetal* acceleration whose magnitude is given by

$$a_c = \frac{v^2}{r},$$

and whose direction is pointed toward the center of the circle. I added the subscript  $c$  on its symbol  $a_c$  to indicate that it is centripetal. Since the centripetal acceleration is proportional to  $v^2$ , if we make the speed bigger, the force becomes bigger. If we double the speed, we need to increase the accelerating force by four times; if we triple the speed the required force goes up by a factor of nine. The centripetal acceleration is *inversely* proportional to  $r$ . Therefore increasing  $r$  results in a lessening of the required force.

Suppose an automobile traveling with a speed  $v = 28\text{m/s}$  (this is  $\approx 100\text{km/hr}$  or  $\approx 60\text{mph}$ ) goes around a curve with radius  $r = 100\text{m}$ , the centripetal acceleration is

$$a_c = \frac{v^2}{r} = \frac{(28\text{m})^2}{100\text{m}} = 7.8\text{m/s}^2,$$

which is 80 percent of the acceleration of gravity. In this case we sometimes say that the acceleration is "0.8g's."

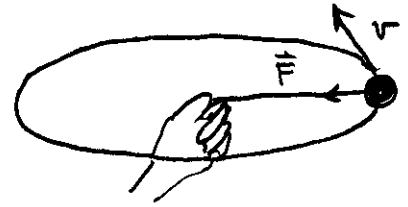
We saw that an astronaut in a circular orbit near the Earth's surface ( $r = 6.4 \times 10^6\text{m}$ ) has a speed of  $8000\text{m/s}$ . Thus, she has a centripetal acceleration of

$$a_c = \frac{v^2}{r} = \frac{(8000\text{m})^2}{6.4 \times 10^6\text{m}} = 10\text{m/s}^2 = g.$$

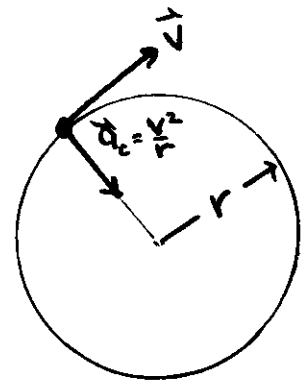
The acceleration of the Astronaut is exactly  $1g$ , which should be expected since we argued above that something in orbit is actually falling around the Earth under the influence of the Earth's gravity.

### 2.4 Inertial Reference Frames

During the first lecture, we noted the law of inertia, which, in somewhat more modern language than that used by Newton, can be stated as



A force is required to keep something moving in a circle. If the force is removed, the object moves in a straight line.



In circular motion with constant speed, the acceleration is directed toward the center of the circle and has magnitude  $a_c = v^2/r$ .



*A body at rest tends to stay at rest, a body in motion tends to keep moving along at a constant speed and in a straight-line path unless interfered with by some external force.*

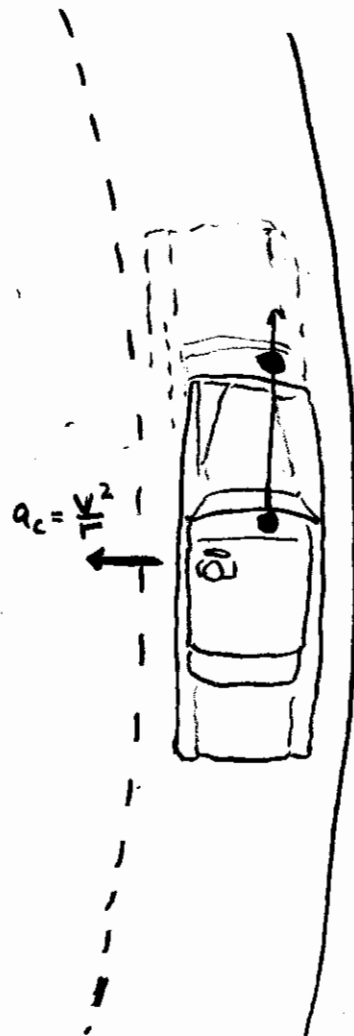
Let us imagine some simple system and how it appears to different observers. Consider a cart sitting at rest on an airtrack. (An airtrack is a device that is designed so that specially shaped things can move along it without much friction.) If the track is perfectly level and the cart on it is at rest, the cart doesn't move and you and I, using coordinate systems at rest in the class room, both see "*a body at rest tend[ing] to stay at rest*" and, thus, we agree that the law of inertia holds. Next, suppose we keep the cart at rest on the track, but I view it while standing on a skate board rolling by it with some constant speed, and measure its motion using a coordinate system that is moving along with me. In my coordinate system, I see the cart (and you, and the rest of the room) move relative to me with the opposite speed, but this motion of the cart relative to me is "*at a constant speed and in a straight-line path,*" so, even though I and my coordinate system are moving, I agree with you, who are not moving, that the law of inertia holds for the cart and it is not "*interfered with by some external force.*" Although we disagree on the value of the cart's velocity, both you and I agree that the law of inertia holds.

Next, let us imagine that I observe the cart when I am moving along on the skate board, but now imagine that I am accelerating and measure things using a coordinate system that accelerates along with me. In this case, I see the cart start moving and see its velocity continue to increase. Although initially at rest, it did not "*tend to stay at rest.*" Therefore, I infer that for the law of inertia to hold the cart must be "*interfered with by some external force.*" However, you, sitting still in the class room still see the cart stay at rest and infer from the law of inertia that the cart is not "*interfered with by some external force.*" Now either there is an external force or there isn't; the presence or absence should not depend upon who is looking at the system.

Well, in fact, you can say that it is me that is accelerating, and not the cart, and it is incorrect for me to expect that the law of inertia should hold for me. The law of inertia

holds for both you and me when I am moving at a constant velocity relative to you, but it doesn't hold for me when I am accelerating relative to you. The law of inertia only works for some observers, or only in some coordinate systems. All those systems in which the law of inertia works are called "inertial coordinate systems" or "inertial reference frames." Note that if you establish that you are in an Inertial Reference Frame, all other inertial reference frames *in the Universe* have a constant velocity relative to you—none of them can be accelerating relative to you. The acid test for an inertial frame is "does the law of inertia hold in this frame?"

Suppose that you are on an airplane cruising along at 1000 km/hr (~630 mi/hr). Does the law of inertia hold? The answer is yes it does (at least to the extent that the airplane is really travelling at a constant speed and not bouncing up and down or the like). If you place a glass of water or a ball on the tray table in front of you, it just sits there and doesn't all of a sudden start to move. However, when the plane is taking off, it has to accelerate and it is no longer an inertial system. If you have a glass of water on your tray table during this time, it's apt to end up in your lap—a body at rest is *not* tending to stay at rest. When you drive along a straight section of the H1 freeway at a constant speed, you are in a (more-or-less) inertial reference frame. You can leave a coke on the dashboard where it will obey the law of inertia and stay still. If, however, you leave your coke there while you exit on an exit loop, your velocity vector changes and, thus, you are accelerating. In that case the car is no longer an inertial reference frame and you had better grab on to the coke before you have a mess. Note that in these cases, the coke and the glass of water try to obey the law of inertia, it is the surroundings that change. Initially, you, the plane, and the glass of water are at rest. When the plane and you start to move, the glass of water "*tends to stay at rest,*" thereby ending up in your lap, which has accelerated forward. Likewise, when the car turns around the clover-leaf, the coke on the dashboard "*tends to keep moving at a constant speed in a straight-line path,*" which causes it to move relative to the rest of the car.



When a car negotiates a turn, an object free to move on the dashboard appears to be pulled outward. The turning car is not an inertial reference frame.