QCD corrections to tri-boson production
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NLO QCD corrections to the production of $t$ anti-$t$ $Z$ in gluon fusion
why NLO?

- new LHC physics need precise SM
- tree level $\rightarrow$ scale dependence
- theoretical uncertainty dominates
- NLO necessary
- wishlists become urgent
why NLO : example

discovery channel ?

No discovery channel ?

Is the shape of the background predicted correctly ??
NLO wishes

An experimenter's wishlist

- Hadron collider cross-sections one would like to know at NLO

<table>
<thead>
<tr>
<th>Process</th>
<th># groups working on</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $pp \rightarrow VV\ jet$</td>
<td>$\sim 4$</td>
</tr>
<tr>
<td>2. $pp \rightarrow t\bar{t}b\bar{b}$</td>
<td>$\sim 1$</td>
</tr>
<tr>
<td>3. $pp \rightarrow t\bar{t} + 2\ jets$</td>
<td></td>
</tr>
<tr>
<td>4. $pp \rightarrow WWW$</td>
<td>$\sim 2$</td>
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<tr>
<td>5. $pp \rightarrow VVb\bar{b}$</td>
<td></td>
</tr>
<tr>
<td>6. $pp \rightarrow VV + 2\ jets$</td>
<td>$\sim 2$</td>
</tr>
<tr>
<td>7. $pp \rightarrow V + 3\ jets$</td>
<td>$\sim 1$ (theor. interest)</td>
</tr>
<tr>
<td>8. $pp \rightarrow b\bar{b}b\bar{b}$</td>
<td>$\sim 1$</td>
</tr>
<tr>
<td>9. $pp \rightarrow 4\ jets$</td>
<td></td>
</tr>
<tr>
<td>10. $gg \rightarrow W^<em>W^</em>$ (NLO, 2 loops)</td>
<td>$\sim 1$</td>
</tr>
<tr>
<td>11. NNLO for $t\bar{t}$</td>
<td>$\sim 1$</td>
</tr>
<tr>
<td>12. NNLO for $Z/\gamma + jet$</td>
<td>(gluon pdfs)</td>
</tr>
</tbody>
</table>

why are they so hard to get ???

almost 6 years to the day and yet not a single calculation finished! Shame
NLO situation

- Real emission corrections relatively well understood
- Virtual problems
  - Too many diagrams
  - Analytical approach = reduction
  - Numerical approach = IR + threshold singularities
The method

- Compute amplitude squared, summed and averaged
- Cast integral in Feynman parameters form
- Perform the loop momentum integration analytically
- UV divergences factorize
- IR divergences treatment: Sector Decomposition
- Threshold singularities: Contour Deformation
- Extract singularities
- Integrate numerically Feynman parameters
The method

Sector decomposition

IR complicated and extractable

Contour Deformation

Feynman variable $x_k$

the position of the threshold depends on the PSP and the sector

This has to happen automagically

IR simple and extractable
ZZZ: first application

<table>
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<th>process $(V \in {Z, W, \gamma})$</th>
<th>background to LES HOUCHES WISHLIST 2005</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $pp \rightarrow V\ V\ jet$</td>
<td>$t\bar{t}H$, new physics</td>
</tr>
<tr>
<td>2. $pp \rightarrow H + 2\ jets$</td>
<td>$H$ production by VBF</td>
</tr>
<tr>
<td>3. $pp \rightarrow t\bar{t}b\bar{b}$</td>
<td>$t\bar{t}H$</td>
</tr>
<tr>
<td>4. $pp \rightarrow t\bar{t} + 2\ jets$</td>
<td>$t\bar{t}H$</td>
</tr>
<tr>
<td>5. $pp \rightarrow V\ V\ bb$</td>
<td>VBF$\rightarrow H \rightarrow VV$, $t\bar{t}H$, new physics</td>
</tr>
<tr>
<td>6. $pp \rightarrow V\ V\ + 2\ jets$</td>
<td>VBF$\rightarrow H \rightarrow VV$</td>
</tr>
<tr>
<td>7. $pp \rightarrow V\ + 3\ jets$</td>
<td>various new physics signatures</td>
</tr>
<tr>
<td>8. $pp \rightarrow V\ V\ V$</td>
<td>SUSY trilepton</td>
</tr>
</tbody>
</table>

Not very urgent physically, but a wise choice as a testbed for our method.
ZZZ production

- 6x8 virtual 'diagrams' – 1x6 pentagons
- ~20' per P.S.P. = a couple of days at the farm (University of Wisconsin)
- mild numerical instabilities easily treated
- real emission diagrams with two-cutoff phase-space slicing method
ZZZ production: results

\[ \sigma_{\text{LO}} = 11.4 \text{fb} \quad \rightarrow \quad \sigma_{\text{NLO}} = 15.2 \text{fb} \]
ZZZ production: results

- enhanced virtual corrections
- K factor reweighing accurate to the percent level
- no significant dependence on kinematics
**tt\bar{Z} production**

- Tevatron: a top pioneer
- But top couplings not accessible directly (ttZ, ttγ, Yukawa)
- New physics at ttZ? Tree level mixing with Z', little Higgs models, ...
- The ttZ coupling needs to be measured
tt$\bar{\ell}$Z production

- LHC will see ttZ, ttH, tt$\gamma$
- Need for accurate SM prediction
- tree-level ttZ is @ order $a_s^2$
- NLO computation needed to reduce uncertainty
t\bar{t}Z production

[Baur et al.]
t\bar{t}Z production

- BG under control thanks to cuts and extrapolations
- Signal theoretical uncertainty dominates
gg2t\bar{t}Z

- 60% of tree level pp2t\bar{t}Z
- NLO corrections expected larger
- technically more involved
gg2t\bar{t}Z

• virtual
  – 162x8 'diagrams' – 12x8 pentagons
  – 45' per P.S.P. = a week at the farm
  – stronger numerical instabilities, treated with standard techniques

• real
  – 50x50=2500 'diagrams'
  – two cutoff slicing method
gg2t\bar{t}Z: results

- NLO corrections effect to $K=1.4$ @\(\mu_0\)
- @\(\mu=m_t\) NLO corrections \(~30\%\)
- if the $q\bar{q}\rightarrow t\bar{t}Z$ contributes less, overall $pp\rightarrow t\bar{t}Z$ corrections \(~20\%)\, so less luminosity needed for the coupling measurement (than 300fb\(^{-1}\)).
gg2t\bar{t}Z: results

Reduction of scale dependence from LO to NLO

residual uncertainty \sim 5\% (\mu_0/4 \text{ to } \mu_0)
The NLO corrections don't change the shape of the distribution.
A K-factor of 1.4 describes well the NLO effect.
Conclusions

- New method for numerical evaluation of loop diagrams
- High degree of automation
- Numerical stability in the cases studied
- Applied in real processes of physical interest
- NLO corrections to $\bar{t}tZ$ from gluon fusion completed
- $q\bar{q}$ channel to follow
- More applications of great interest ahead!
extra: method checks

- three different codes in FORTRAN, C
- $1/\varepsilon$, $1/\varepsilon^2$ poles cancel exactly at the differential level
- contour deformation parameter independence
Extra: Primary Sector Decomposition

- The integration volume is divided into $N$ sectors and each sector is mapped back to the unit hypercube.
- The $\delta$-function is eliminated.

$$\sum \prod_{i \neq j} \theta(x_i > x_j) \quad \text{Sector of } x_2$$

$$\theta(x_2 > x_1)\theta(x_2 > x_3)\theta(x_2 > x_4) \ldots \theta(x_2 > x_N)$$
Extra: Secondary Sector Decomposition

\[ I(N) \sum_{a,j,m,n} \int [dy] \frac{y_m^{R-1}}{y_m^{\lambda(N-a-2+\epsilon)}} \frac{y_n^{R'-1}}{y_n^{\lambda'(N-a-2+\epsilon)}} \frac{\tilde{N}_{a,j,mn}(y, \epsilon)}{\Delta_{jmn}(y)^{N-a-2+\epsilon}} \]

\[ \Delta_{jmn}(y) = y_i y_k y_m y_n S_{ik} + \ldots + C \]

quartic term comes from cubic terms without the particular \( t_m \) in the case that \( \lambda = 1 \)

There can be at most two iterations before a finite \( \Delta \) emerges in every sector
extra: A general integral

\[
\int [dy] J(y) \frac{1}{(y_m V_m)^{a_m+b_m \epsilon}} \frac{1}{(y_n V_n)^{a_n+b_n \epsilon}} \frac{\tilde{N}_{a,j,mn}(y V(y), \epsilon)}{\Delta_{jmn}(y)^{N-a-2+\epsilon}}
\]

\[V_m(y) = 1 - \lambda y_m (1 - y_m) w_m\]

\[
\int [dy] \frac{1}{(y_m)^{a_m+b_m \epsilon}} \frac{1}{(y_n)^{a_n+b_n \epsilon}} F(y, \epsilon)
\]
extra: Pole extraction and $\varepsilon$-expansion

$$\int [dy] \frac{1}{(y_m)^{a_m+b_m\varepsilon}} \frac{1}{(y_n)^{a_n+b_n\varepsilon}} F(y, \varepsilon)$$

In case any of the exponents is $1+b\varepsilon$, the integral over the corresponding $y$-variable gives a $1/\varepsilon$ pole that is extracted at this point with the help of

$$\int dx \frac{f(x)}{x^{1+b\varepsilon}} = -\frac{1}{b\varepsilon} f(0) + \int dx \frac{f(x) - f(0)}{x} - b\varepsilon \int dx \frac{(f(x) - f(0)) \log(x)}{x} + O(\varepsilon^2)$$

The function $F(y, \varepsilon)$ can then be safely expanded over $\varepsilon$. 