

Take-Home Midterm Exam #3, Part A

NO exam time limit. Calculator required. All books and notes are allowed, and you may obtain help from others. Complete all of Part A **AND** Part B.

For multiple-choice questions, circle the letter of the one best answer (unless more than one answer is asked for). *For fill-in-the-blank and multiple-choice questions, you do NOT need to show your work.*

Show your work on all free-response questions. Be sure to use **proper units** and **significant figures** in your final answers.

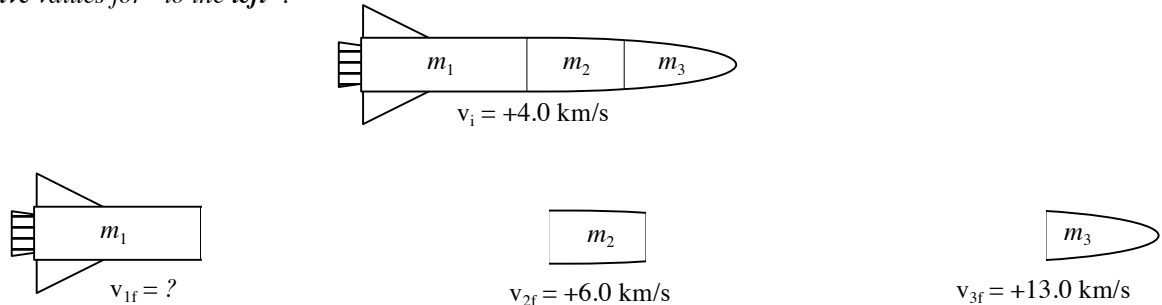
Ignore friction and air resistance in all problems, unless told otherwise.

Physical constants: It's an open-book test, so you can look them up in your textbook!

Useful conversions: It's an open-book test, so you can look them up in your textbook!

1. A three-part rocket begins intact as a single object in distant outer space, traveling to the right at 4.0 km/s. The first "stage" ($m_1 = 950,000$ kg) explodes away from the rear of the rocket, with an unknown final velocity (v_{1f}). Later, the second stage ($m_2 = 550,000$ kg) explodes away from the rear, with a final velocity to the right at 6.0 km/s. The third stage ($m_3 = 350,000$ kg) ends up with a final velocity to the right of 13.0 km/s.

(All three masses move only along the x -axis. Ignore gravity throughout this problem. Assume that all parts of the rocket have constant masses.) For ALL answers to this problem, use **positive** values for "to the **right**," and **negative** values for "to the **left**":



a. (3 pts.) What is the **final velocity** v_{1f} of the first stage? _____

b. (2 pts.) Find the **impulse** received by the first rocket stage (in its explosion): _____

c. (2 pts.) Find the **total impulse** received by the second rocket stage (after both explosions): _____

d. (2 pts.) Find the **total impulse** received by the third rocket stage (after both explosions): _____

e. (1 pt.) Find the **sum** of these three impulses: _____

Hint: According to conservation of momentum, what should the sum of all impulses be equal to?

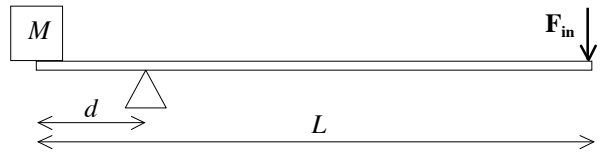
f. (1 pt.) During the second explosion, the **force** acting on m_2 was _____ the **force** acting on m_3 .
A. less than B. the same as C. greater than

g. (1 pt.) After both explosions, the **total kinetic energy** of the system...
A. increased B. decreased C. remained unchanged

h. (1 pt.) After both explosions, what is the **final velocity of the center-of-mass** of the three parts? _____

2. A simple lever gives the user a mechanical advantage: pushing down with a small force F_{in} on the long end of the lever creates a large upward force F_{out} on the short end.

Suppose that you want to support mass M at rest by using a lever of total length L whose fulcrum is located at a distance d from the short end of the lever. (Assume that the mass of the lever itself is negligibly small.)



a. (2 pts.) What is the **mechanical advantage** ($F_{out} \div F_{in}$) of this lever? Express your answer **ONLY** in terms of L , d , and mathematical constants:

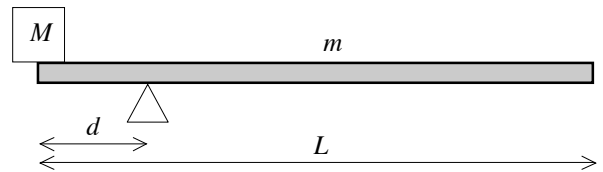
b. (1 pt.) At what **distance** d should the fulcrum be positioned so that the lever has a mechanical advantage equal to

exactly 1? Express your answer **ONLY** in terms of L and mathematical constants: _____

c. (1 pt.) As $d \rightarrow 0$, what **value** does the **mechanical advantage** approach?

- A. 0 B. 1 C. ∞

We can eliminate the need for a human F_{in} by instead using a thick, massive lever with mass m , so that the weight of the lever itself balances M . (Assume that the lever has uniform thickness and density.)



d. (2 pts.) At what **distance** d should the fulcrum be positioned so that the lever's mass m exactly balances M ? Express your answer **ONLY** in terms of L , m , M , and mathematical constants:

e. (2 pts.; -1 for each error) Which of the following statements is/are **TRUE** for a balanced system? **Circle ALL that apply:**

- A. If $M \gg m$, $d \rightarrow 0$.
 B. If $M = m$, $d = L/2$.
 C. If $M \ll m$, $d \rightarrow L$.

f. (3 pts.) Suppose that d is positioned *incorrectly*, so that the lever system is *NOT* balanced. Let $d = 25$ cm, $L = 100$ cm, $M = 7.0$ kg, and $m = 5.0$ kg. Immediately after the system is released from horizontal rest

(as shown in the diagram above), what is the **net torque** about the fulcrum? _____

g. (3 pts.) In part (f), what is the system's **angular acceleration** immediately after it is released? (Assume that the lever is a thin rod with $I = \frac{7}{48} mL^2$, and that M is a "point mass" located at the very end of the lever.)

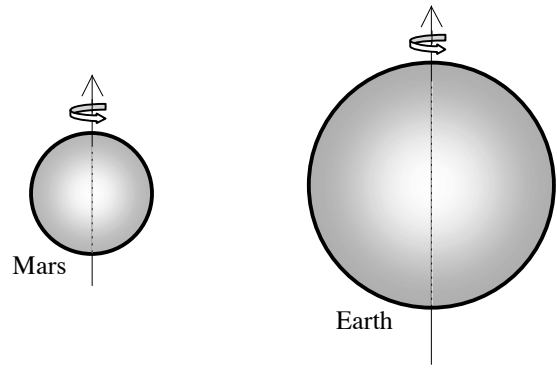
h. (1 pt.) The angular acceleration in part (g) is...

- A. **clockwise** B. **counter-clockwise**

3. Earth (“E”) and Mars (“M”) have rotation periods that are surprisingly similar, even though Mars has a radius that is only about half as large as Earth’s.

For parts (a)–(e) of this problem, assume that both planets are solid, uniform-density spheres with the following properties (*NOT* actually true, but use *these* values):

rotation periods: $T_E = T_M$
planetary radii: $R_E = 2R_M$
planetary masses: $M_E = 10M_M$



You may give the next five answers as pure integers, as simplified pure rational numbers, or as 3-sig.-fig. decimals:

- a. (1 pt.) The **angular speed** of Earth = _____ times the **angular speed** of Mars.
- b. (1 pt.) The **moment of inertia** of Earth = _____ times the **moment of inertia** of Mars.
- c. (1 pt.) The **angular momentum** of Earth = _____ times the **angular momentum** of Mars.
- d. (1 pt.) The **rotational kinetic energy** of Earth = _____ times the **rotational kinetic energy** of Mars.
- e. (1 pt.) The **density** of Earth = _____ times the **density** of Mars.

For the remaining parts of this question, use the *actual* values for Mars’s physical properties (again, assume Mars is a solid sphere of uniform density):

$T_M = 24.6 \text{ h}$
 $R_M = 3.40 \times 10^3 \text{ km}$
 $M_M = 6.42 \times 10^{23} \text{ kg}$

BONUS (+2 pts.) Calculate Mars’s **angular momentum**, including MKS units: _____

BONUS (+2 pts.) Calculate Mars’s **rotational kinetic energy**, including MKS units: _____

f. (3 pts.) Calculate Mars’s **density**, including MKS units: _____

g. (1 pt.) A typical density for rock is $\sim 2500 \text{ kg/m}^3$, while iron (the most common metal in the solar system) has a density of $\sim 9000 \text{ kg/m}^3$. We can conclude that Mars’s interior mass consists, very roughly, of a mixture of...

- A. 99% rock and 1% metal
- B. 80% rock and 20% metal
- C. 50% rock and 50% metal
- D. 20% rock and 80% metal
- E. 1% rock and 99% metal

4. A water polo ball (which looks like a waterproof yellow volleyball) is a hollow sphere of radius 11.1 cm and mass 450. grams. Assume that the density of water is 1.000 g/cm^3 .



a. (2 pts.) Find the **volume** of the ball: _____

b. (1 pt.) While floating, what **mass** of water does the ball displace? _____

c. (2 pts.) While floating, what **volume** of water does the ball displace? _____

d. (3 pts.) Suppose that a polo player starts pushing downward on the floating ball, gradually increasing his force. What is the magnitude of the **athlete's downward force** when the ball has exactly *half* of its volume

submerged below the surface? _____

e. (3 pts.) Later, the polo player *completely* submerges the ball, pushing it a short depth below the surface of the water. Then he releases the ball from rest, underwater. *Immediately* afterward, what is the ball's upward

acceleration? (Neglect any drag force.) _____

Suppose the water polo ball has an internal pressure (absolute pressure, *not* "gauge pressure") of 185 kPa.

f. (2 pts.) Find the **surface area** of the ball: _____

g. (2 pts.) While the ball sits at rest in 1.00-atm air, what is the **net outward force** acting on the inside

of the ball? _____

h. (3 pts.) At what **depth** in a swimming pool would the absolute pressure on the outside of the ball exceed

185 kPa, causing the ball to start to collapse? _____

(Assume that the pool is located near sea level, so that there is 1.00 atm of *atmospheric* pressure at the water's surface. Your answer will be deeper than most swimming pools... but not by much!)

Take-Home Midterm Exam #3, Part B

1. A bowling ball (solid, uniform-density sphere of mass M and radius R) rolls *without slipping* toward a hill of maximum height H and varying slope.

a. (5 pts.) If the bowling ball starts with linear speed v_0 at the bottom of the hill, what is its **linear speed** as it rounds the crest of the hill? *Show your work completely.*

*Express your final answer algebraically **ONLY** in terms of “known” variables M , R , H , v_0 , g , and any necessary numerical constants (but **NOT** ω or any other variables!). **Simplify** your final answer as much as possible!*

b. (1 pt.) Suppose someone makes a “hollow” bowling ball whose center is mostly empty, but whose outer edge is loaded with dense metal. Overall, it has the same mass M and outer radius R as a standard bowling ball.

If you roll the *hollow* bowling ball with the *same* initial linear speed v_0 toward the same hill H , the hollow ball will reach the top of the hill with _____ **linear speed** than the standard bowling ball.

- A. **greater**
- B. **less**
- C. **the same**

2. An ice skater begins by spinning with a rotation period of 1.20 s when her arms and legs are outstretched, giving her whole body an initial moment-of-inertia of $2.56 \text{ kg}\cdot\text{m}^2$. By pulling her arms and legs in close to her spin axis, her moment of inertia decreases to $0.850 \text{ kg}\cdot\text{m}^2$. (Ignore all friction.)

a. (5 pts.) What is the skater's **final rotation period**? *Show your work completely.*

b. (4 pts.) How much total **work** did the skater's muscles perform while pulling in her arms and legs? (*Hint: What is her increase in kinetic energy?*)

c. (5 pts.) If the skater undergoes a constant angular acceleration over a span of 7.0 s, how many (fractional) **revolutions** does she execute during the acceleration? (blaarrp... how dizzying!) *Show your work completely.*