

## Take-Home Midterm Exam #3, Part A

**NO exam time limit. Calculator required. All books and notes are allowed, and you may obtain help from others.** Complete all of Part A *AND* Part B.

For multiple-choice questions, circle the letter of the one best answer (unless more than one answer is asked for). For fill-in-the-blank and multiple-choice questions, you do NOT need to show your work.

**Show your work** on all free-response questions. Be sure to use **proper units** and **significant figures** in your final answers.

Ignore friction and air resistance in all problems, unless told otherwise.

Physical constants: It's an open-book test, so you can look them up in your textbook!

Useful conversions: It's an open-book test, so you can look them up in your textbook!

1. (4 pts.) **Convert** the following quantities into the given units. Fill in the blanks. (You do NOT need to show your work.) Use *scientific notation* where appropriate (very large or very small values), and express all final values to *THREE significant figures*.

a.  $9.85 \times 10^{-7} \text{ MW} = \text{_____} \text{ mW}$

b.  $3.00 \times 10^5 \text{ } \mu\text{m/ns} = \text{_____} \text{ km/h}$

c.  $12,500 \text{ rpm} = \text{_____} \text{ kHz}$  ("rpm" = rev/min)

d.  $19,300 \text{ kg/m}^3 = \text{_____} \text{ mg/mm}^3$  (this is the density of gold)

2. A 430-g soccer ball flying directly to the left at speed  $v_b$  strikes the head of a soccer player. The entire impact lasts 15 ms. Afterward, the ball rebounds directly to the right at the same speed  $v_b$ . Express all answers in **MKS units**:

a. (2 pts.) If  $v_b = 21 \text{ m/s}$ , what is the total **impulse** that the ball receives during the impact? \_\_\_\_\_

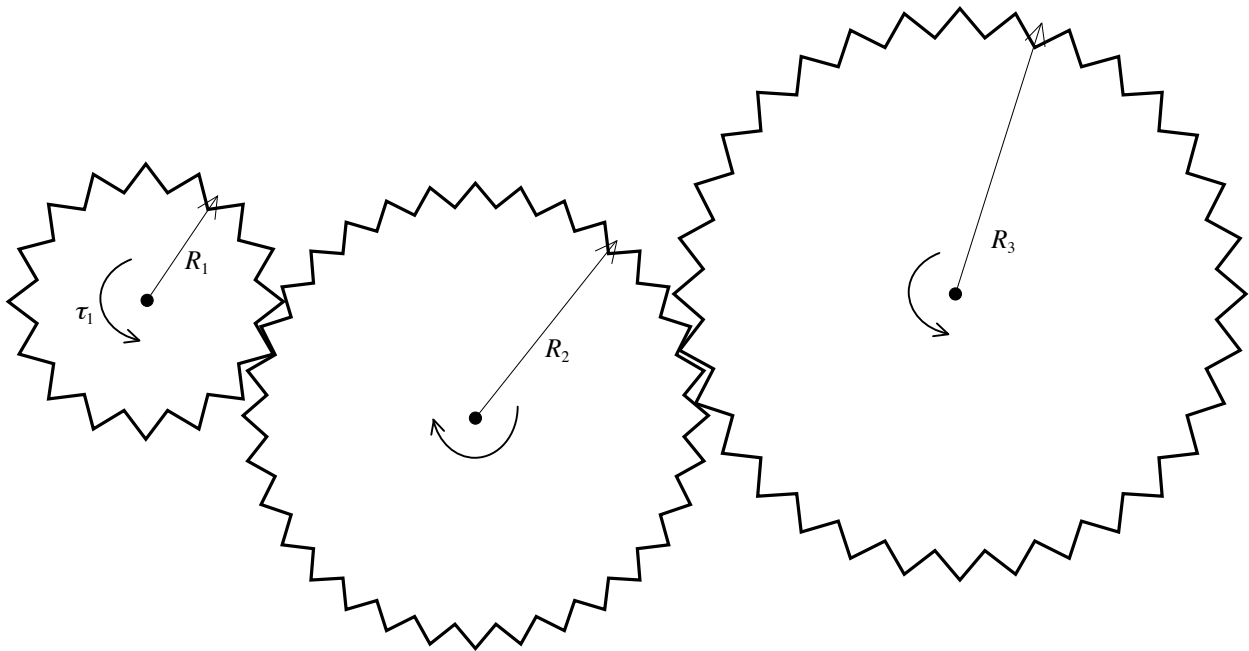
b. (2 pts.) What is the **average force** that the ball exerts on the player's head during part (a)? \_\_\_\_\_

c. (2 pts.) The player's head has a mass of 4.8 kg. The player will experience a concussion (brain injury) if his head undergoes an average acceleration greater than 75 gees. (1 gee =  $9.80 \text{ m/s}^2$ . Ignore the head's attachment to the rest of the player's body; just consider the head alone.) Assuming that an impact always lasts 15 ms

regardless of the value of  $v_b$ , what is the **minimum ball speed  $v_b$**  that will cause a concussion? \_\_\_\_\_  
(Fortunately, speeds this large are *not* normally attained by soccer balls during play.)

3. An airplane propeller with a moment of inertia (about its center) equal to  $55 \text{ kg}\cdot\text{m}^2$  starts out spinning at 330 rpm. The airplane's engine then takes 25 s to accelerate the propeller (at a constant acceleration) up to a new rate of 1600 rpm. Express all answers in **MKS units**:

- a. (2 pts.) What is the propeller's **angular acceleration** during this time? \_\_\_\_\_
- b. (2 pts.) If the propeller measures 1.5 m from center to tip, what is the **total distance traveled** by the tip of the propeller during the acceleration in part (a)? \_\_\_\_\_  
(You may express your answer in either meters or kilometers.)
- c. (2 pts.) Suppose that air resistance opposes the spinning propeller with a constant torque of  $85 \text{ N}\cdot\text{m}$ , regardless of the propeller's speed. (Not actually true! But we'll assume it for simplicity.) In that case, what is the **engine's torque** acting on the propeller during the acceleration in part (a)? \_\_\_\_\_
- d. (2 pts.) What is the total **average power output** of the engine during this acceleration? Be sure to include the energy spent overcoming air resistance. Convert your answer to units of horsepower: \_\_\_\_\_ **hp**

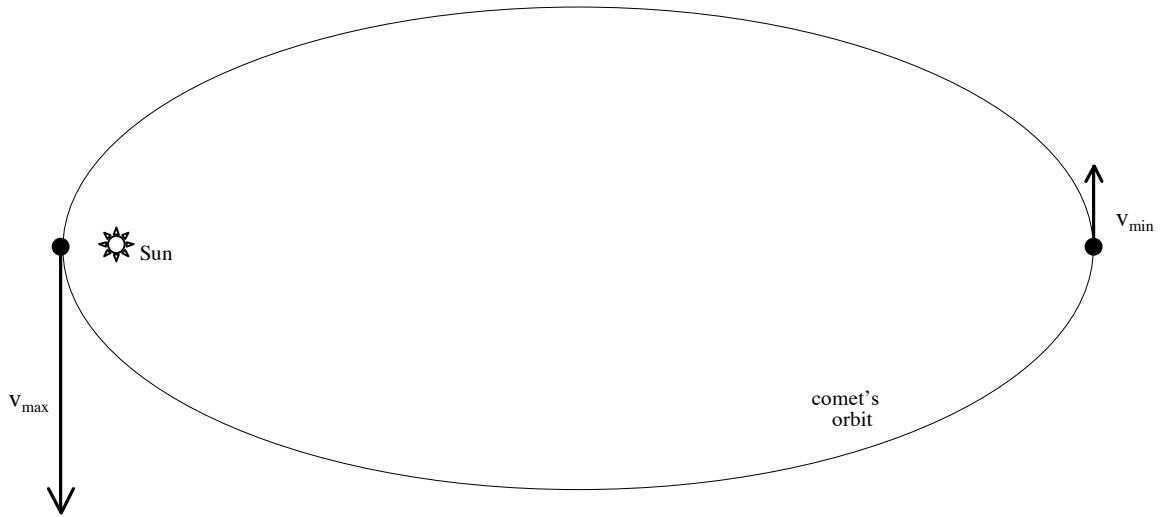


4. Three interlocking gears have average radii of  $R_1$ ,  $R_2$  and  $R_3$ , with relative sizes as shown above. (Their centers are all fixed, and there is no friction at their pivots. Assume that the size of the gears' teeth is much smaller than the gears' radii.)

- a. (2 pts.) An electric motor exerts torque  $\tau_1$  on Gear #1, ultimately resulting in torque  $\tau_3$  on Gear #3. If the system's mechanical advantage is equal to the ratio  $\tau_{\text{out}}/\tau_{\text{in}}$ , what is the **mechanical advantage** ( $\tau_3/\tau_1$ ) of this gear system?

Express your answer *ONLY* in terms of  $R_1$ ,  $R_2$ ,  $R_3$ , and numerical constants: \_\_\_\_\_

- b. (1 pt.) Suppose that the electric motor rotates Gear #1 at a constant angular speed. As you watch all three gears turn, which gear will exhibit the **greatest angular speed**?  
A. Gear #1    B. Gear #2    C. Gear #3    D. all three will have equal angular speeds
- c. (1 pt.) Again, Gear #1 is rotated at a constant angular speed. Each gear has a small dot painted near its outer edge. During a particular time interval, which gear's dot travels the **greatest distance**?  
A. Gear #1    B. Gear #2    C. Gear #3    D. all three will travel equal distances



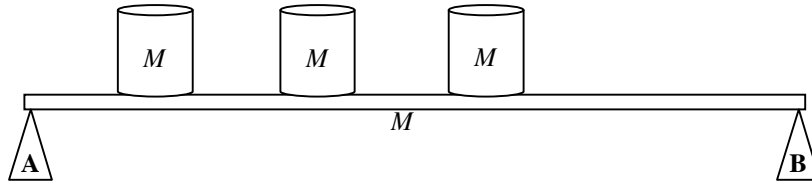
5. Halley's Comet (mass =  $1.7 \times 10^{15}$  kg) has a very elliptical orbit: it varies from a distance of 0.586 AU (at closest approach) to 35.1 AU (at greatest distance) away from the Sun. (Recall: 1 AU = average Earth-Sun distance =  $1.50 \times 10^{11}$  m.) Assume that the comet is a "point mass" in space.

Kepler's 2<sup>nd</sup> Law tells us that any body orbiting the Sun should have greatest velocity when closest to the Sun and slowest velocity when farthest. Kepler discovered this by trial-and-error, but Newton later showed that this law can be derived as the result of conservation of angular momentum:

a. (3 pts.) If the comet's speed is 54 km/s at its closest approach to the Sun, find its **angular momentum** at that point. Express your answer in **MKS units**: \_\_\_\_\_

b. (1 pt.) In which **direction** does the comet's **angular momentum vector** point?  
 A. into the page      B. out of the page      C. direction varies during orbit

c. (2 pts.) Knowing that the comet's angular momentum remains constant as it orbits the Sun, find the comet's **speed** when it reaches its **farthest point** from the Sun, in [km/s]: \_\_\_\_\_ **km/s**



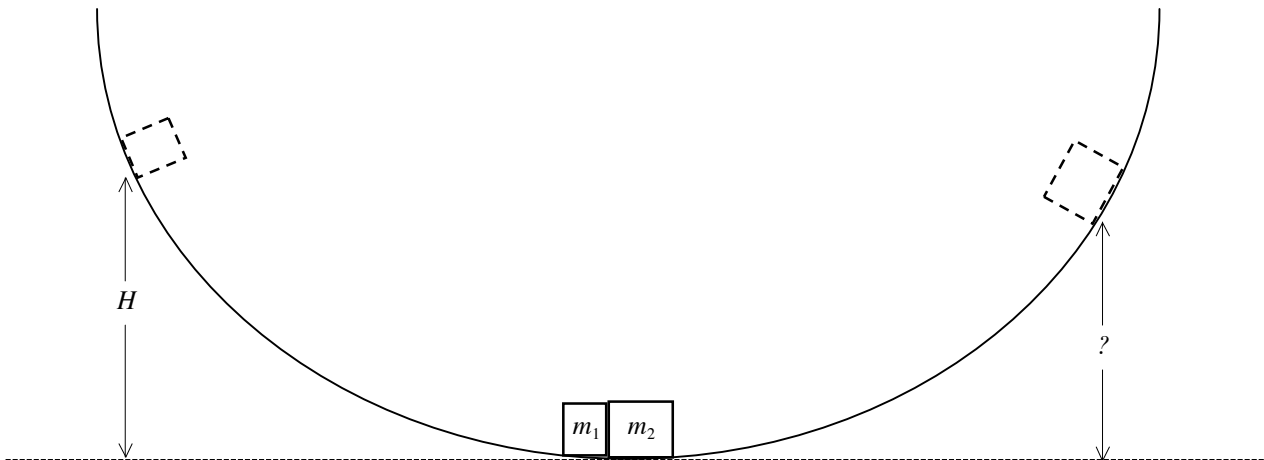
6. A shelf is supported by two posts at a distance  $L$  apart from each other. Three canisters, each of mass  $M$ , are located at distances of  $(1/5)L$ ,  $(2/5)L$ , and  $(3/5)L$  from post A. The shelf itself also has mass  $M$ . The entire configuration is at rest in equilibrium. For both parts (a) and (b):

- Express each final answer **ONLY** in terms of  $M$ ,  $L$ ,  $g$ , and any necessary mathematical constants.
- **Simplify** each final answer to its most compact algebraic form.

a. (2 pts.) Find the upward **force** exerted by **post A** on the shelf: \_\_\_\_\_

b. (2 pts.) Find the upward **force** exerted by **post B** on the shelf: \_\_\_\_\_

### Take-Home Midterm Exam #3, Part B



1. Two *unequal* masses  $m_1$  and  $m_2$  are located in a large, frictionless bowl, and both start at rest at the bottom of the bowl, touching each other. An explosion between the two masses pushes them apart horizontally in opposite directions. Mass  $m_1$  slides to a maximum height  $H$  up the left side of the bowl.

(8 pts.) For both parts of this question, your answers will be algebraic expressions. You may solve parts (a) and (b) in whichever order you wish. For both parts:

- Express each final answer **ONLY** in terms of  $m_1$ ,  $m_2$ ,  $H$ ,  $g$ , and any necessary mathematical constants.
- Show all steps of your derivation.
- **Simplify** each final answer to its most compact algebraic form.

- To what **maximum height** does **mass  $m_2$**  slide up the right side of the bowl?
- Find the **total** amount of **kinetic energy** gained by both masses in the explosion.

1. *continued:*

---

Suppose that  $m_2 > m_1$ . Use this fact along with your work above to answer the following questions:

c. (1 pt) *Immediately* after the explosion...

- A.  $m_1$  has a *faster* **speed** than  $m_2$
- B.  $m_1$  has the *same* **speed** as  $m_2$
- C.  $m_1$  has a *slower* **speed** than  $m_2$

d. (1 pt) *Immediately* after the explosion...

- A.  $m_1$  has *more* **kinetic energy** than  $m_2$
- B.  $m_1$  has the *same* **kinetic energy** as  $m_2$
- C.  $m_1$  has *less* **kinetic energy** than  $m_2$

2. You want to determine the total moment of inertia of an entire car wheel (rim, tire, and all). Although it is a complicated arrangement of many parts, it is still radially symmetric about its center. You measure the wheel's overall outer radius to be 25.0 cm, and you weigh it to find that its total mass is 23.0 kg.

You release the wheel from rest at the top of a 1.50-meter-tall ramp, and it rolls down without slipping. Averaging together several trials, you find that the wheel's final linear speed is 4.90 m/s at the bottom of the ramp.

a. (5 pts.) Find the **moment of inertia** of the car wheel. Express your final answer in MKS units. Show your work completely.

*continued on next page...*

**2. continued:**

**b.** (3 pts.) If the ramp is a  $30.0^\circ$  incline of constant slope, find the total **time of descent** for the car wheel. Show your work.