

## Take-Home Midterm Exam #2, Part A

**NO exam time limit. Calculator required. All books and notes are allowed, and you may obtain help from others.** Complete all of Part A **AND** Part B.

*You do NOT need to show your work for fill-in-the-blank or multiple-choice questions.* For multiple-choice questions, circle the letter of the one best answer (unless more than one answer is asked for). For all fill-in-the-blank answers, be sure to provide **proper units** and **significant figures**!

**Show your work** on all free-response questions. Be sure to use **proper units** and **significant figures** in your final answers.

Ignore friction and air resistance in all problems, unless told otherwise.

Physical constants: It's an open-book test, so you can look them up in your textbook!

Useful conversions: It's an open-book test, so you can look them up in your textbook!

1. (8 pts.) **Convert** the following quantities into the given units. Fill in the blanks. Use *scientific notation* where appropriate (very large or very small values). Express all final values to **THREE significant figures**.

a.  $6.50 \times 10^3 \text{ rpm} = \underline{\hspace{2cm}}$  kHz      (“rpm” = rev/min)

b.  $7.17 \times 10^{-4} \text{ rad/h} = \underline{\hspace{2cm}}$  °/d      (“h” = hour; “d” = day)

c.  $7.40 \times 10^{-5} \text{ kg}\cdot\text{m}^2 = \underline{\hspace{2cm}}$  g·cm<sup>2</sup>

d.  $5.84 \times 10^8 \text{ mi/y} = \underline{\hspace{2cm}}$  nm/μs      (“y” = year)

2. A tetherball of mass  $m$  (and negligibly small radius) hangs at the end of a tether of length  $L$  from the top of a tall narrow pole. (Assume that the tether cord is very lightweight and does not stretch.)

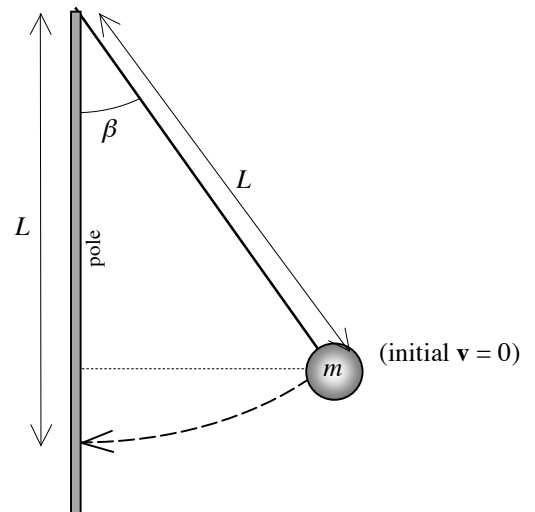
The ball and tether are pulled aside to an angle  $\beta$ , where they are released *from rest*. The ball falls back toward the pole.

**Express your answer algebraically ONLY in terms of  $m$ ,  $g$ ,  $\beta$ ,  $L$ , and any necessary numerical constants (but NOT  $v$  or any other variables!). Simplify your final answer as much as possible!**

a. (2 pts.) Using conservation of energy, find the **speed** of the ball immediately before it hits the pole:

\_\_\_\_\_

**SIDE VIEW:**

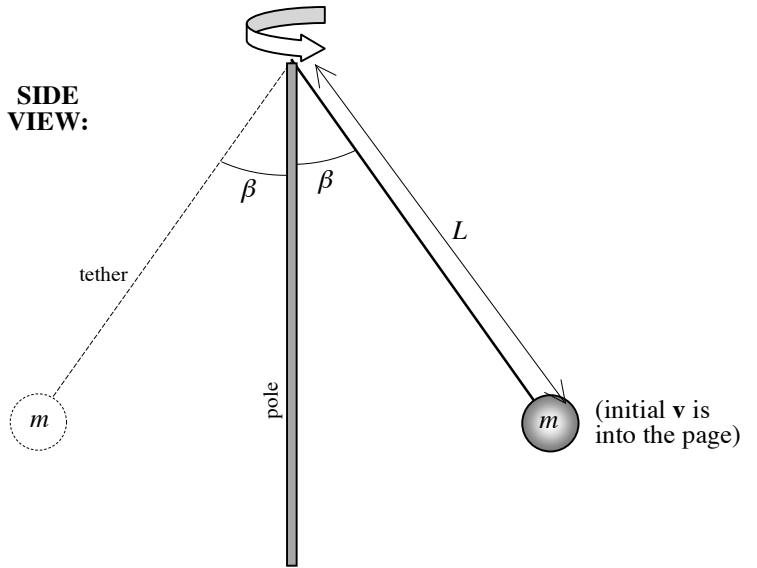


**2. continued:**

If the ball is hit tangentially with *just* the right speed  $v$ , it will circle around the pole in uniform circular motion (UCM), parallel to the ground, at a constant vertical angle  $\beta$  (as shown in the diagrams here). Assuming that  $L$  remains constant (i.e., the tether does *NOT* wrap around the pole or get any shorter), this “just right” UCM speed is:

$$v = \sqrt{gL \sin \beta \tan \beta}$$

b. (2 pts.) Draw a vector showing the **direction of the NET force** on the ball as it undergoes UCM, on **EACH** of the two diagrams shown at right (top view AND side view). If the net force is zero, write “ $\Sigma F = 0$ .”



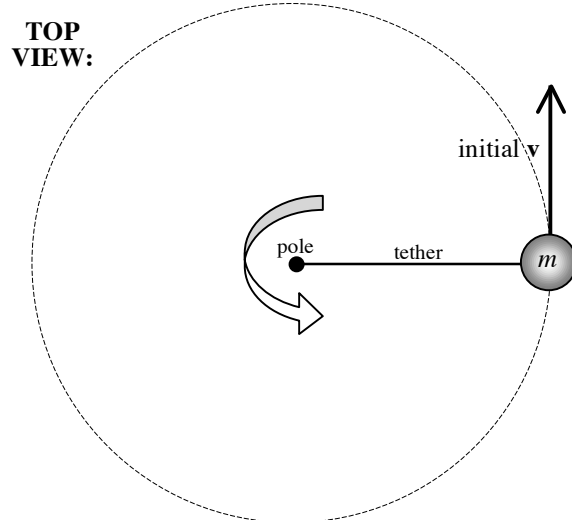
Express all answers algebraically **ONLY** in terms of  $m$ ,  $g$ ,  $\beta$ ,  $L$ , and any necessary numerical constants (but *NOT*  $v$  or any other variables!). *Simplify your final answers as much as possible!*

c. (2 pts.) Find an expression for the **period** of the ball:

\_\_\_\_\_

d. (2 pts.) Find an expression for the magnitude of the **tension** in the tether cord:

\_\_\_\_\_



e. (2 pts.) As  $\beta \rightarrow 0^\circ$  (the ball hangs almost vertically downward while circling), what values do the **period** and **tension** approach? (*Thought question:* Does this limiting expression for the period look familiar? What is the period of a “regular” pendulum?)

period  $\rightarrow$  \_\_\_\_\_

tension  $\rightarrow$  \_\_\_\_\_

f. (2 pts.) As  $\beta \rightarrow 90^\circ$  (the ball hangs almost horizontally outward while circling), what values do the **period** and **tension** approach? (*Thought question:* What is the speed  $v$  in this case? Can this actually be accomplished?)

period  $\rightarrow$  \_\_\_\_\_

tension  $\rightarrow$  \_\_\_\_\_



4. After its discovery in 2004, the asteroid Apophis was predicted to have a possibility of colliding with Earth in the year 2029, or possibly 2036, because its orbit crosses the Earth's orbit. Although more precise observations have since all but ruled out those potential impacts, Apophis still serves as a good example of the sort of object that poses a major hazard to life on Earth, and it raises the question of what we should do to prevent future impacts.

The radius of Apophis is 140 m, and its mass is  $2.1 \times 10^{10}$  kg. (You may assume that it is spherical and uniform-density.)

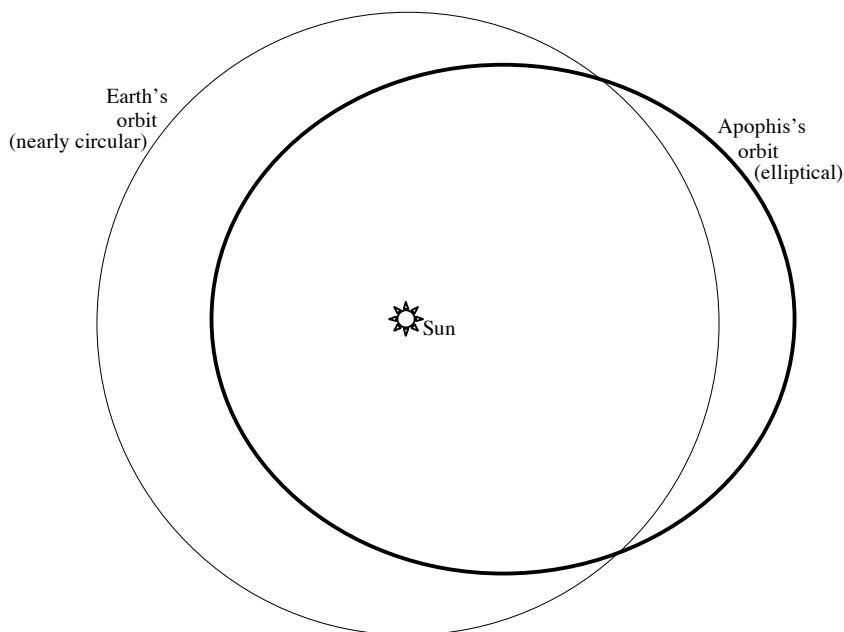
a. (2 pts.) If a squad of astronauts were sent to Apophis, perhaps to plant a bomb on the asteroid, what would be the magnitude of the **“surface gravity”** (acceleration due to gravity on the surface of the asteroid) that they would experience, **in gees**? (1 “gee” =  $9.80 \text{ m/s}^2$ ) \_\_\_\_\_ gee

b. (2 pts.) Another possibility is to use a “gravity tractor”: a little spacecraft that hovers or orbits near Apophis, very slowly tugging on Apophis gravitationally, until Apophis's orbital path is nudged away from a collision-course. If the gravity tractor has a mass of 1100 kg, and it hovers 250 m away from the center of Apophis, what is the magnitude of the **gravitational force** that it exerts on Apophis? \_\_\_\_\_

c. (2 pts.) Suppose that Apophis orbits the Sun with a period of 0.94 year (not actually true, but use that value for this question). Assuming that Apophis's orbit is *circular* (and ignoring the gravitational influence of all other planets), what is its **orbital distance** from the Sun, in kilometers? \_\_\_\_\_ km

d. (1 pt.) If both Apophis and Earth have circular orbits, which one of the following has the **faster linear speed**?  
 A. Apophis                      B. Earth

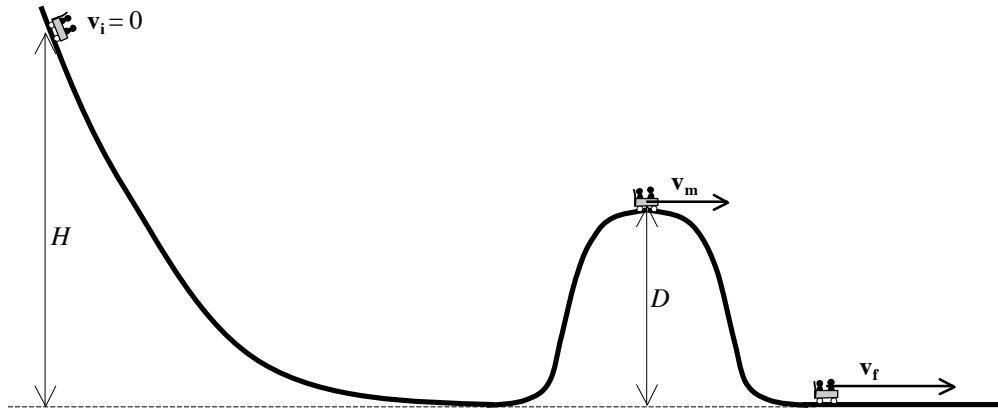
e. (2 pts.) Apophis's orbit is *actually* an ellipse, closer to the Sun on one side, and farther from the Sun (and crossing the Earth's orbit) on the other. **Label** the two points on Apophis's orbit below where it has its **slowest** and its **fastest linear speeds**:



## Take-Home Midterm Exam #2, Part B

1. Some dedicated physics students take along handheld speed sensors on their trip to a local amusement park. First, they ride a roller coaster (mass  $m$ ) with a frictionless “hill” track, as shown here:

(Note: This sketch is NOT to scale.)



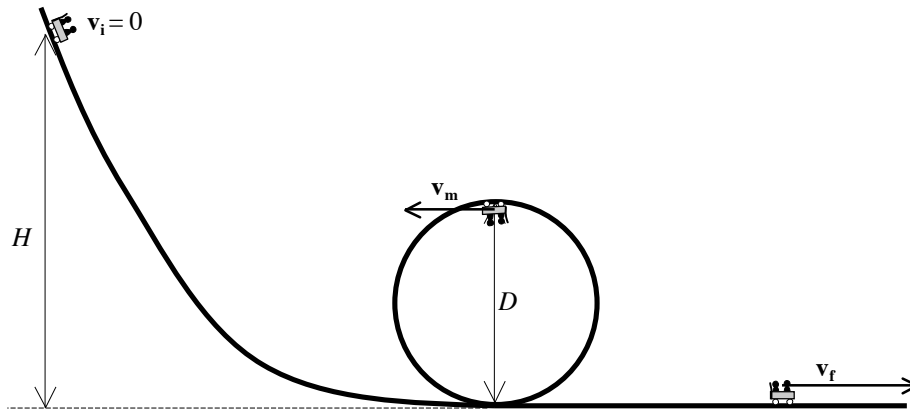
a. (6 pts.) The students find that  $v_f = 2v_m$ . Knowing this, find an expression for their **initial height  $H$** :

**Express your answer algebraically ONLY in terms of  $m$ ,  $g$ ,  $D$ ,** and any necessary numerical constants (but *NOT* any other variables!). *Simplify your final answer as much as possible! Show your work completely.*

**1. continued:**

Later, the same students ride a *different* roller coaster (same mass  $m$ ) with a frictionless “circular loop” track, as shown here:

(Note: The values of  $m$ ,  $H$ , and  $D$  are the SAME here as for the previous part.  
This sketch is NOT to scale.)



When the students start from the *same height*  $H$  as they did in part (a), they will find that...

b. (1 pt.)  $v_m$  on the “loop” track is \_\_\_\_\_  $v_m$  on the “hill” track.  
A. greater than      B. less than      C. the same as

c. (1 pt.)  $v_f$  on the “loop” track is \_\_\_\_\_  $v_f$  on the “hill” track.  
A. greater than      B. less than      C. the same as

d. (1 pt.) their potential energy at the top of the “loop” is \_\_\_\_\_ their potential energy at the top of the “hill.”  
A. greater than      B. less than      C. the same as

e. (1 pt.) they \_\_\_\_\_ remain pressed against their seats at the top of the loop.  
A. do      B. do not

f. (4 pts.) The *minimum* necessary speed at the top of the loop to prevent the students from falling out of their seats is:  $v'_m = \sqrt{\frac{1}{2}gD}$ . Find an expression for the **minimum starting height  $H'$**  that still allows the students to successfully round the top of the loop:

**Express your answer algebraically ONLY in terms of  $m$ ,  $g$ ,  $D$ , and any necessary numerical constants (but NOT any other variables!). Simplify your final answer as much as possible! Show your work completely.**

[Important note:  $H'$  and  $v'_m$  here are NOT equal to  $H$  or  $v_m$  in parts (a)–(e).]

2. During an episode of the animated TV show *Family Guy*, Brian tries to demonstrate to Peter just how badly Peter needs to go on a diet: Brian takes an apple and tosses it into a circular orbit around Peter's midsection. (To see this short joke on YouTube, visit: <http://www.youtube.com/watch?v=MD7urmuSeEg> or else search the [www.youtube.com](http://www.youtube.com) website for "family guy gravitational".)

Suppose the apple has a mass of 250 g, and that it orbits Peter's center-of-mass in a circular orbit of radius 1.0 m and period 5.0 s. Assume that both the apple and Peter are spherical and uniform-density. (*Assume that NO other masses, including the Earth, are nearby.*)

a. (4 pts.) Find the apple's **linear speed**. *Show your work.*

b. (5 pts.) What is **Peter's mass**, in kilograms? *Show your work completely.* (Your answer will be many orders-of-magnitude greater than the mass of a typical human... more like the mass of a small mountain!)

3. (6 pts.) A bartender slides a large mug of beer ( $m = 1.8 \text{ kg}$ ) down a long, level countertop, hoping that it will stop gracefully in front of a customer 4.2 m away. From long experience, he knows the mug will require an initial velocity of 3.6 m/s (about 8 mi/h). Find the **coefficient of kinetic friction** between the mug and the countertop. You may use either kinematics/dynamics or work/energy, but you must show your work completely.

