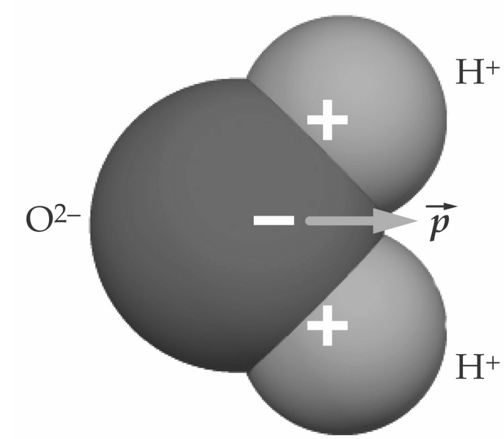
Today's Plan

- Electric Dipoles
- More on Gauss' Law
- Comment on PDF copies of Lectures
- Final iclicker roll-call

Electric Dipoles

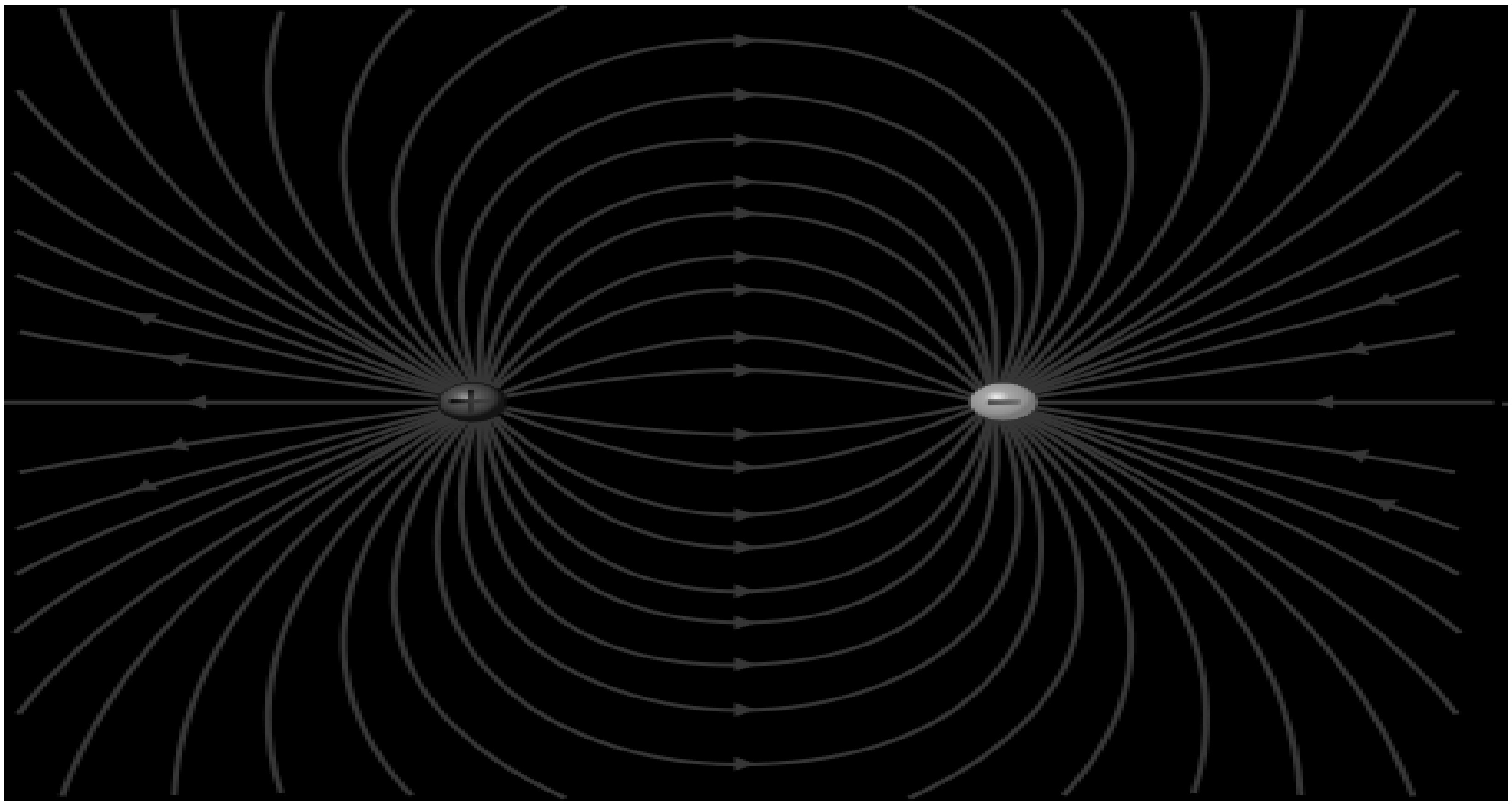
- A positive (q) and negative charge (-q) separated by a small distance d.

- Electric dipole moment vector with length p=q d, from the negative charge to the positive charge.

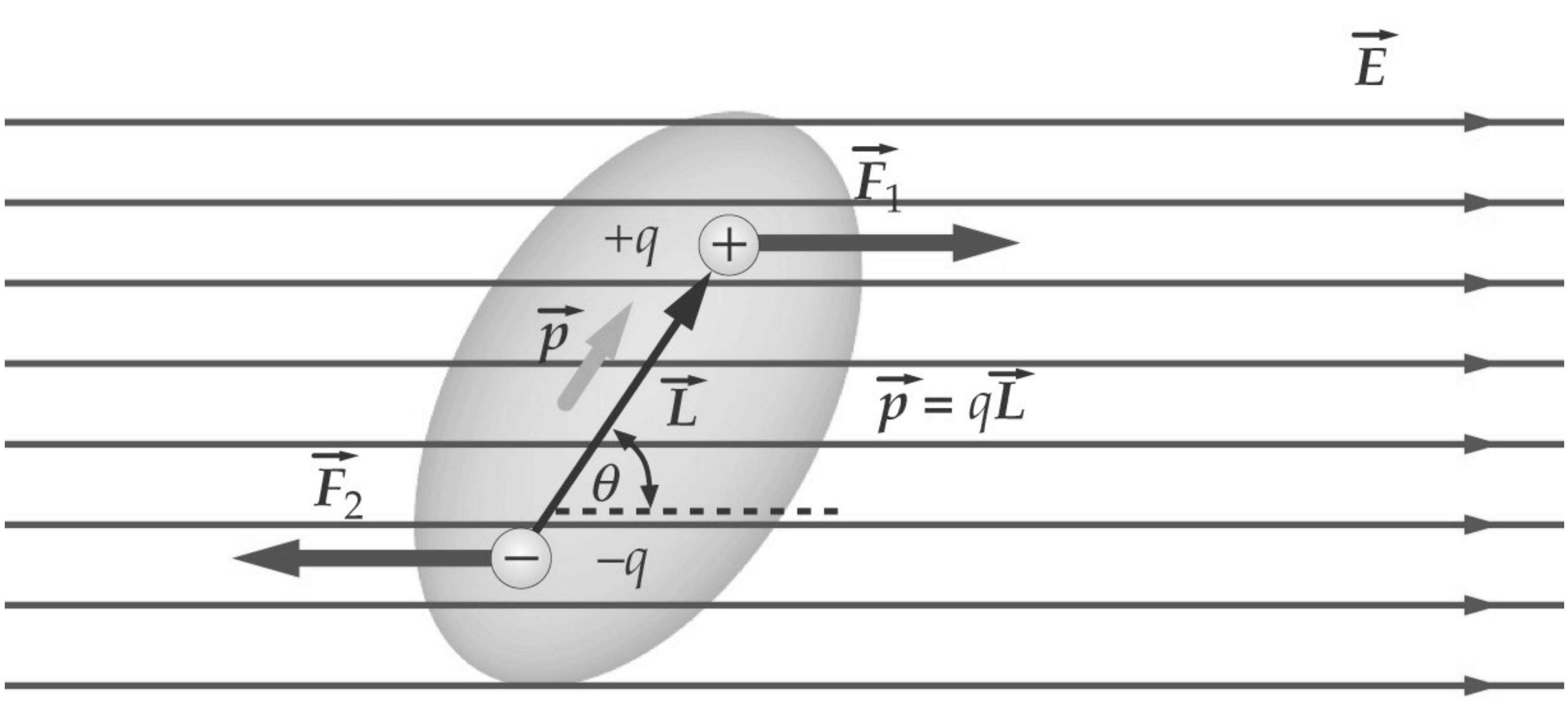


- Examples include TV antennas and on the microscopic scale (H_2O).

Electric Field Lines of a Dipole



Torque and Forces on a Dipole



N.B. No net force on the dipole

Properties of electric dipoles

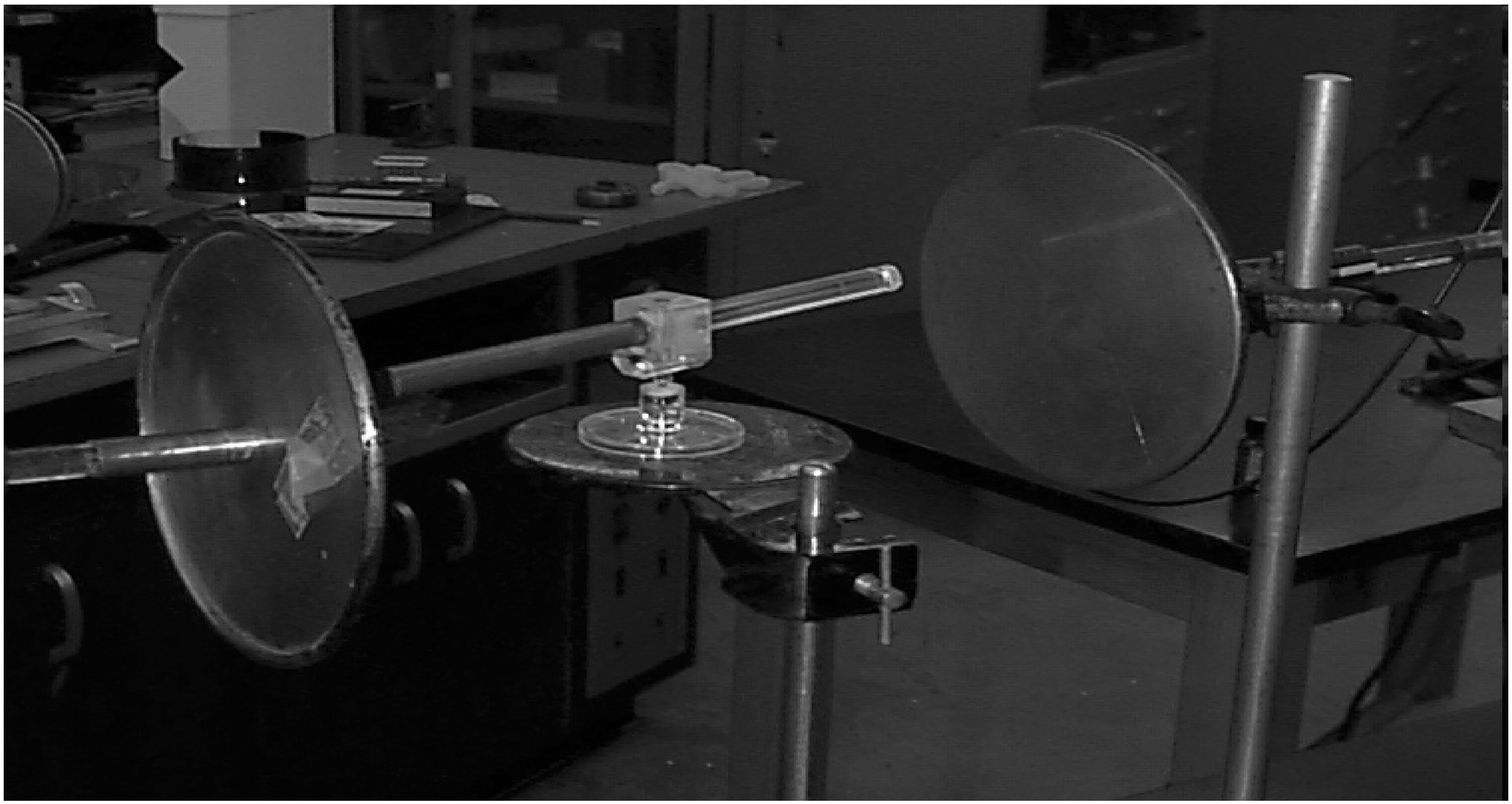
- Potential energy

$$\boxed{U = -\vec{p} \bullet \vec{E}} \qquad \boxed{U = -pE\cos(\theta)}$$

- Torque $\boxed{\vec{\tau} = -\vec{p} \times \vec{E}}$ $\boxed{|\tau| = pE\sin(\theta)}$

- In a uniform electric field, the field exerts a torque (but no net force) on the dipole to minimize its potential energy and align it with the field.

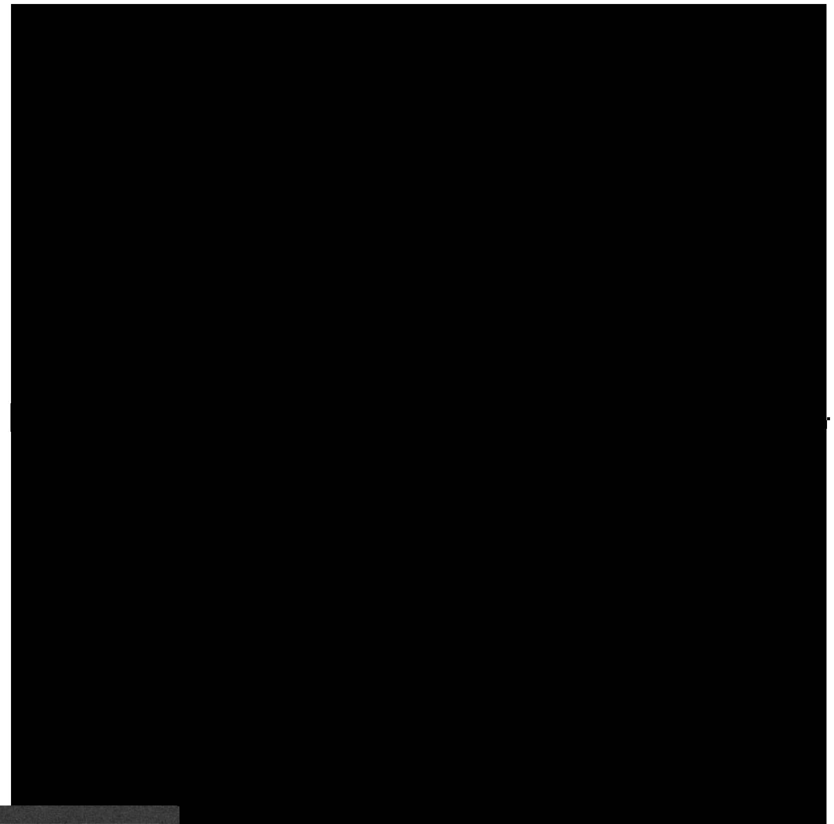
Experimental setup for torque on a dipole



Torque on dipoles (water molecules) provides heating in microwave ovens

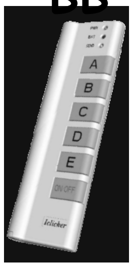
Alternating electric field

Gauss' Law



$$\oint \vec{E} \bullet d\vec{A} = \Phi_E = \frac{q_{enclosed}}{\varepsilon_0}$$

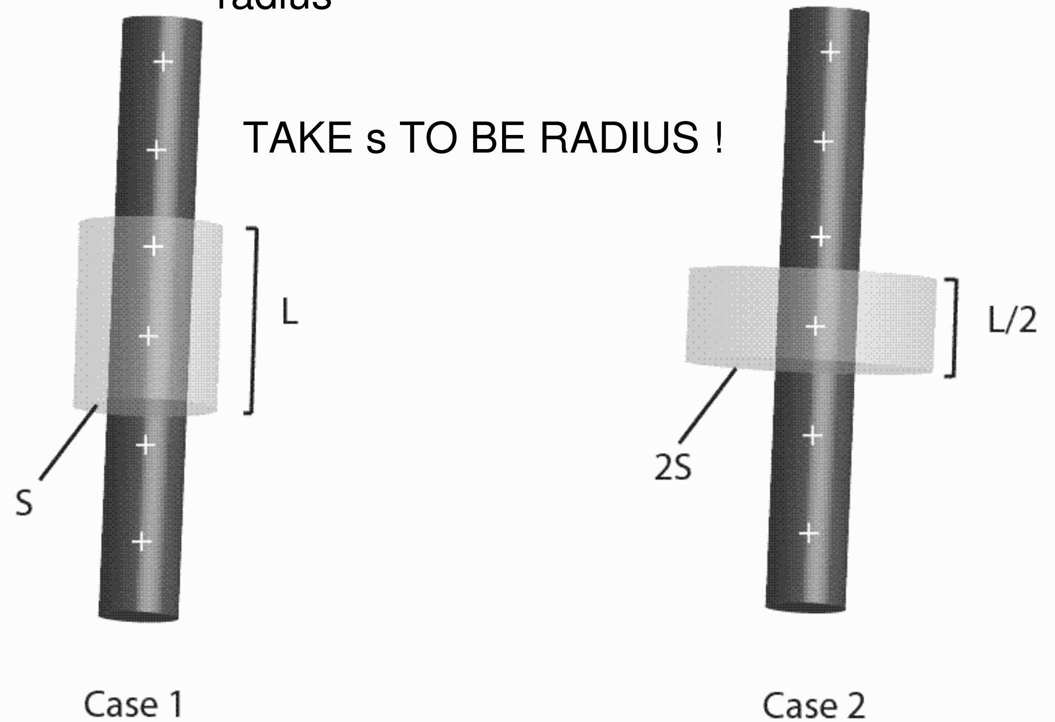
BB



Clicker Prob

2) An infinitely long charged rod has uniform charge density of λ, and passes through a cylinder (gray). The cylinder in case 2 has twice the ~~cross sectional area~~ radius and half the length compared to the cylinder in case 1.

TAKE s TO BE RADIUS !

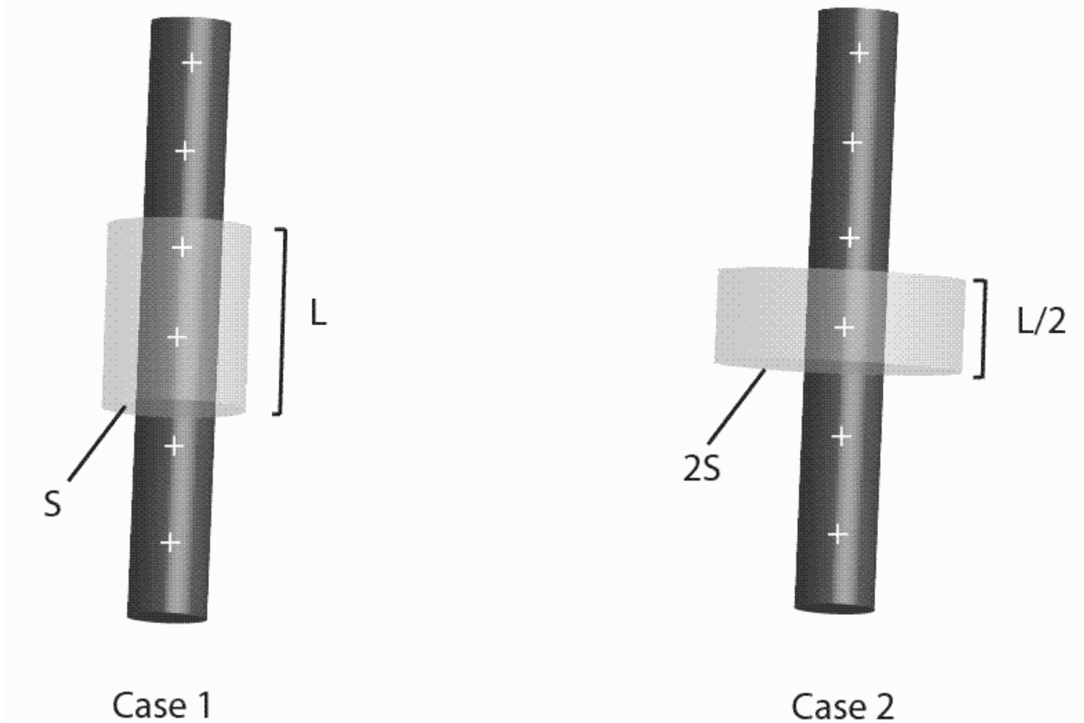


$\boxed{\Phi_1=2\Phi_2 \text{ (A)}}$	$\Phi_1=\Phi_2$ (B)	$\Phi_1=1/2\Phi_2$ (C)	none (D)

Explanation

Definition of Flux:

2) An infinitely long charged rod has uniform charge density of λ, and passes through a cylinder (gray). The cylinder in case 2 has twice the cross sectional area and half the length compared to the cylinder in case 1.



$\boxed{\Phi_1 = 2\Phi_2 \text{ (A)}}$	$\Phi_1 = \Phi_2$ (B)	$\Phi_1 = 1/2\Phi_2$ (C)	none (D)

E constant on barrel of cylinder
E perpendicular to barrel surface
(E parallel to dA)

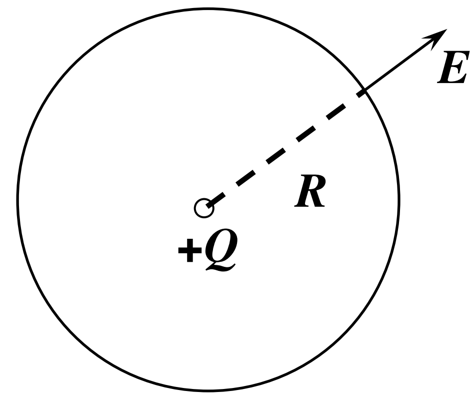
RESULT: GAUSS' LAW
Φ proportional to charge enclosed !

Case 1

Case 2

Review: Gauss' Law $\Rightarrow$ Coulomb's Law

- **We now illustrate this for the field of the point charge and prove that Gauss' Law implies Coulomb's Law.**



- **Symmetry $\Rightarrow$ *E*-field of point charge is radial and spherically symmetric**
- **Draw a sphere of radius *R* centered on the charge.**
- **Why?**

  E **normal to every point on the surface**

  $\Rightarrow \vec{E} \bullet d\vec{A} = E\,dA$

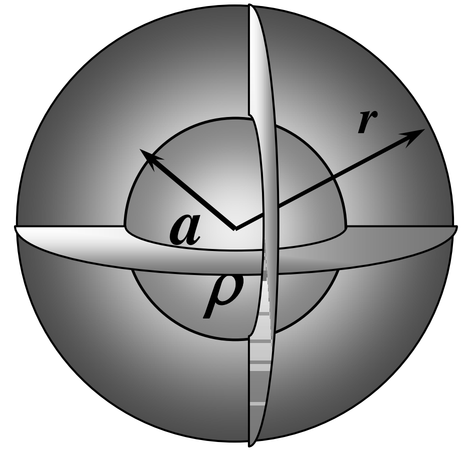
  E **has same value at every point on the surface**

  $\Rightarrow$ **can take *E* outside of the integral!**

- **Therefore,** $\oint \vec{E} \bullet d\vec{A} = \oint E dA = E \oint dA = 4\pi R^2 E$ **!**
  - **Gauss' Law** $\varepsilon_0 4\pi R^2 E = Q$ $\boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R^2}}$
  - **We are free to choose the surface in such problems… we call this a "Gaussian" surface**

Uniform charged sphere

What is the magnitude of the electric field due to a <u>solid</u> sphere of radius a with uniform charge density ρ (C/m³)?



- **Outside sphere: ($r>a$)**
  - **We have spherical symmetry centered on the center of the sphere of charge**
  - **Therefore, choose Gaussian surface = hollow sphere of radius r**

$$\oint \vec{E} \bullet d\vec{A} = 4\pi r^2 E = \frac{q}{\varepsilon_0}$$

$$q = \frac{4}{3}\pi a^3 \rho$$

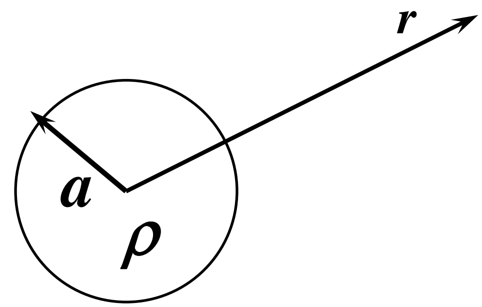
$\Longrightarrow$ **Gauss' Law**

$$\boxed{E = \frac{\rho a^3}{3\varepsilon_0 r^2}} = \frac{1}{4\pi\,\varepsilon_0}\frac{q}{r^2}$$

same as point charge!

Uniform charged sphere

- **Outside sphere: ($r > a$)** $\boxed{E = \dfrac{\rho a^3}{3\varepsilon_0 r^2}}$

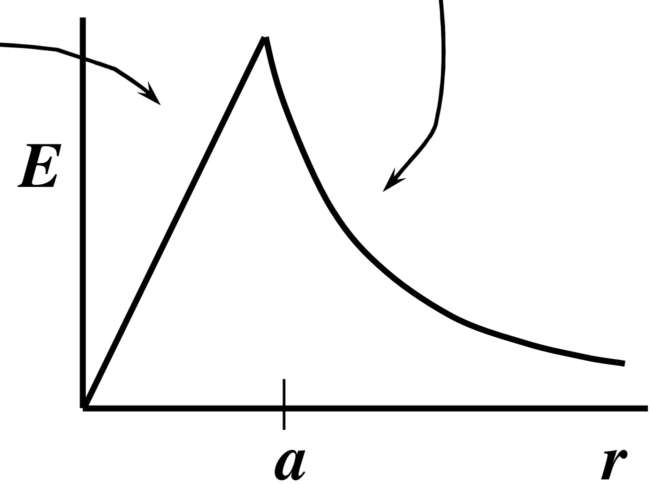


- **Inside sphere: ($r < a$)**
  - **We still have spherical symmetry centered on the center of the sphere of charge.**
  - **Therefore, choose Gaussian surface = sphere of radius r**

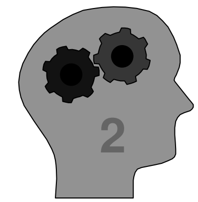
Gauss' Law $\oint \vec{E} \bullet d\vec{A} = 4\pi r^2 E = \dfrac{q}{\varepsilon_0}$

But, $q = \dfrac{4}{3}\pi \mathrm{r}^3 \rho$

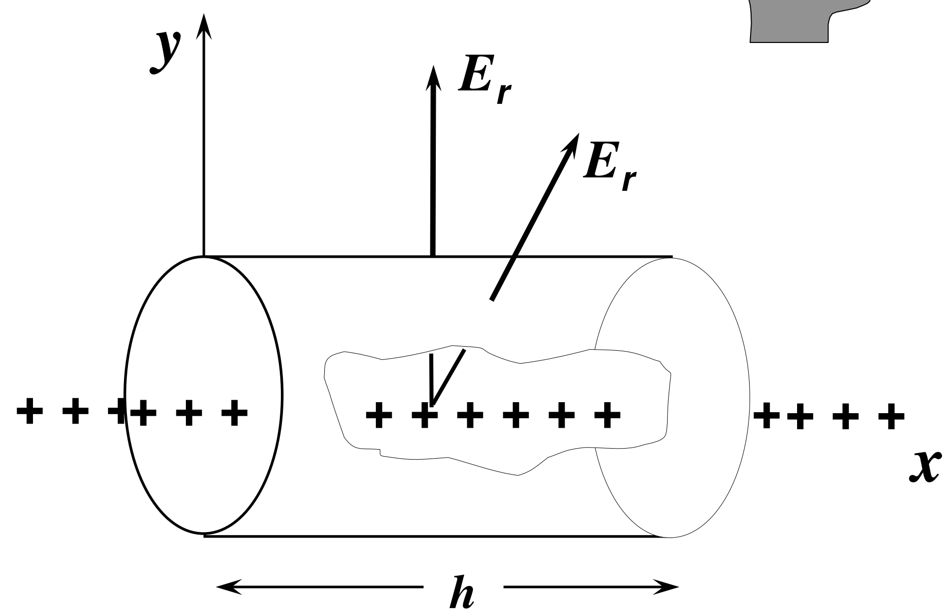
Thus: $E = \dfrac{\rho}{3\varepsilon_0} r$



Infinite Line of Charge



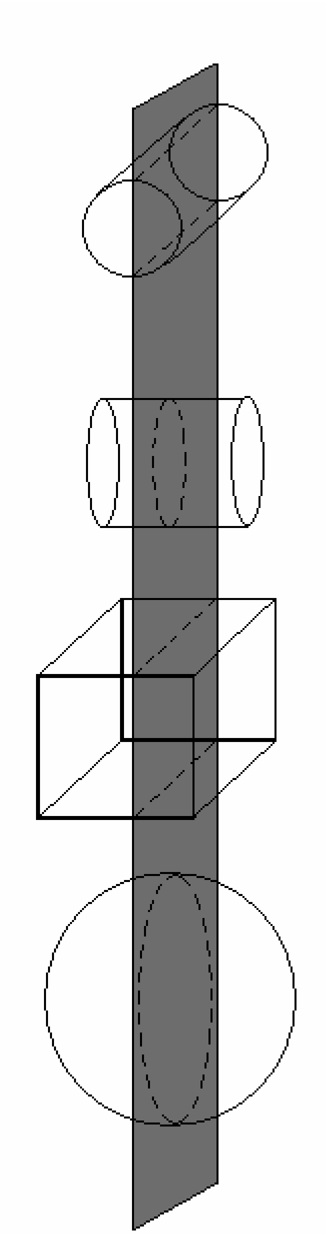
- **Symmetry ⇒ *E*-field must be ⊥ to line and can only depend on distance from line**
- **Therefore, CHOOSE Gaussian surface to be a cylinder of radius *r* and length *h* aligned with the *x*-axis.**



- **Apply Gauss' Law:**
  - **On the ends,** $\vec{E} \bullet d\vec{A} = 0$
  - **On the barrel,** $\oint \vec{E} \bullet d\vec{A} = 2\pi r h E$ **AND** $q = \lambda h \Rightarrow$ $\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$

NOTE: we have obtained here the same result as we did last lecture using Coulomb's Law. The symmetry makes today's derivation easier.

UI4PF4:



5) Given an infinite sheet of charge as shown in the figure. You need to use Gauss' Law to calculate the electric field near the sheet of charge. Which of the following Gaussian surfaces are best suited for this purpose?

Note: you may choose more than one answer

a) a cylinder with its axis along the plane

b) a cylinder with its axis perpendicular to the plane

c) a cube

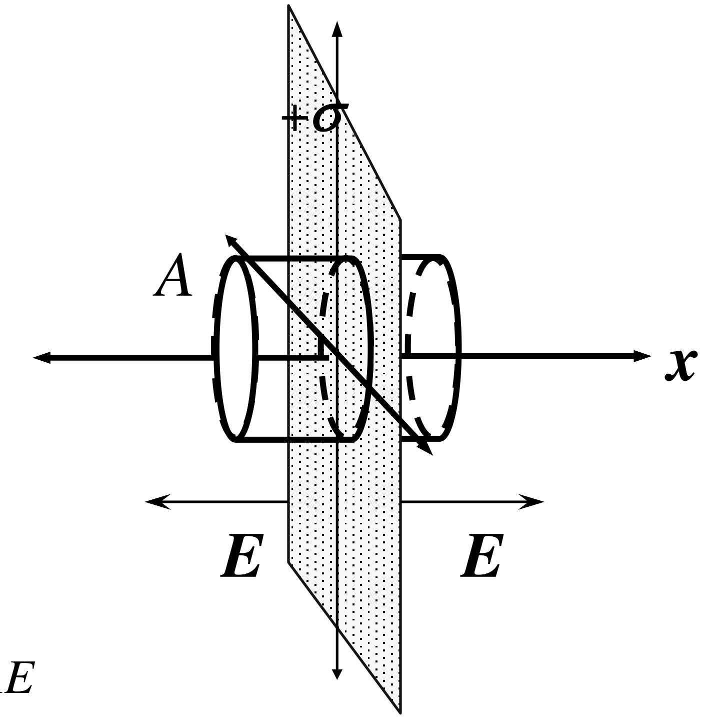
d) a sphere

Infinite sheet of charge, surface charge density σ

- **Symmetry:**

  direction of E = x-axis

- **Therefore, CHOOSE Gaussian surface to be a cylinder whose axis is aligned with the x-axis.**



- **Apply Gauss' Law:**
  - **On the barrel,** $\vec{E} \bullet d\vec{A} = 0$
  - **On the ends,** $\oint \vec{E} \bullet d\vec{A} = 2AE$
  - **The charge enclosed =** σA

Therefore, Gauss' Law $\Rightarrow \varepsilon_0 \left(2EA\right) = \sigma A \quad \Longrightarrow \quad \boxed{E = \dfrac{\sigma}{2\varepsilon_0}}$

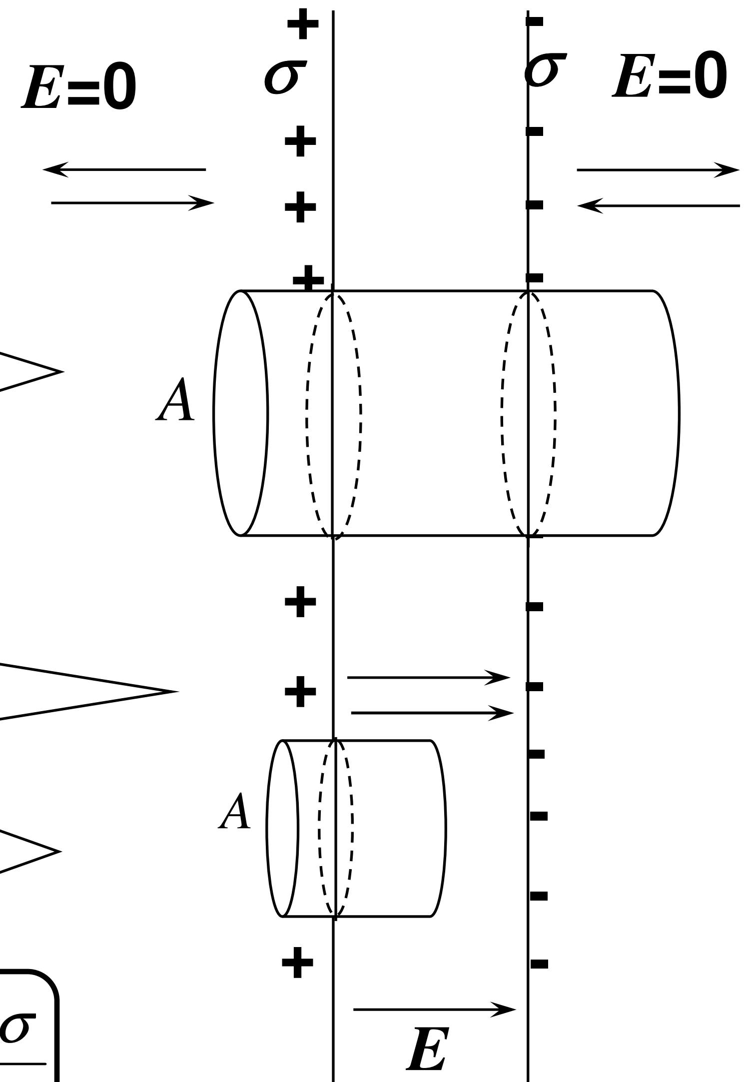
Conclusion: An infinite plane sheet of charge creates a CONSTANT electric field .

Same as integrating charge distribution.

Two Infinite Sheets

(into the screen)

- **Field outside must be zero. Two ways to see:**
  - **Superposition**
  - **Gaussian surface encloses zero charge**



- **Field inside is NOT zero:**
  - **Superposition**
  - **Gaussian surface encloses non-zero charge**

$$Q = \sigma A$$

$$\oint \vec{E}\bullet d\vec{A} = \cancelto{0}{AE_{outside}} + AE_{inside}$$

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

Gauss' Law: Help for the Homework Problems

- **Gauss' Law is ALWAYS VALID!** $\boxed{\varepsilon_0 \oint \vec{E} \bullet d\vec{A} = q_{enclosed}}$

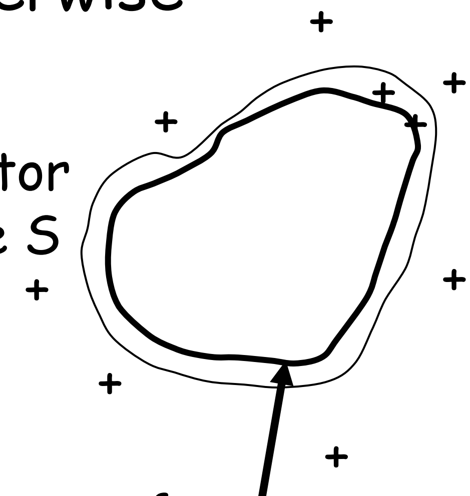
- **What Can You Do With This?**
  If you have (a) spherical, (b) cylindrical, or (c) planar symmetry AND:
  - **If you know the charge (RHS), you can calculate the electric field (LHS)**
  - **If you know the field (LHS, usually because E=0 inside conductor), you can calculate the charge (RHS).**

- **Spherical Symmetry: Gaussian surface = sphere of radius r**
  LHS: $\varepsilon_0 \oint \vec{E} \bullet d\vec{A} = 4\pi\varepsilon_0 r^2 E$
  RHS: q **= ALL charge inside radius r** $\Rightarrow$ $\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}}$
- **Cylindrical symmetry: Gaussian surface = cylinder of radius r**
  LHS: $\varepsilon_0 \oint \vec{E} \bullet d\vec{A} = \varepsilon_0 2\pi r L E$
  RHS: q **= ALL charge inside radius r, length L** $\Rightarrow$ $\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$
- **Planar Symmetry: Gaussian surface = cylinder of area A**
  LHS: $\varepsilon_0 \oint \vec{E} \bullet d\vec{A} = \varepsilon_0 2 A E$
  RHS: q **= ALL charge inside cylinder =σA** $\Rightarrow$ $\boxed{E = \frac{\sigma}{2\varepsilon_0}}$

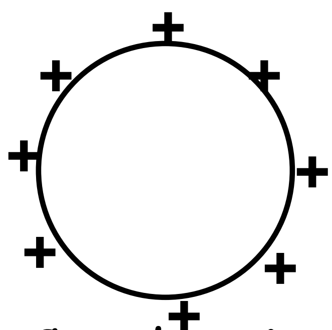
Gauss' Law and Conductors

- We know that E=0 inside a conductor (otherwise the charges would move). Electrostatics!

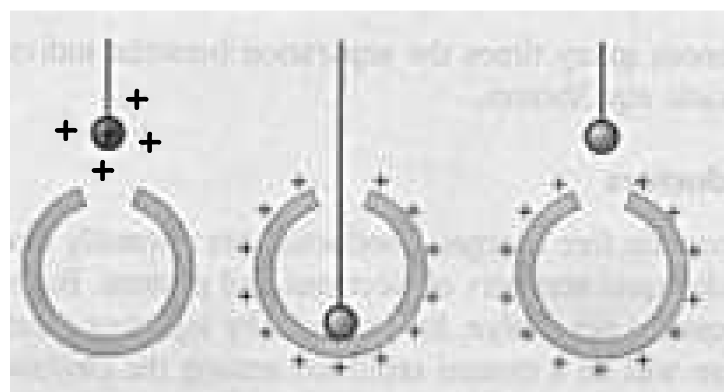
- But since $\oint \vec{E} \bullet d\vec{A} = 0$ →



Charges on a conductor only reside on the surface(s)!

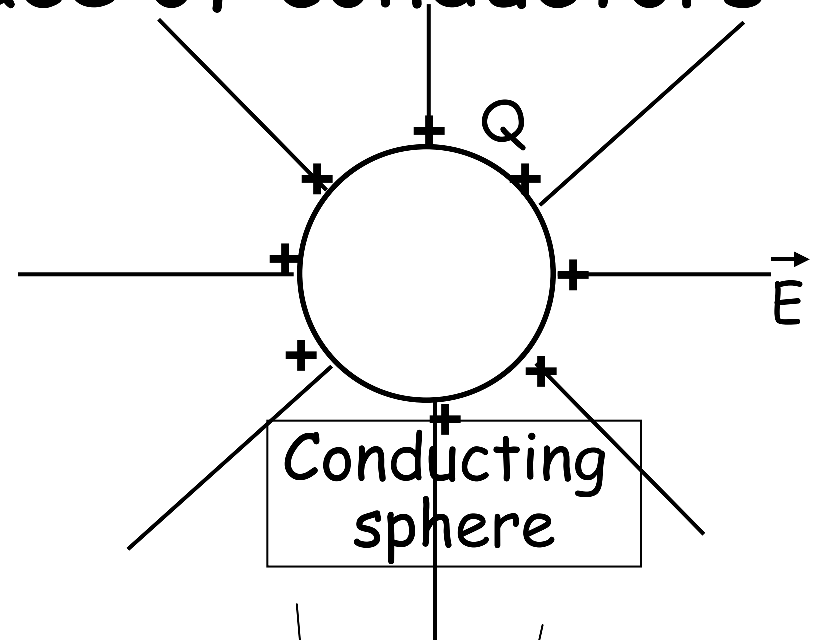


Conducting sphere



Fields at surface of conductors

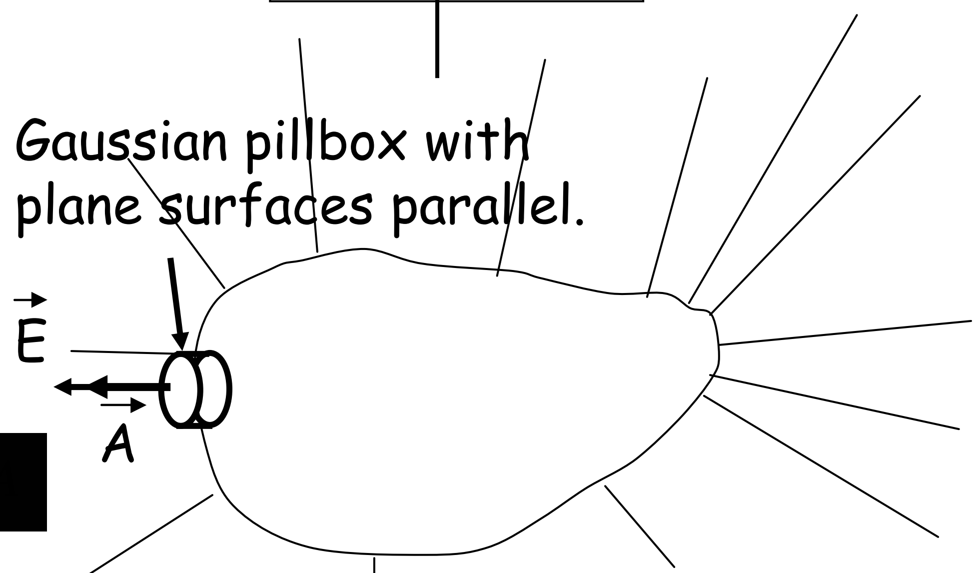
Conducting sphere: charge distributes uniformly. $\vec{E}$ outside just like point charge Q.



More general shape:

1. E is ⊥ to surface since there can be no component tangent.
2. Flux on cyl. surface is zero.
3. Flux on inside is zero.
4. Therefore

[redacted]



$\vec{E}$ at surface of conductor is normal and $E = \sigma/\varepsilon_0$.

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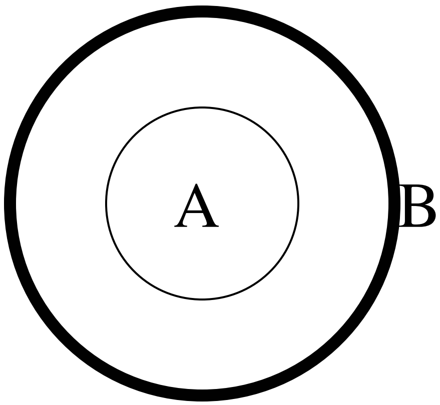


A blue sphere A is contained within a red spherical shell B. There is a charge Q_A on the blue sphere and charge Q_B on the red spherical shell.

7) The electric field in the region between the spheres is completely independent of Q_B the charge on the red spherical shell.

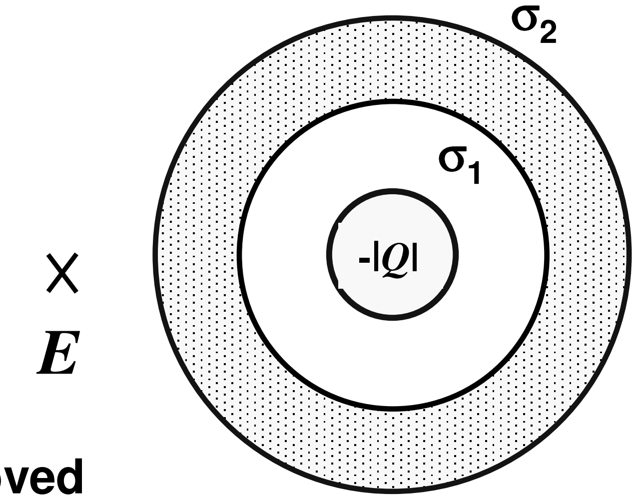
True

False

UI4ACT1

Consider the following two topologies:

A) A solid non-conducting sphere carries a total charge Q = -3 μC distributed evenly throughout. It is surrounded by an *uncharged* conducting spherical shell.



B) Same as (A) but conducting shell removed

1A •**Compare the electric field at point X in cases A and B:**

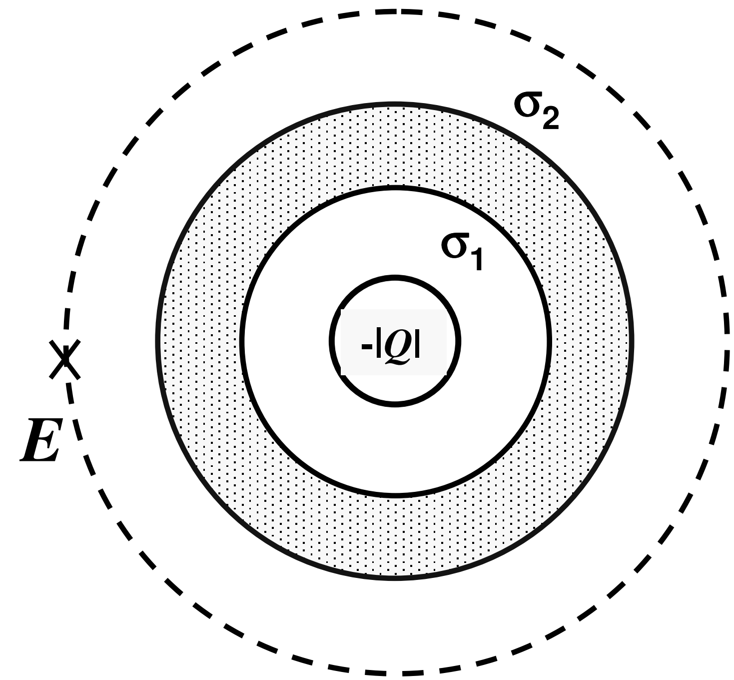
(a) $E_A < E_B$ **(b)** $E_A = E_B$ **(c)** $E_A > E_B$

1B •**What is the surface charge density σ_1 on the inner surface of the conducting shell in case A?**

(a) $\sigma_1 < 0$ **(b)** $\sigma_1 = 0$ **(c)** $\sigma_1 > 0$

UI4ACT1

Consider the following two topologies:



A) A solid non-conducting sphere carries a total charge Q = -3 μC distributed evenly throughout. It is surrounded by an *uncharged* conducting spherical shell.

1A •Compare the electric field at point X in cases A and B:

(a) $E_A < E_B$ $\boxed{\text{(b) } E_A = E_B}$ (c) $E_A > E_B$

- Select a sphere passing through the point X as the Gaussian surface.
- How much charge does it enclose?
  - Answer: $-|Q|$, whether or not the uncharged shell is present.

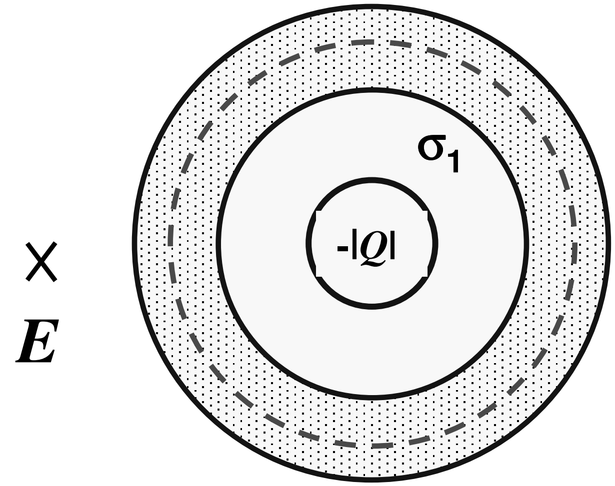
(The field at point X is determined only by the objects with NET CHARGE.)

UI4ACT1

Consider the following two topologies:

> **A solid non-conducting sphere carries a total charge Q = -3 μC and is surrounded by an *uncharged* conducting spherical shell.**

B) Same as (A) but conducting shell removed



1B

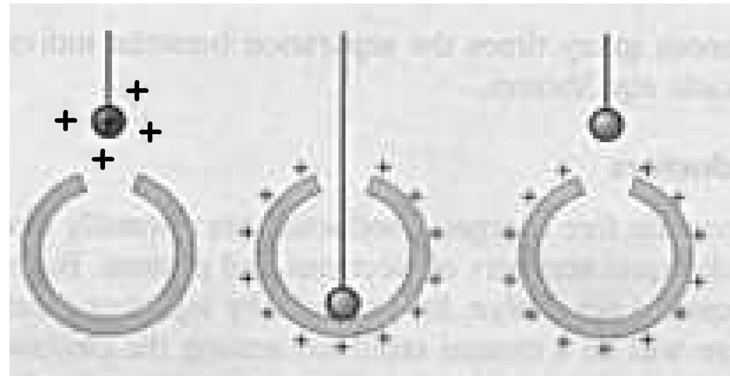
•What is the surface charge density σ_1 on the inner surface of the conducting shell in case A?

(a) $\sigma_1 < 0$ **(b)** $\sigma_1 = 0$ $\boxed{\textbf{(c)}\ \sigma_1 > 0}$

- **Inside the conductor, we know the field E = 0**
- **Select a Gaussian surface inside the conductor**
  - **Since E = 0 on this surface, the total enclosed charge must be 0**
  - **Therefore, σ_1 must be positive, to cancel the charge -|Q|**
- **By the way, to calculate the actual value: $\sigma_1 = -Q / (4\pi r_1^2)$**

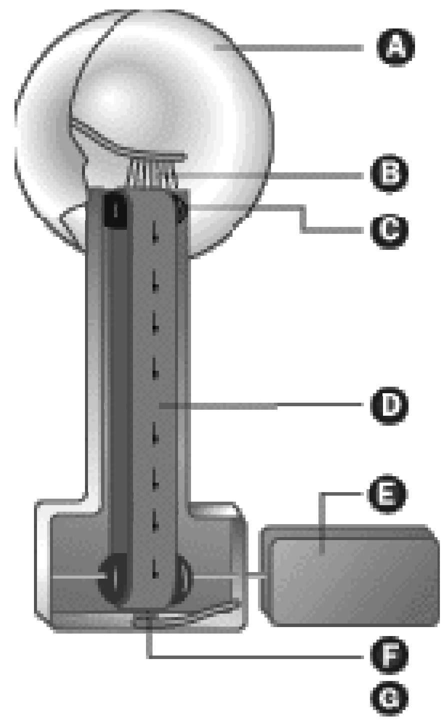
Testing Gauss's Law

Faraday's ice pail experiment.
If Gauss's Law correct, we will never detect any charge on the inside.



Application: Van der Graaf generator

Ⓐ **Output terminal — An aluminum or steel sphere**
Ⓑ **Upper brush — A piece of fine metal wire**
Ⓒ **Upper roller — A piece of nylon**
Ⓓ **Belt — A piece of surgical tubing**
Ⓔ **Motor**
Ⓕ **Lower brush**
Ⓖ **Lower roller — A piece of nylon covered with silicon tape**



Coaxial cable: signal is shielded from external noise by Gauss' Law