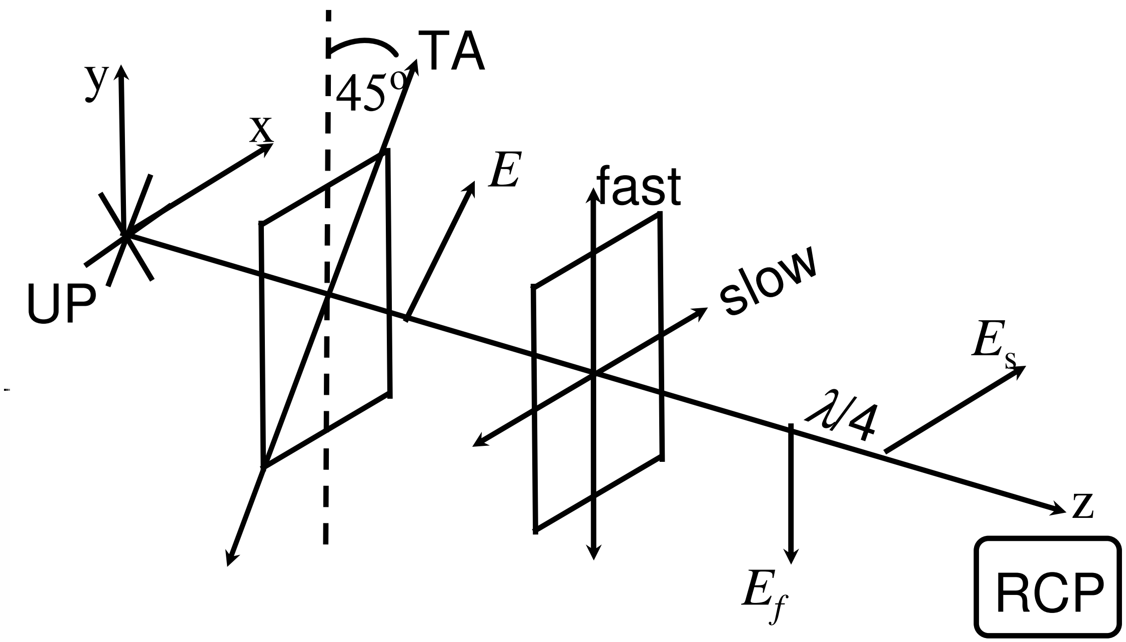
Phys-272   Lecture 23

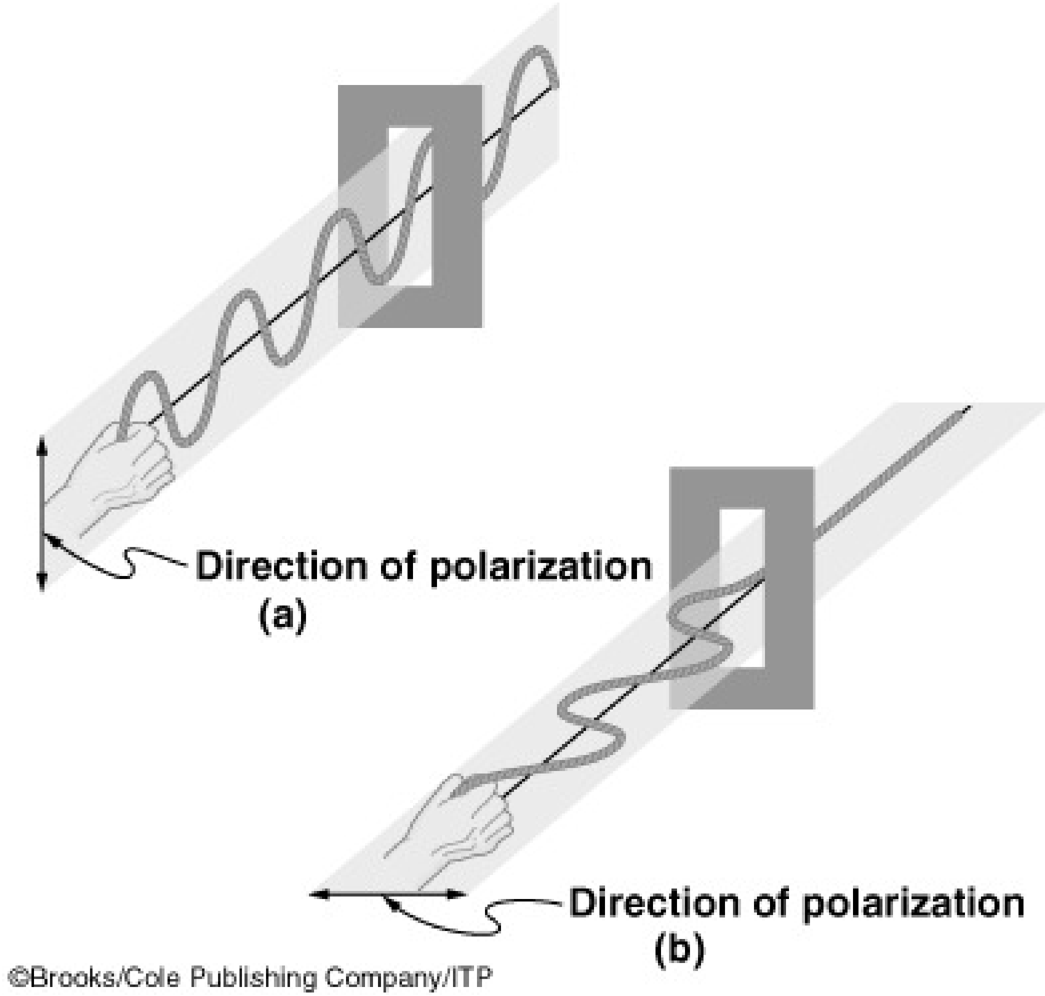
Polarization

Birefringence

Polarization



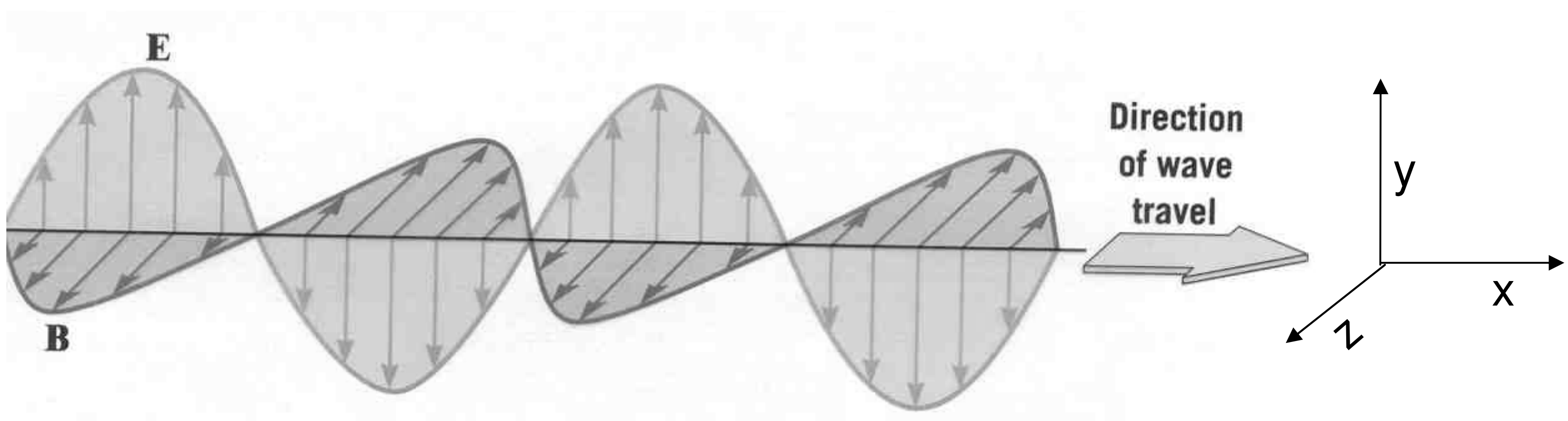
Polarization



Orientation of E field matters when the EM wave traverses matter

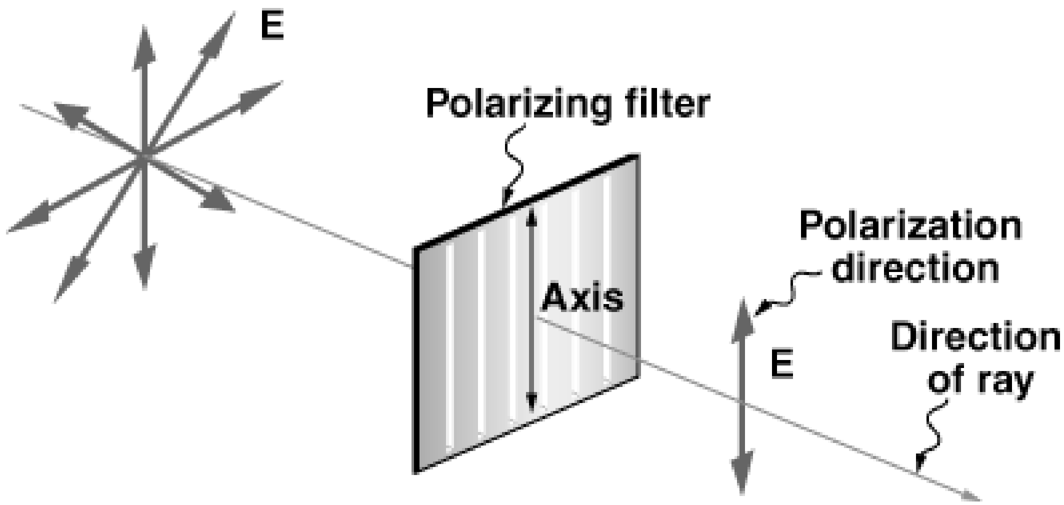
Polarization

- **Transverse waves have a polarization**
  - **(Direction of oscillation of E field for light)**
- **Types of Polarization**
  - **Linear   (Direction of E is constant)**
  - **Circular (Direction of E rotates with time)**
  - **Unpolarized (Direction of E changes randomly)**



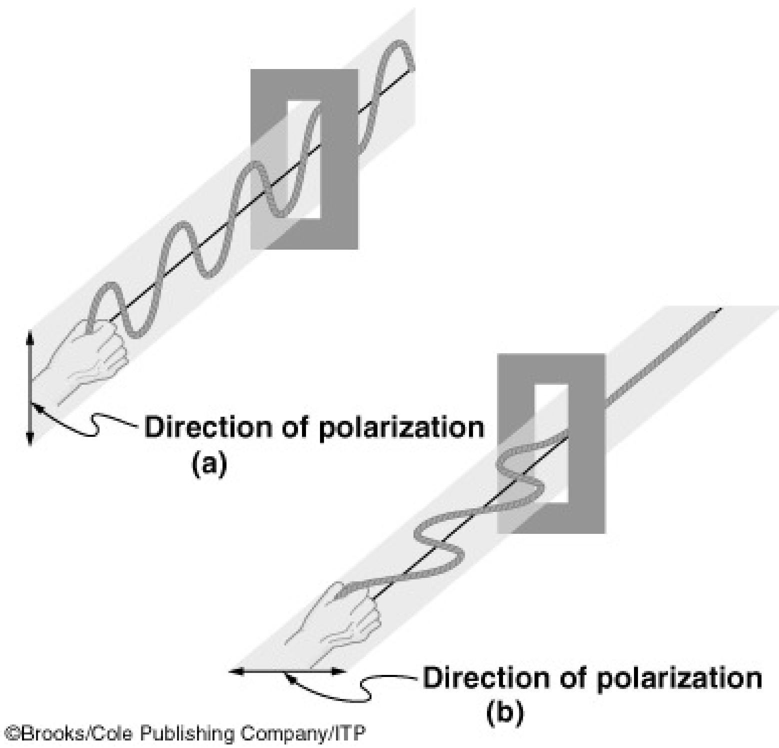
Natural Light is Unpolarized

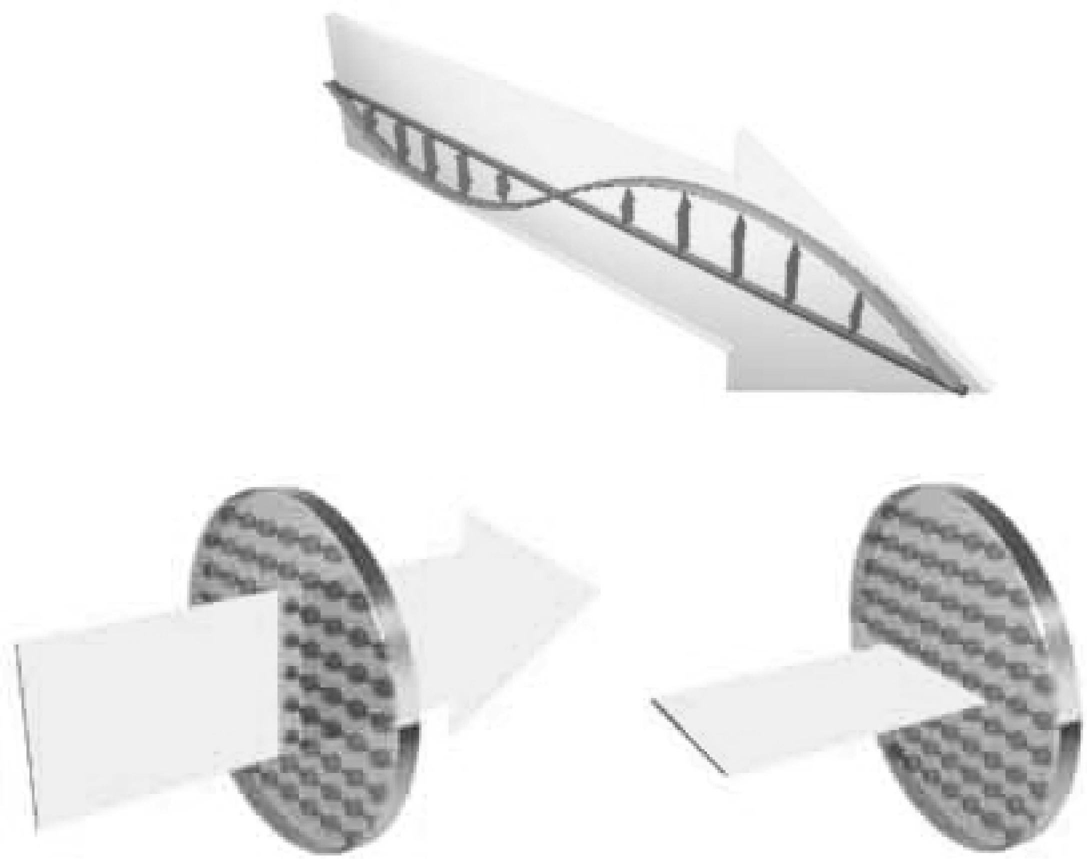
[redacted]



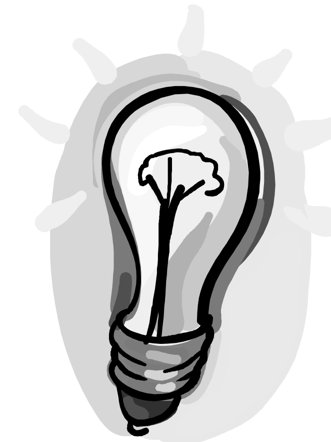
Linear Polarizers

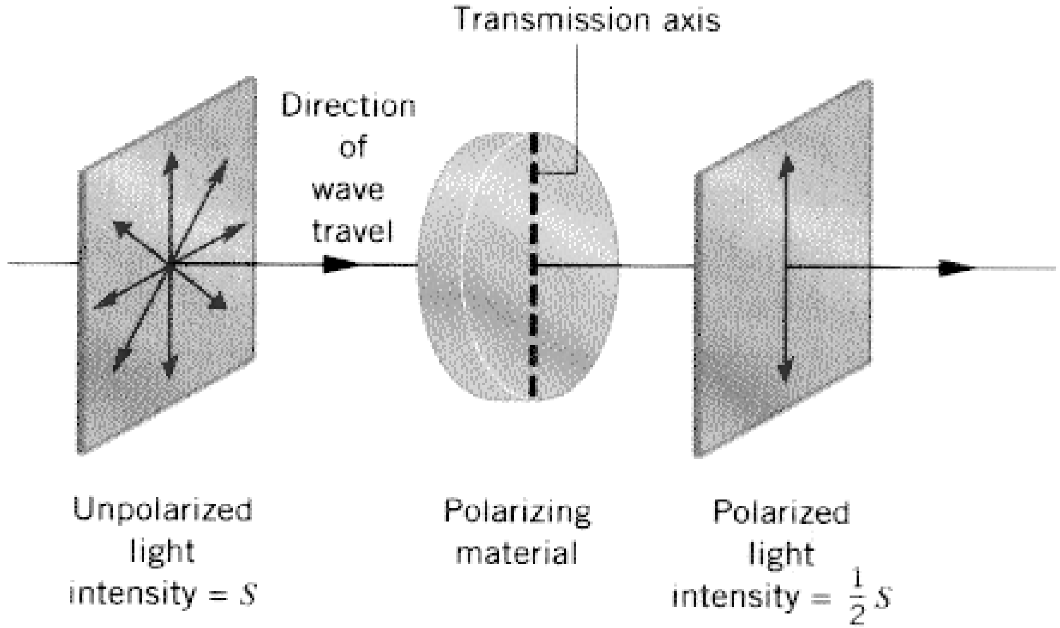
- **Linear Polarizers absorb all electric fields perpendicular to their transmission axis.**





Unpolarized Light on Linear Polarizer





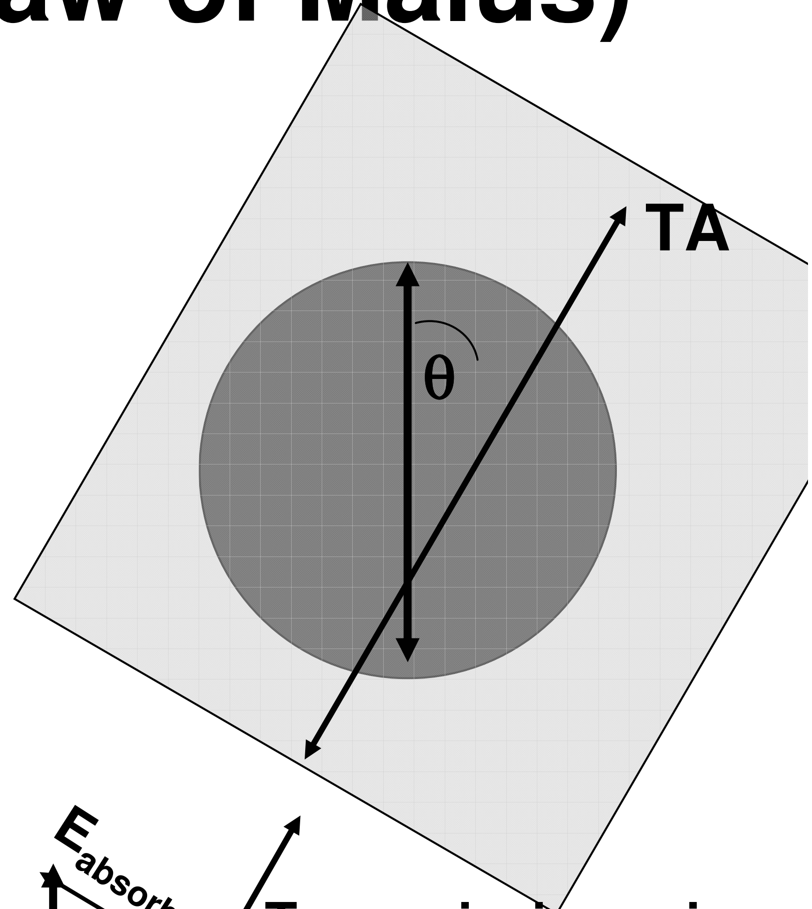
- Most light comes from electrons accelerating in random directions and is unpolarized.
- Averaging over all directions, intensity of transmitted light reduces due to reduction in E

$$I = \frac{c\varepsilon_0 E^2}{2}$$

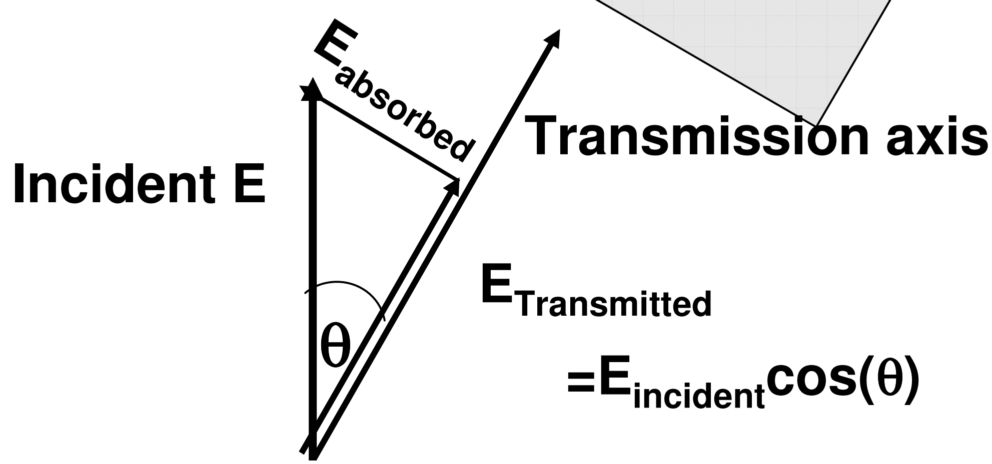
Linearly Polarized Light on Linear Polarizer (Law of Malus)

$E_{tranmitted} = E_{incident} \cos(\theta)$

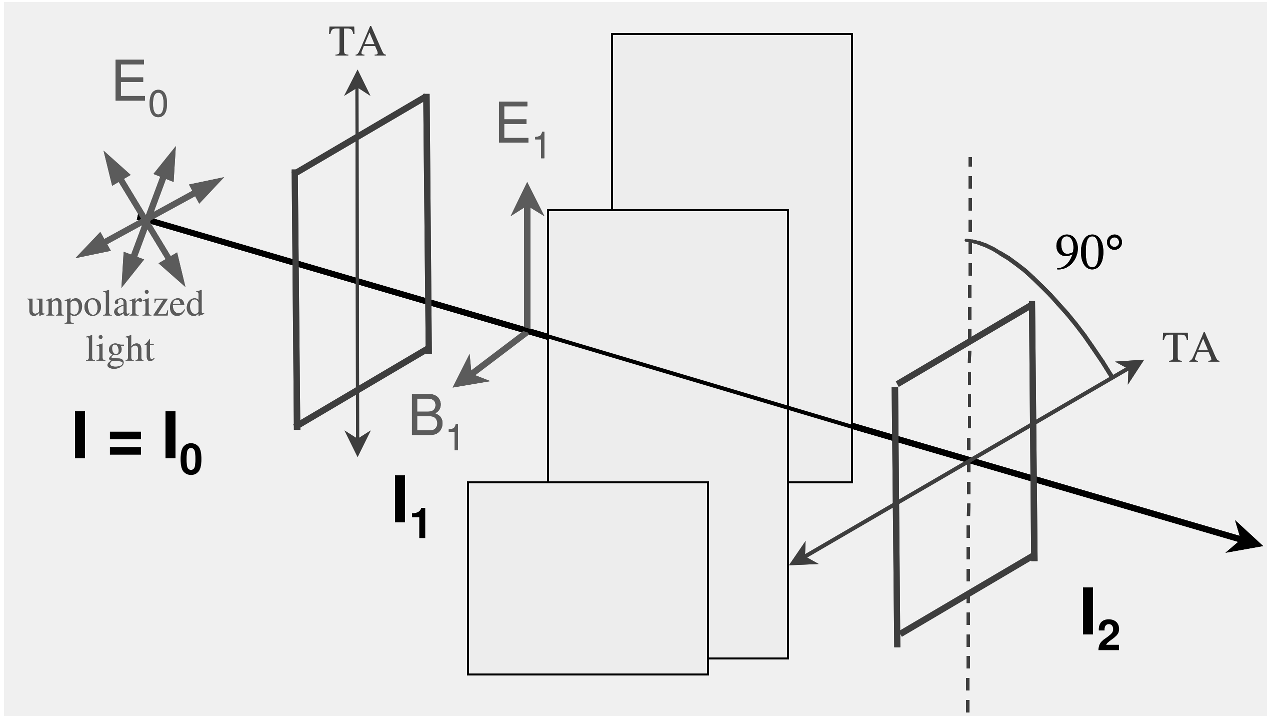
$I_{transmitted} = I_{incident} \cos^2(\theta)$



θ is the angle between the incoming light's polarization, and the transmission axis



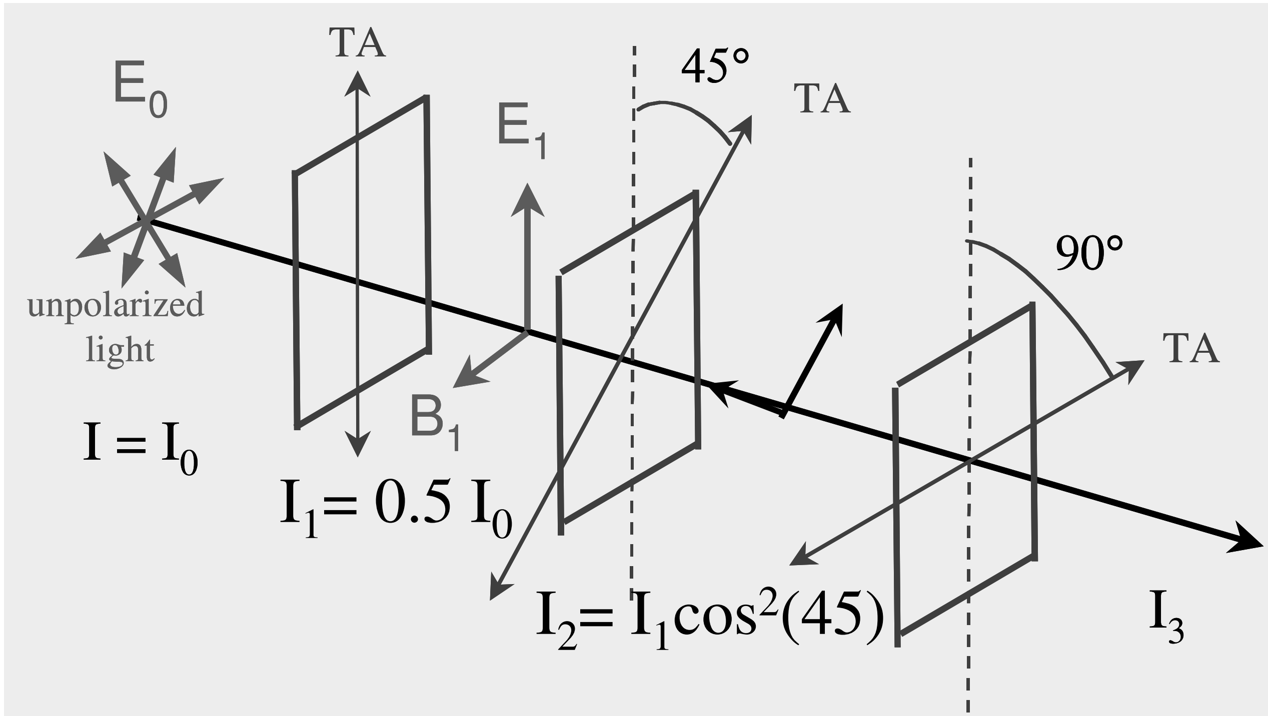
Law of Malus Example



1) Intensity of unpolarized light incident on linear polarizer is reduced by half . $I_1 = I_0 / 2$

2) Light transmitted through first polarizer is vertically polarized. Angle between it and second polarizer is θ=90º . $I_2 = I_1 \cos^2(90º) = 0$

Law of Malus Example

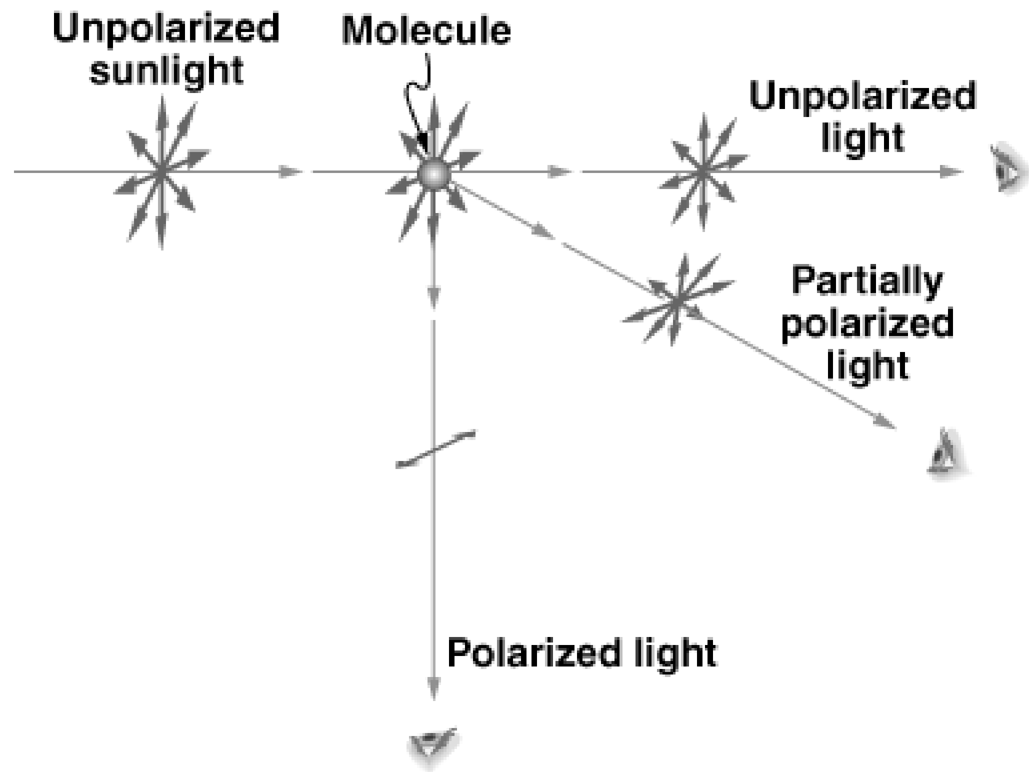


2) Light transmitted through first polarizer is vertically polarized. Angle between it and second polarizer is θ=45º. I_2= I_1 $\cos^2$ (45º)=0.5I_1=0.25 I_0

3) Light transmitted through second polarizer is polarized 45º from vertical. Angle between it and third polarizer is θ=45º. I_3= I_2 $\cos^2$(45º) =0.125 I_0

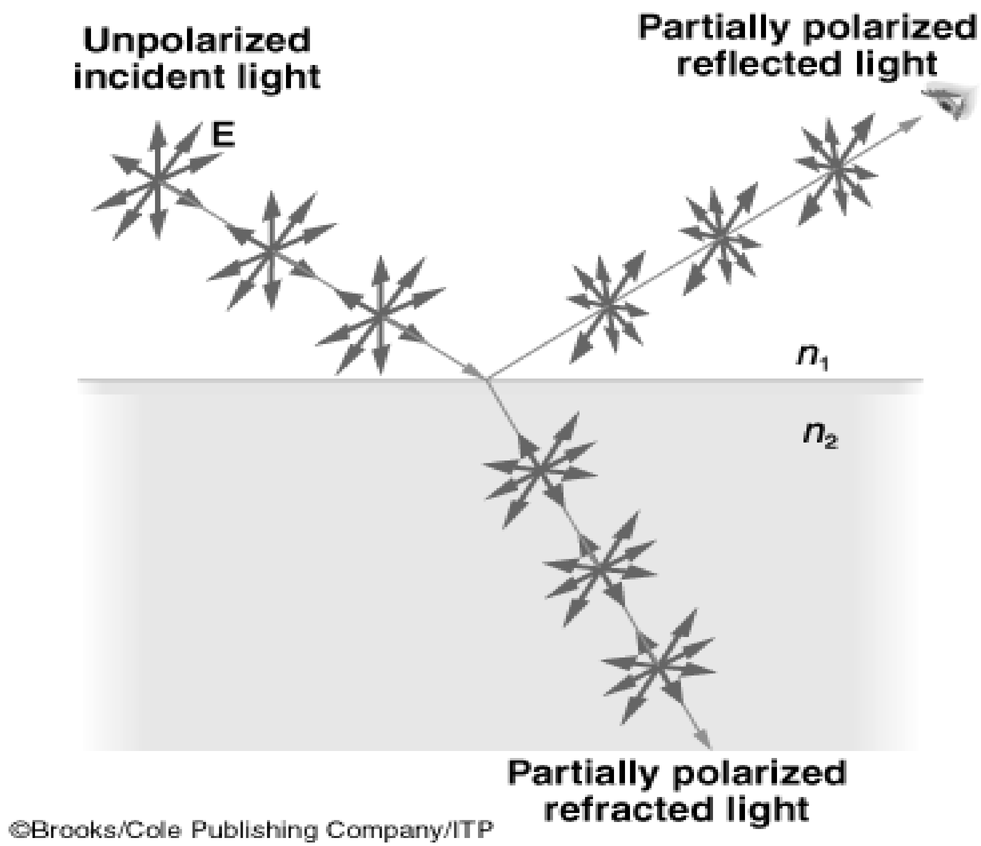
Polarization by Scattering

- **Light can also be polarized in scattering processes**
  - **Light scatters of molecules in air**
  - **Rotate polarized sunglasses while looking at blue sky**
    - » **Light gets cut off - blue sky turns brighter and dimmer as you rotate indicating polarization**



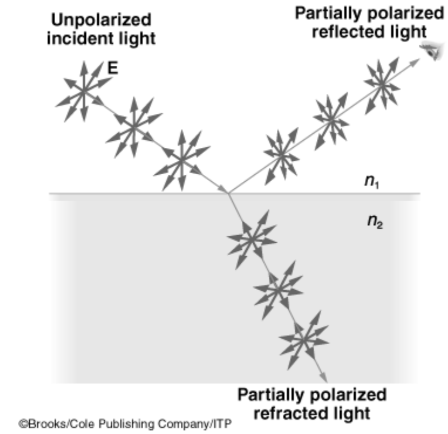
Polarization by Reflection

- **Light can by polarized by reflection**



This is why polarized sunglasses are effective in reducing "glare"

Brewster's angle



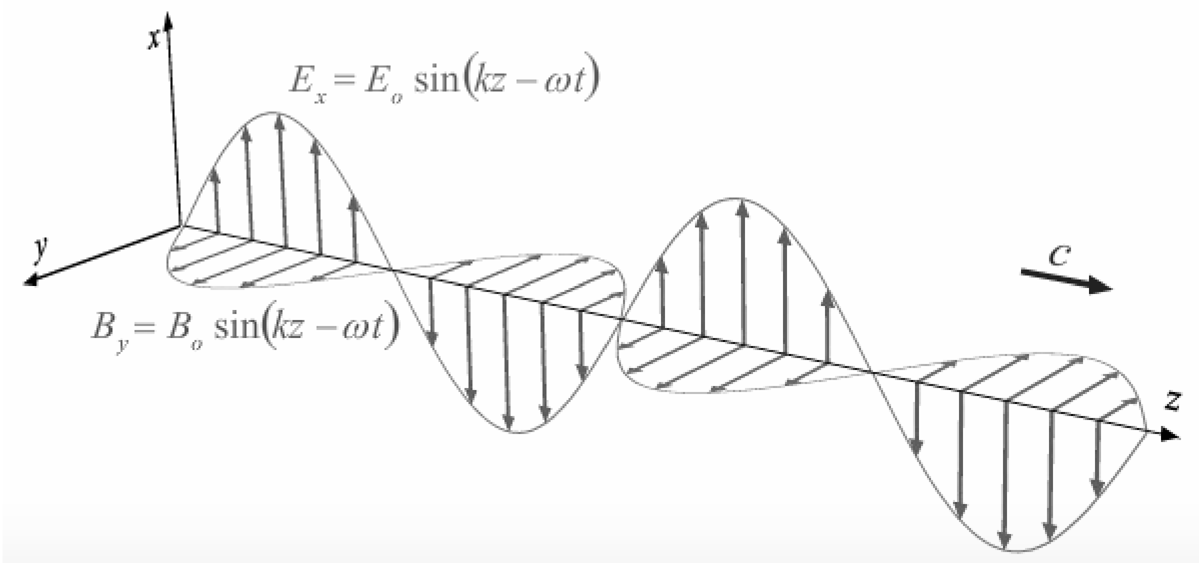
- **Partial polarization on reflection**
- **Part of the light is refracted (also polarized)**
- **Depends on refractive indices**
  - **Brewster's law**

Complete polarization for Brewster's angle

$$\tan\theta_b = \frac{n_2}{n_1}$$

when reflected and refracted light paths are perpendicular to one another

So far we have considered plane waves that look like this:



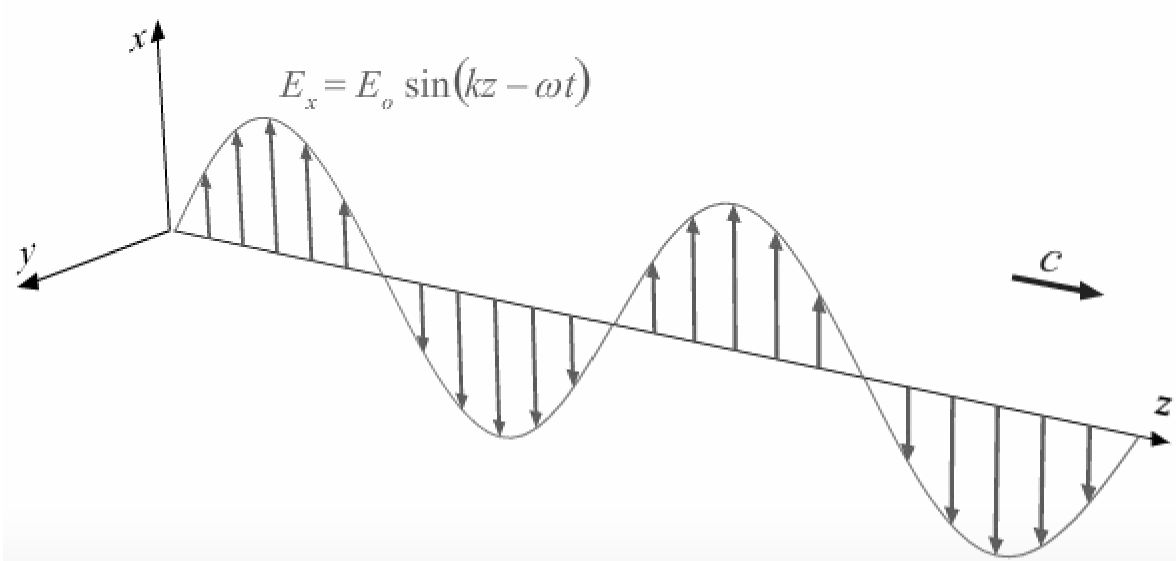
$$\omega = kc$$

$$E_o = cB_o$$

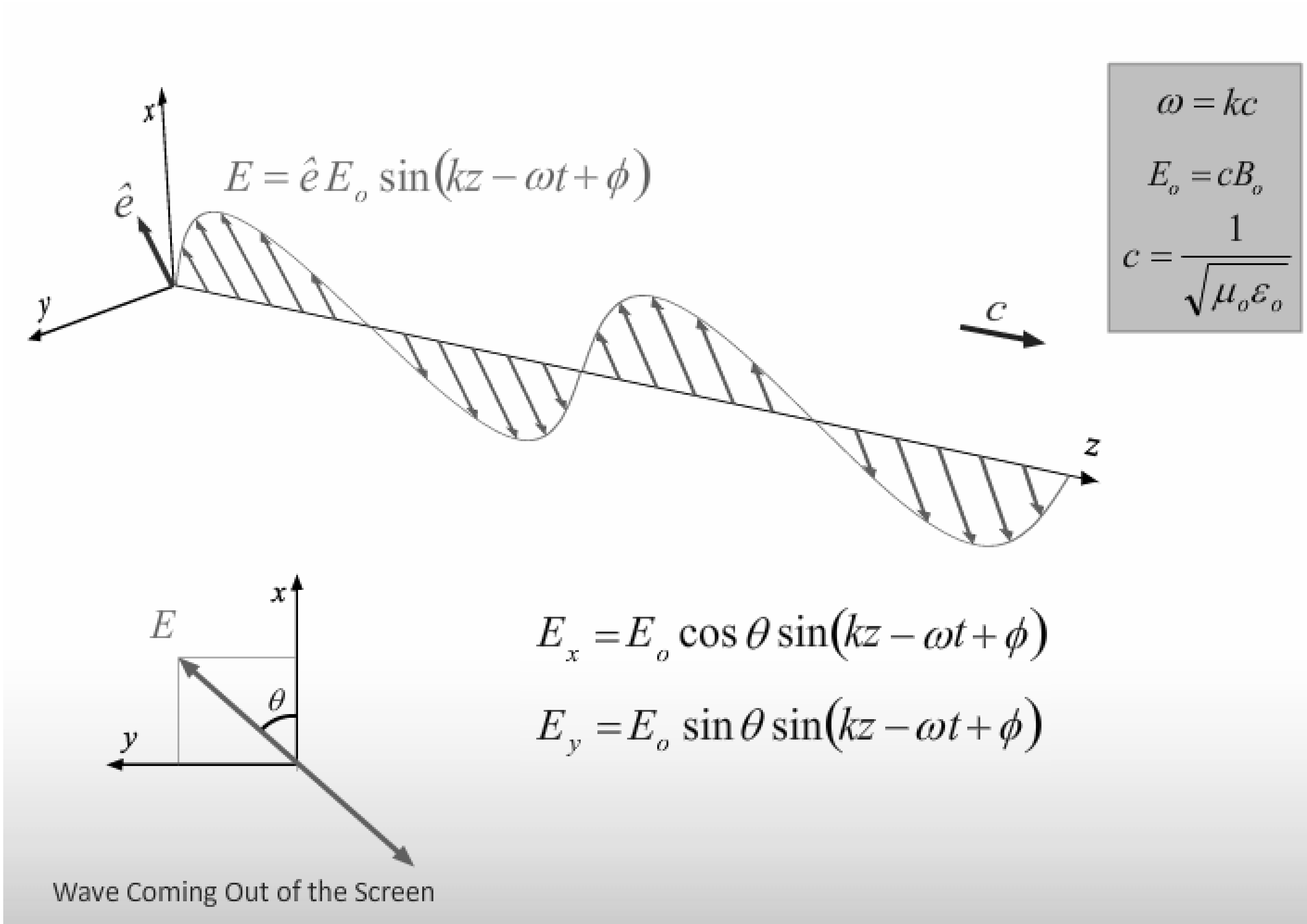
$$c = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$$

From now on just draw E and remember that B is still there:

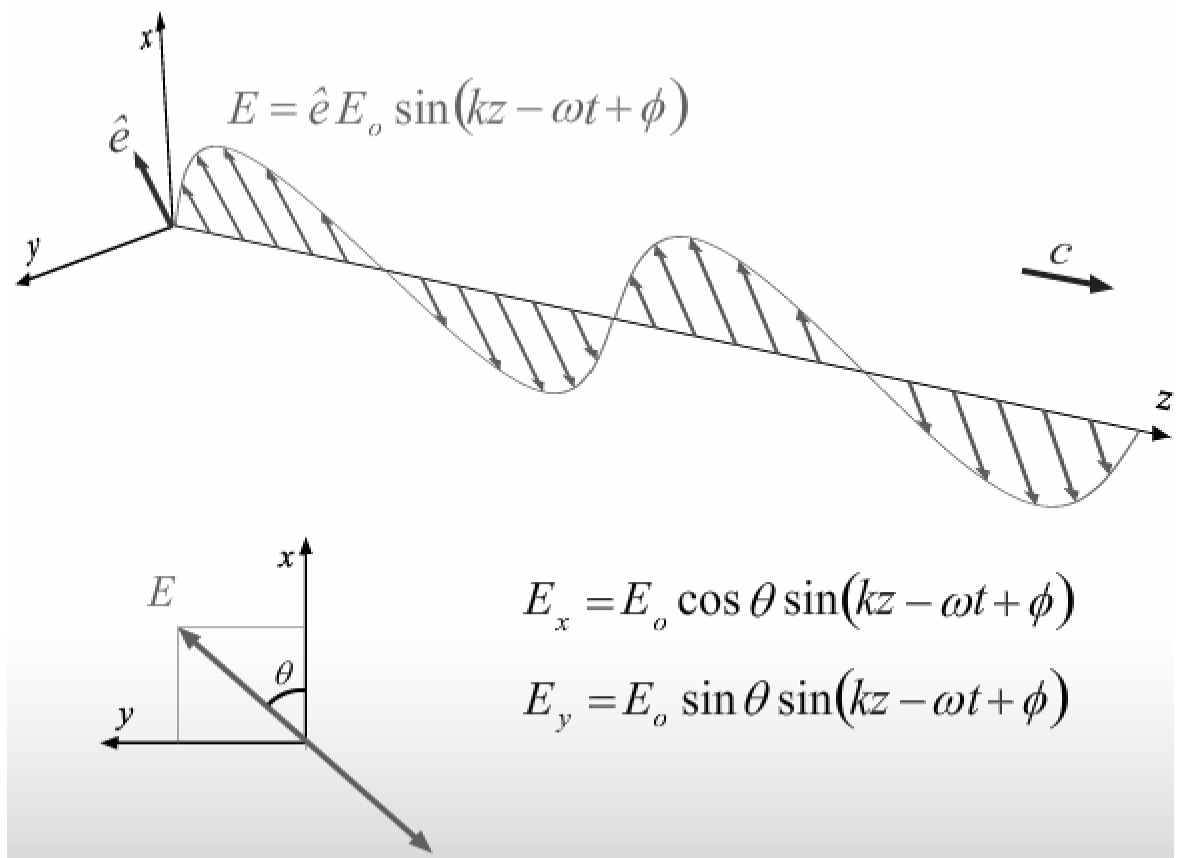
$\vec{E}$ Field determines Polarization



Linear Polarization



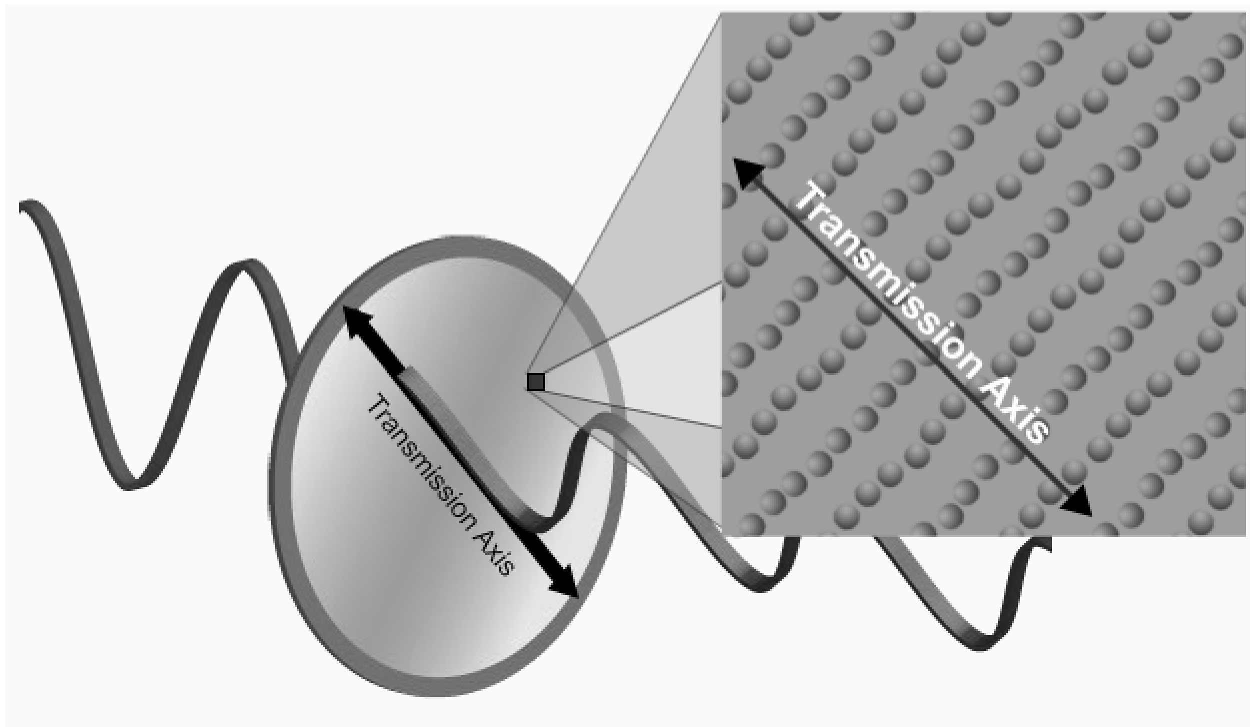
The intensity of a wave does not depend on its polarization:



$$
\begin{aligned}
I &= \varepsilon_0 c \left\langle E^2 \right\rangle \\
&= \varepsilon_0 c \left\langle E_x^2 + E_y^2 \right\rangle \\
&= \varepsilon_0 c \left[\left\langle E_x^2 \right\rangle + \left\langle E_y^2 \right\rangle \right] \\
&= \varepsilon_0 c E_0^2 \underbrace{\left\langle \sin^2(kz-\omega t-\phi) \right\rangle}_{1/2} \underbrace{(\sin^2\theta + \cos^2\theta)}_{1}
\end{aligned}
$$

So $I = \frac{1}{2}\varepsilon_0 c E_0^2$ Just like before

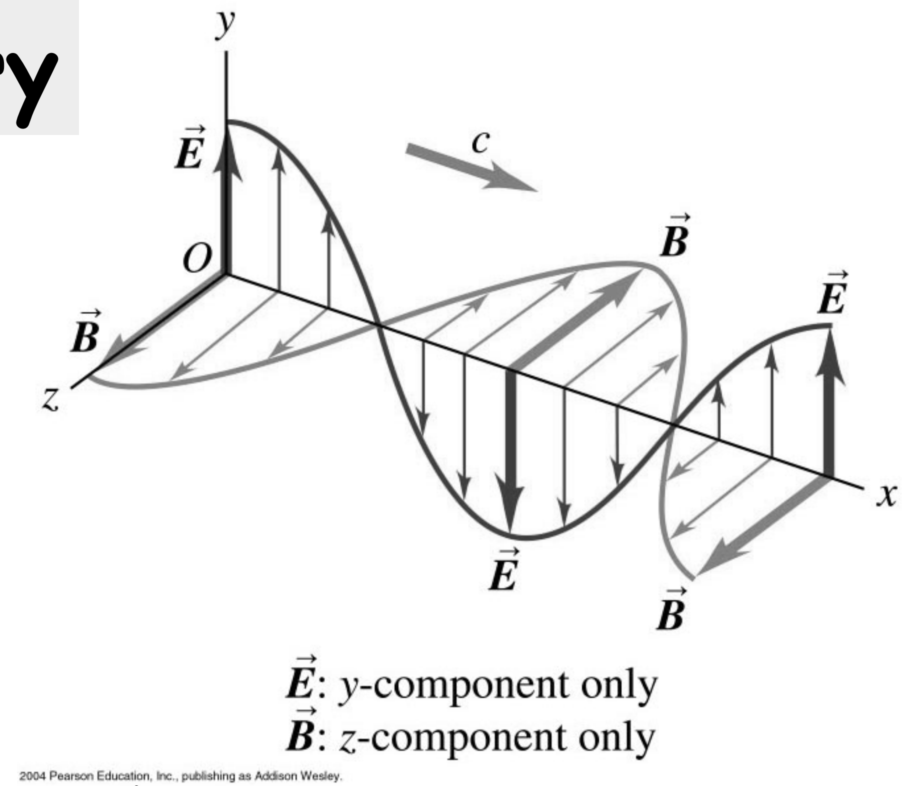
Polarizers



The molecular structure of a polarizer causes the component of the E field perpendicular to the Transmission Axis to be absorbed.

Polarization summary

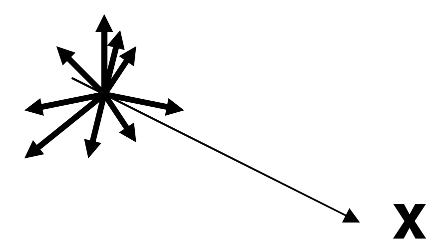
Consider an EM plane wave. The E field is polarized in the Y-direction. This is called "linearly Polarized light".



$$E_y = E_0 \sin(kx - \omega t)$$

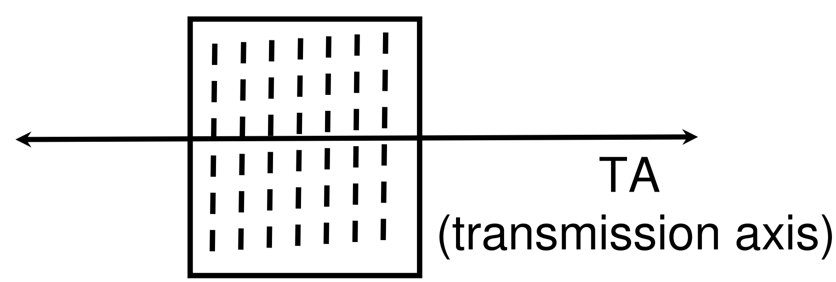
$$B_z = \frac{E_0}{c} \sin(kx - \omega t)$$

Most light sources are not polarized in a particular direction. They produce unpolarized light or radiation.



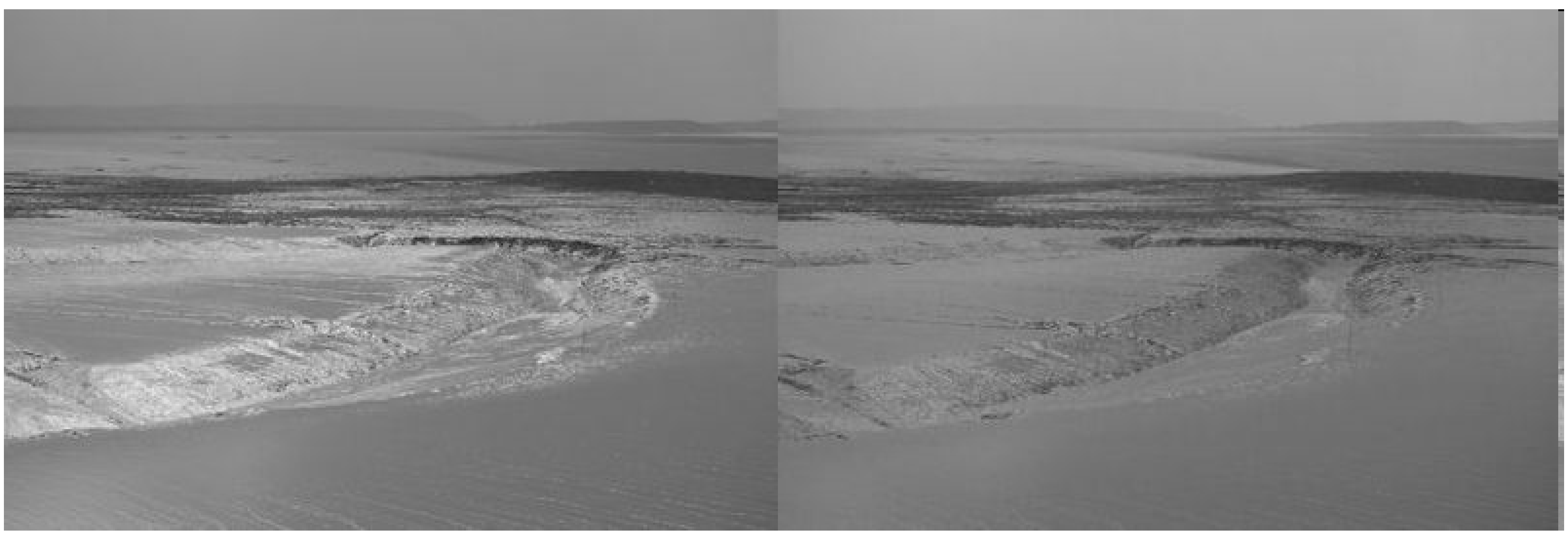
polaroid (sunglasses)

Long molecules absorb E-field parallel to molecule.

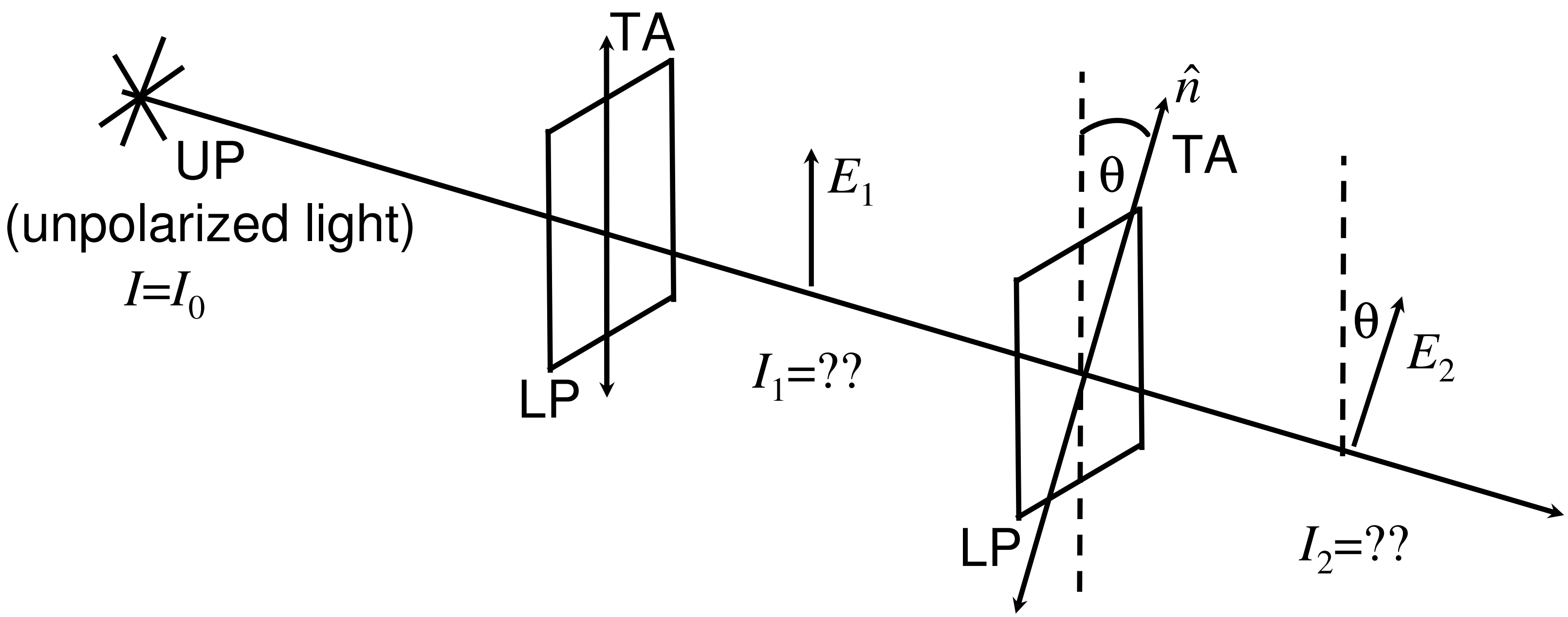


With and without a polarizing filter

Polarizers at 90 and 0 degrees



Linear Polarizers/Malus' Law



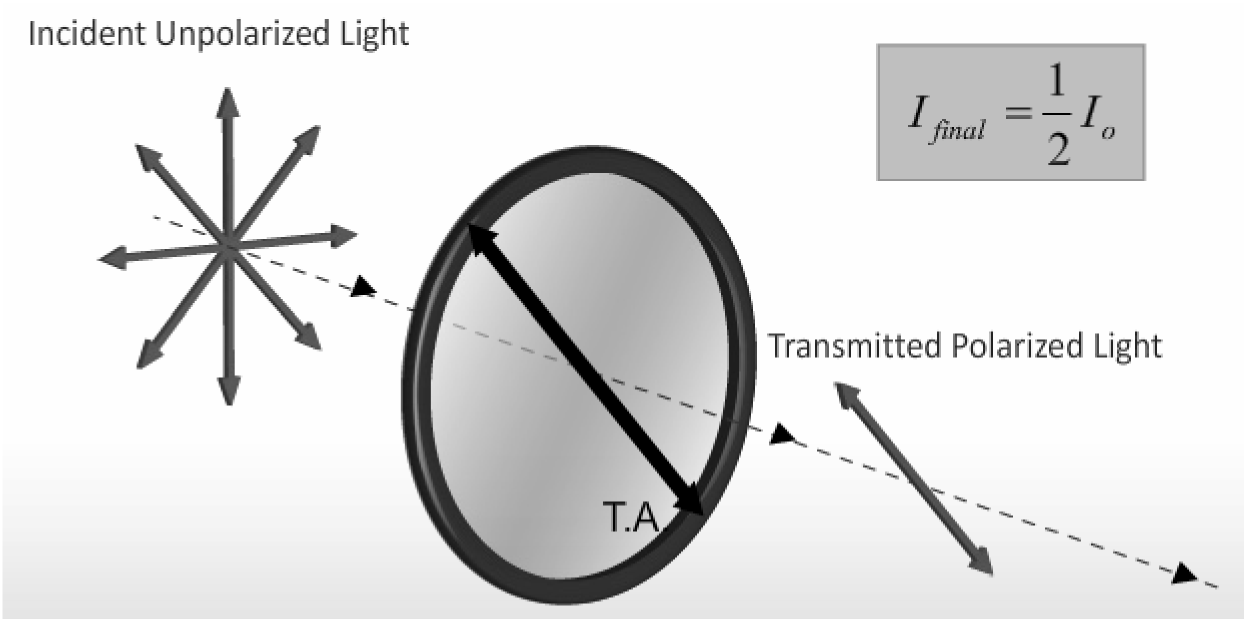
- **This set of two linear polarizers produces LP light. What is the final intensity?**
  - **First LP transmits 1/2 of the unpolarized light: $I_1 = 1/2\, I_0$**
  - **Second LP projects out the E-field component parallel to the TA:**

$$E_2 = E_1 \cos\theta$$

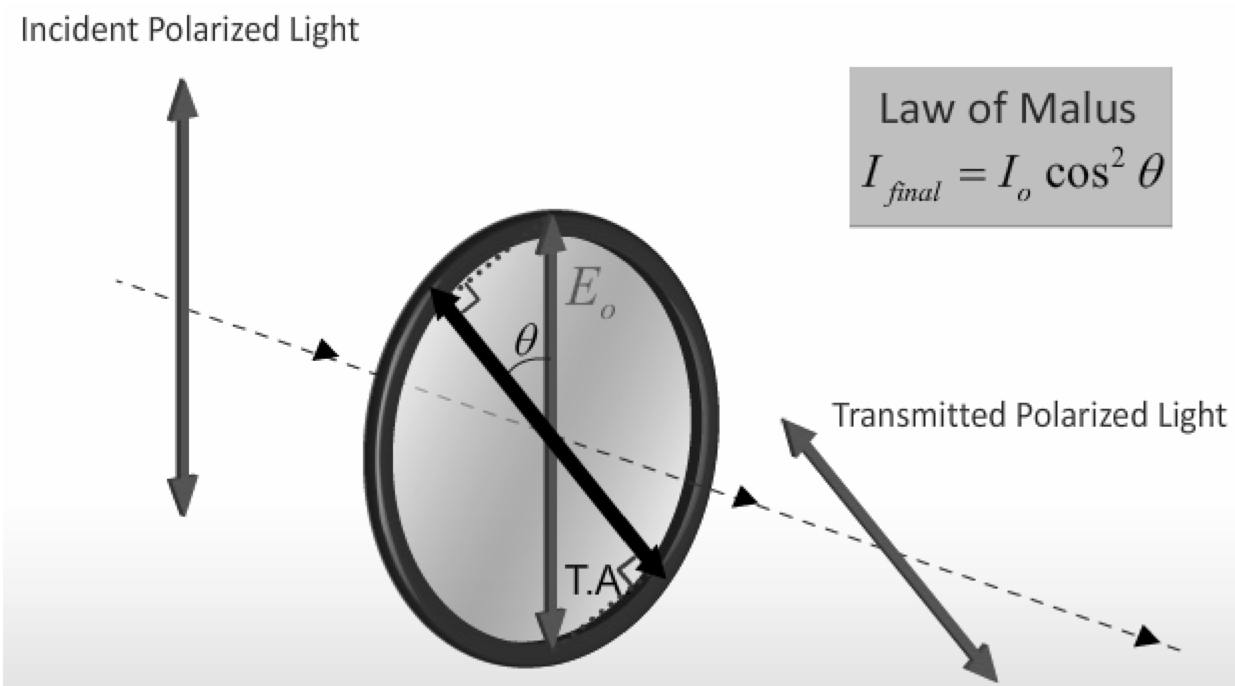
$$I \propto E^2$$

$$\Rightarrow \quad I_2 = I_1 \cos^2\theta$$

Law of Malus

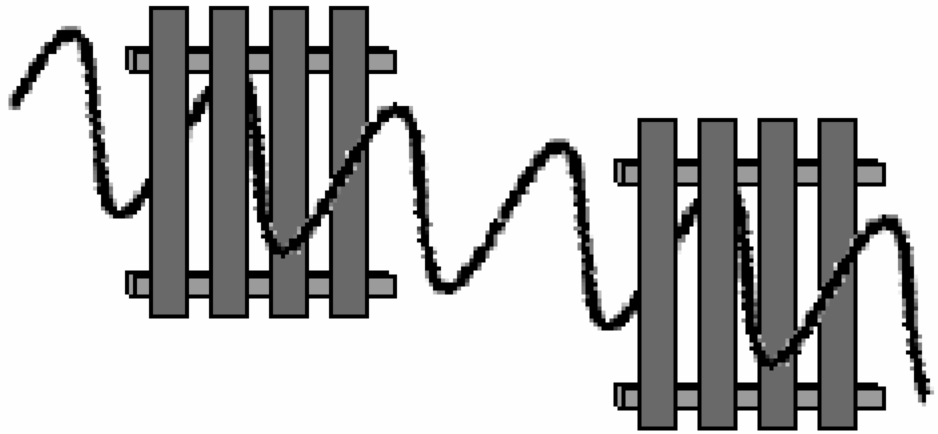


Incident Unpolarized Light
$I_{final} = \frac{1}{2} I_o$
Transmitted Polarized Light
T.A

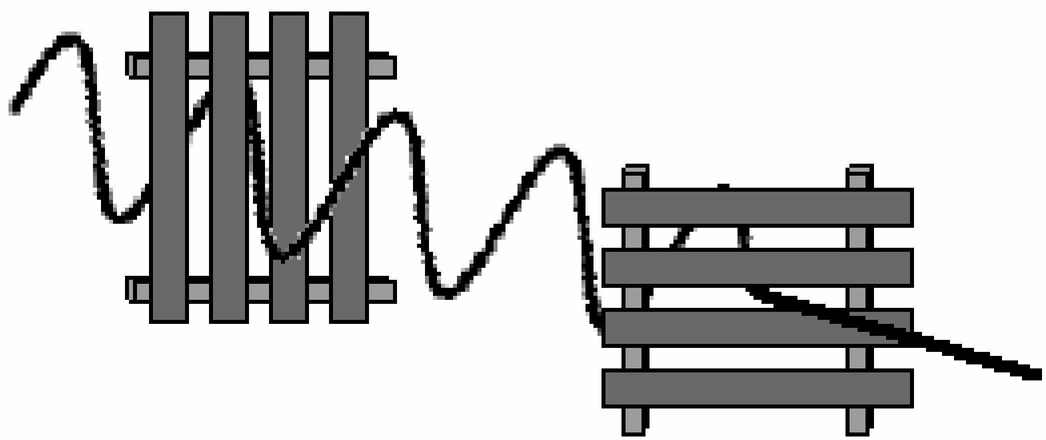


Incident Polarized Light
Law of Malus
$I_{final} = I_o \cos^2 \theta$
E_o
θ
Transmitted Polarized Light
T.A

The Picket Fence Analogy

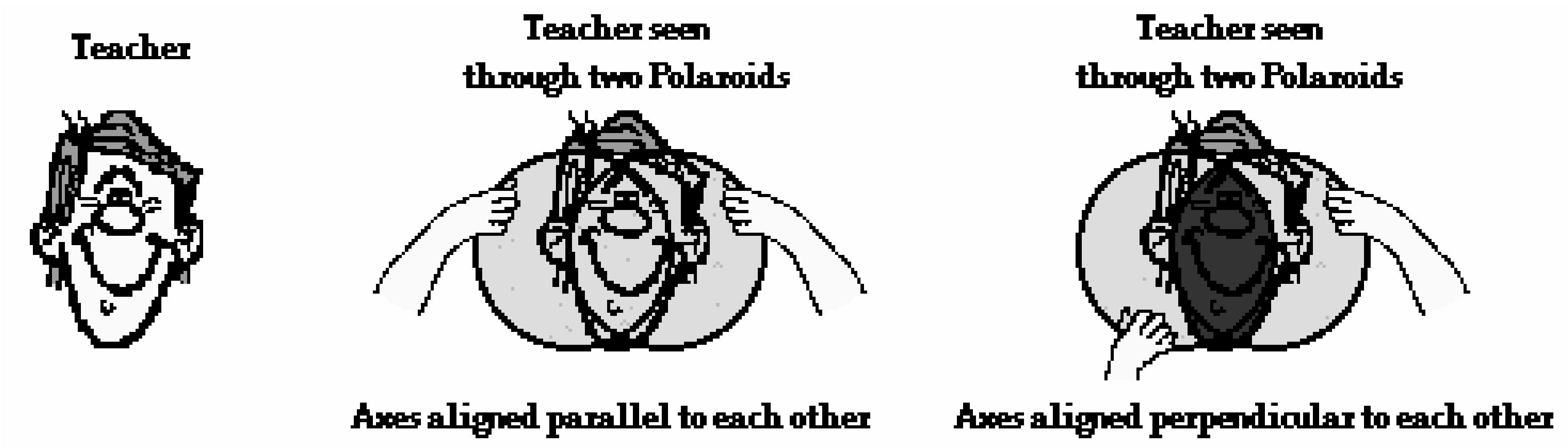


When the pickets of both fences are aligned in the vertical direction, a vertical vibration can make it through both fences.



When the pickets of the second fence are horizontal, vertical vibrations which make it through the first fence will be blocked.

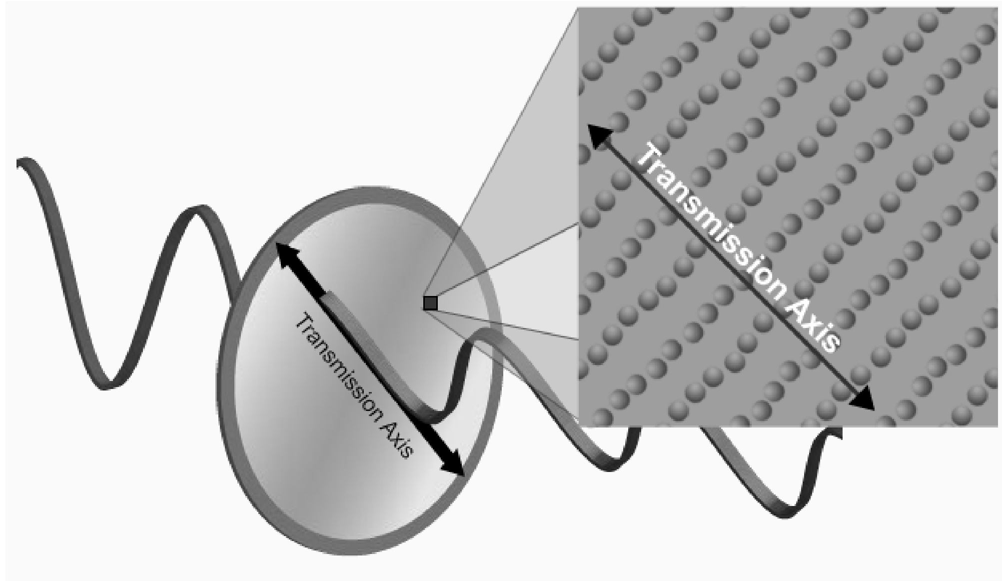
An example of Malus' Law



Parallel axes

Perpendicular axes

Clicker

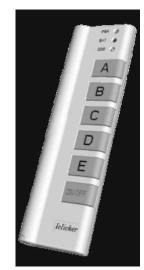


3) An EM wave is passed through a linear polarizer. Which component of the *E*-field is absorbed? The component of the *E*-field which is absorbed is ____________ .

a) perpendicular to the transmission axis

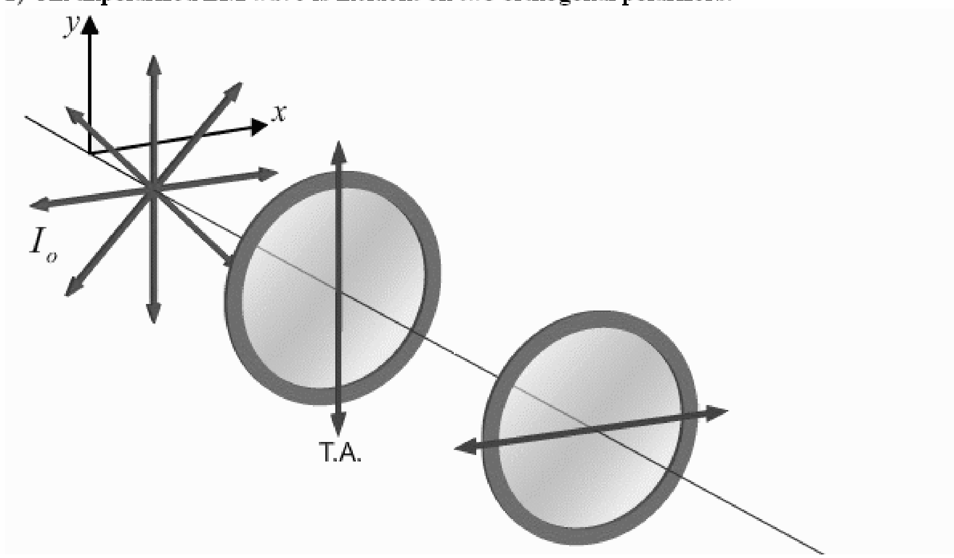
b) parallel to the transmission axis

c) both components are absorbed



Clicker

2) An unpolarized EM wave is incident on two orthogonal polarizers.



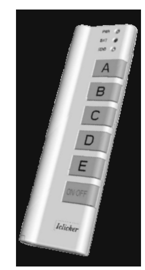
An EM wave polarized along the *y*-axis, is incident on two orthogonal polarizers.

I) What percentage of the intensity gets through both polarizers?

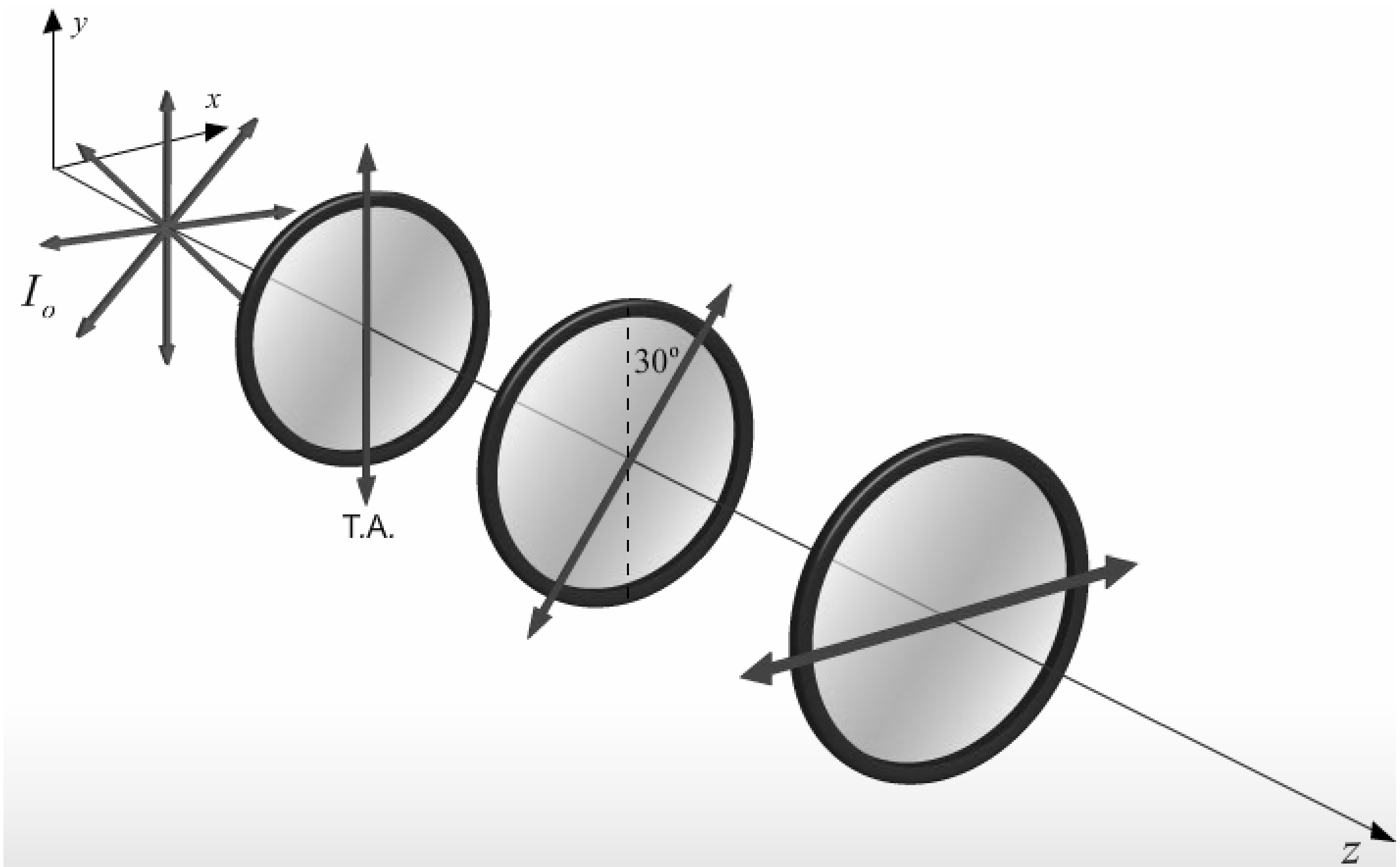
a) 50%

b) 25%

c) 0%



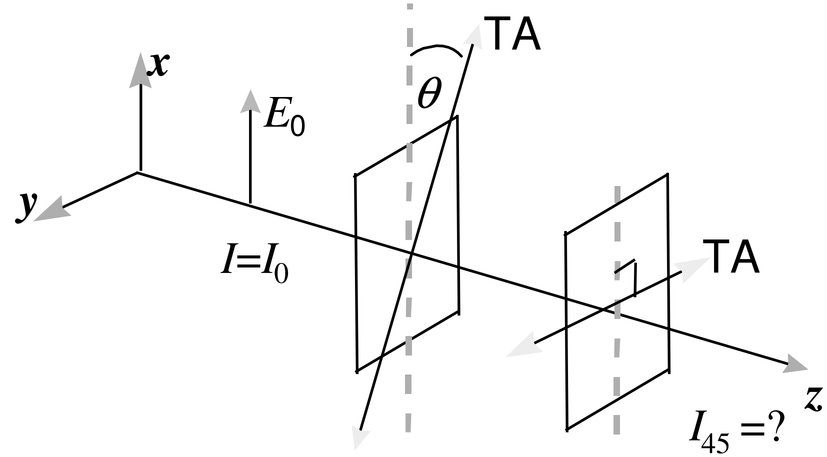
Q) Is it possible to increase this percentage by inserting another polarizer between the original two? [not a clicker problem (yet)]



Will any light be transmitted through the crossed polarizers ? Why or why not ?

Multi-part clicker

- **Light of intensity I_0, polarized along the *x* direction is incident on a set of 2 linear polarizers as shown.**

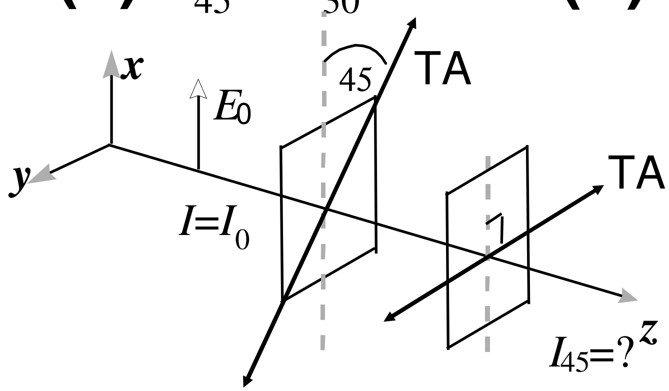


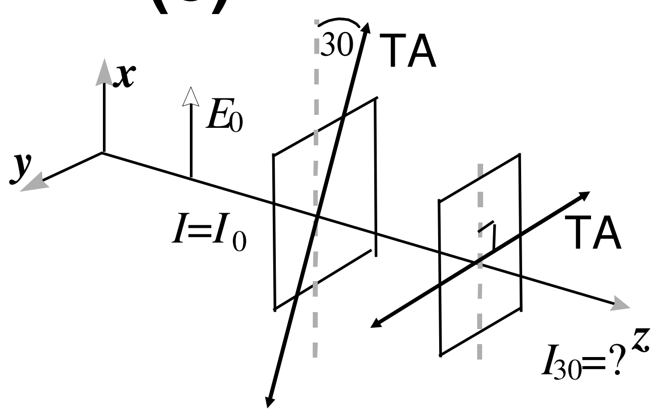
1A – **Assuming $\theta = 45°$, what is I_{45}, the intensity at the exit of the 2 polarizers, in terms of I_0?**

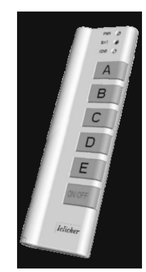
(a) $I_{45} = \frac{1}{2} I_0$ **(b)** $I_{45} = \frac{1}{4} I_0$ **(c)** $I_{45} = 0$

1B • **What is the relation between I_{45} and I_{30}, the final intensities in the situation above when the angle $\theta = 45°$ and $30°$, respectively?**

(a) $I_{45} < I_{30}$ **(b)** $I_{45} = I_{30}$ **(c)** $I_{45} > I_{30}$

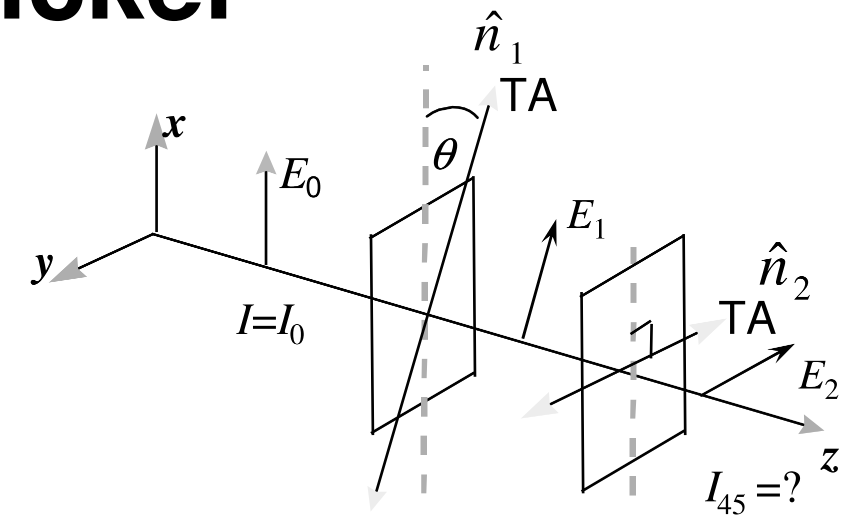






Multi-part clicker

- **Light of intensity I_0, polarized along the *x* direction is incident on a set of 2 linear polarizers as shown.**



1A **Assuming θ = 45°, what is I_{45}, the intensity at the exit of the 2 polarizers, in terms of I_0?**

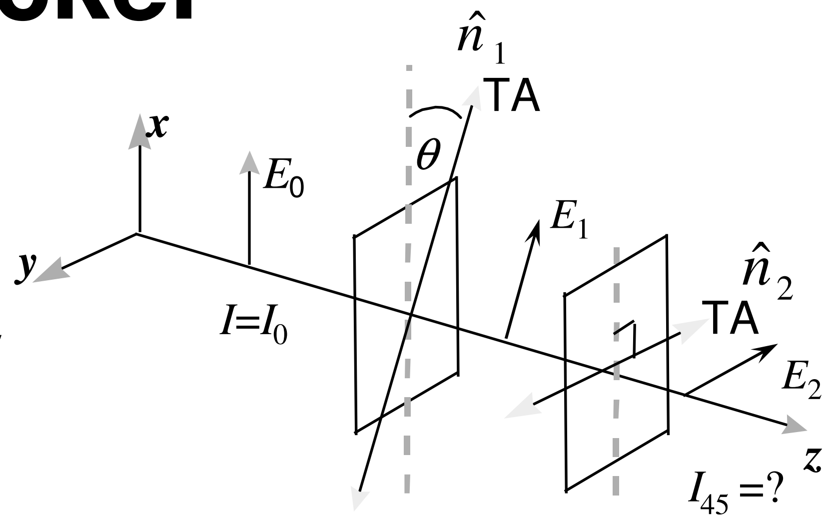
(a) $I_{45} = \frac{1}{2} I_0$ **(b)** $I_{45} = \frac{1}{4} I_0$ **(c)** $I_{45} = 0$

- **We proceed through each polarizer in turn.**
  - **The intensity after the first polarizer is:** $I_1 = I_0 \cos^2(45-0) = \frac{I_0}{2}$
  - **The electric field after the first polarizer is LP at θ_1 = 45.**
  - **The intensity after the second polarizer is:**

$$I_{45} = I_1 \cos^2(90-45) = \frac{I_1}{2} \Rightarrow I_{45} = \frac{1}{4} I_0$$

Multipart clicker

- **Light of intensity I_0, polarized along the *x* direction is incident on a set of 2 linear polarizers as shown.**



1A **Assuming θ = 45°, what is the relation between the I_{45}, the intensity at the exit of the 2 polarizers, in terms of I_0?**

(a) $I_{45} = \frac{1}{2} I_0$ **(b)** $I_{45} = \frac{1}{4} I_0$ **(c)** $I_{45} = 0$

1B **• What is the relation between I_{45} and I_{30} , the final intensities in the situation above when the angle $\theta = 45°$ and $30°$, respectively?**

(a) $I_{45} < I_{30}$ **(b)** $I_{45} = I_{30}$ **(c)** $I_{45} > I_{30}$

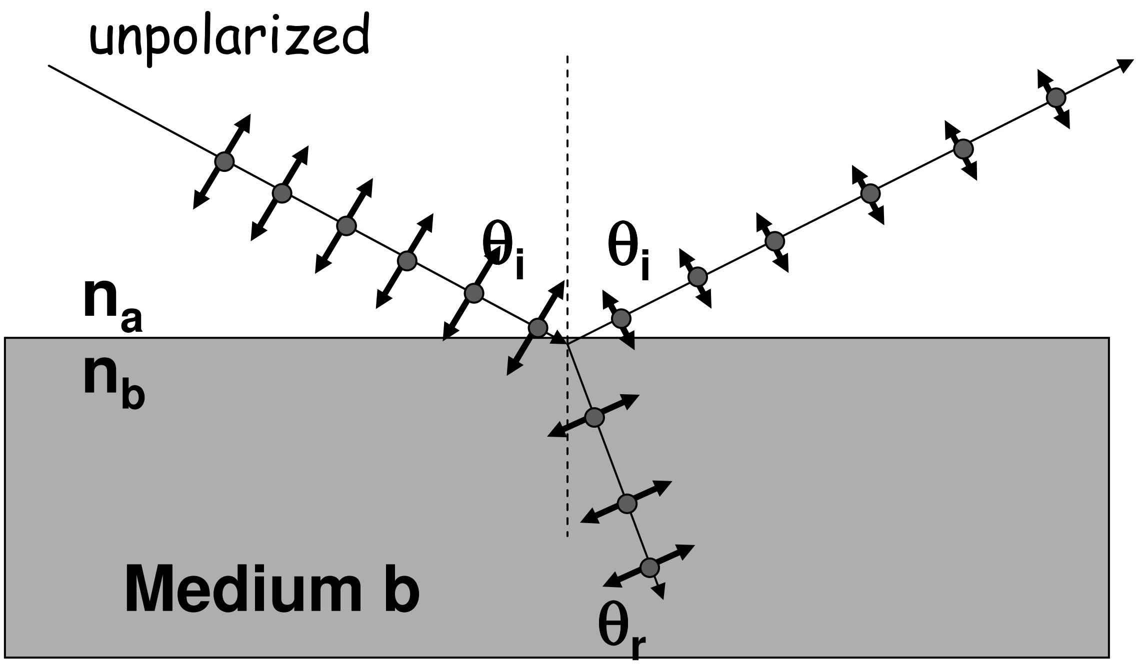
• In general, the first polarizer reduces the intensity by $\cos^2\theta$, while the second polarizer reduces it by an additional factor of $\cos^2(90 - \theta)$.
• Thus, the final output intensity is given by:

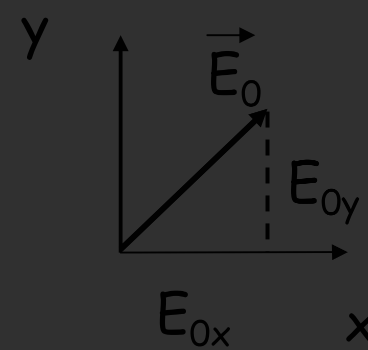
$$I_{30} = I_0 \cos^2(30) \cos^2(90 - 30) = 0.1875$$

In general: $$I_{out} = I_0 \cos^2(\theta) \cos^2(90 - \theta) = I_0 \cos^2(\theta) \sin^2(\theta) \propto \sin^2(2\theta)$$

This has a maximum when θ = 45°.

Polarization by reflection

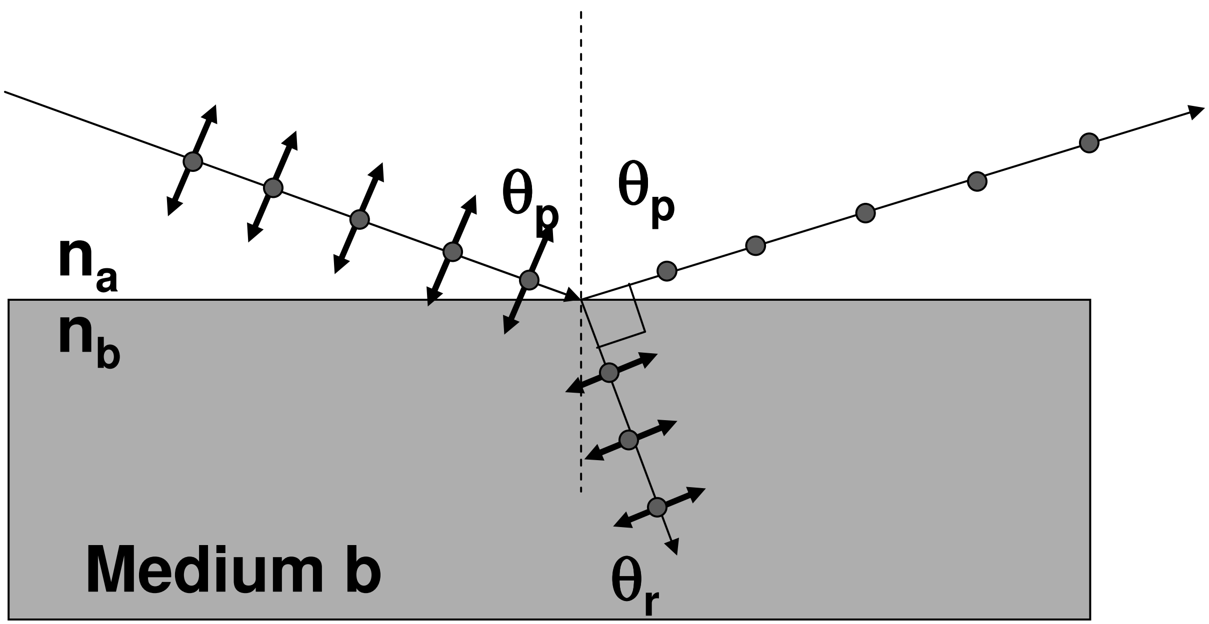




Can always describe E in terms of components in two arbitrary directions. The components are equal for unpolarized light.

The reflected rays are partially polarized in the horizontal plane. The transmitted rays are also partially polarized.

Polarization by reflection

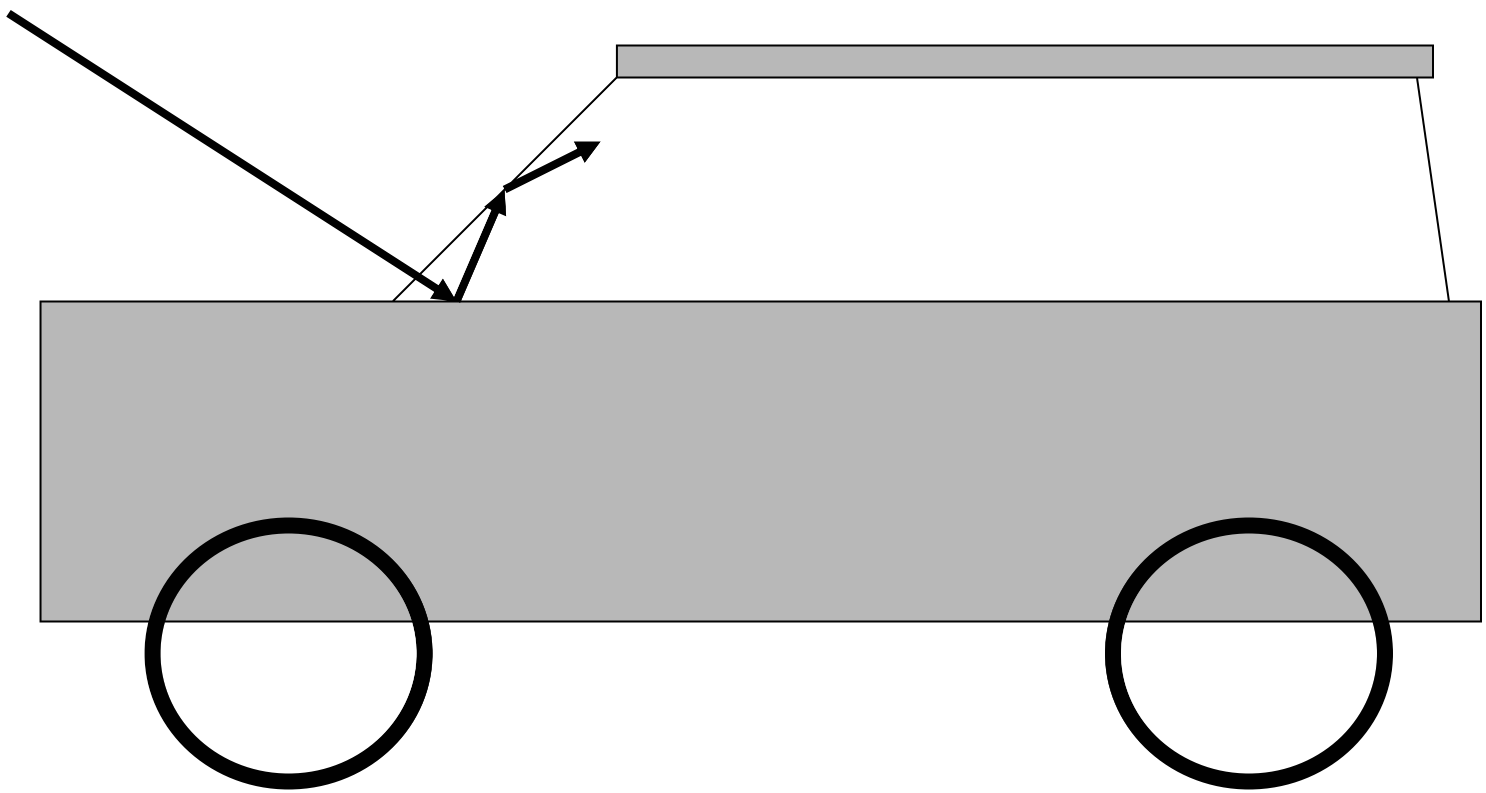


For a certain angle, the Brewster angle, the reflected light is completely polarized in the horizontal plane. This occurs when the angle between the refl. and refr. rays is 90°.

From Maxwell's eqn. it can be shown that Brewster's angle is given by

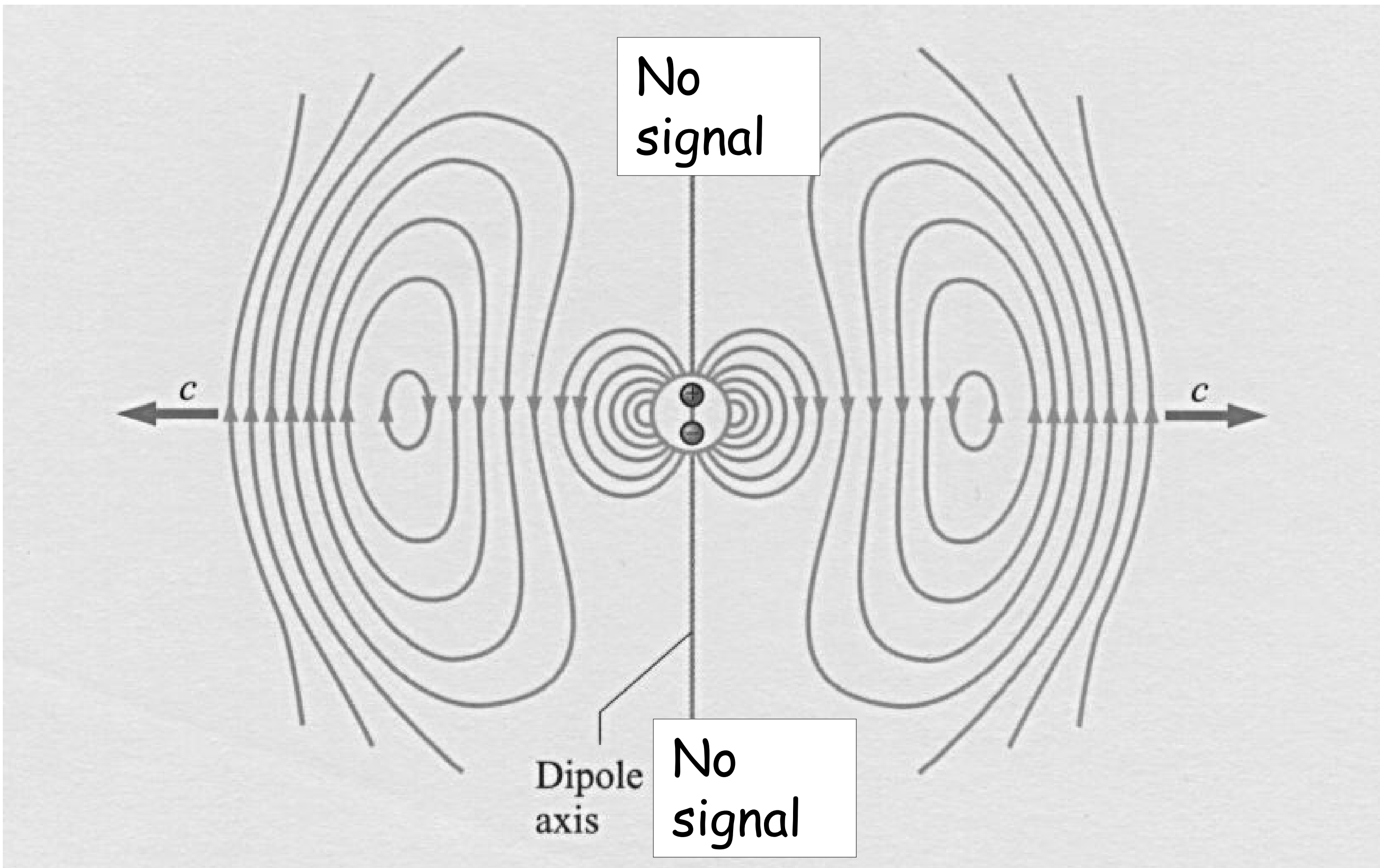
$$\tan\theta_p = \frac{n_b}{n_a}$$

Light reflected on dashboard to the windshield will be polarized in the horizontal plane. Using polaroid dark glasses with a vertical polarization axis will remove most of reflected light (glare).



Electric field lines from oscillating dipole

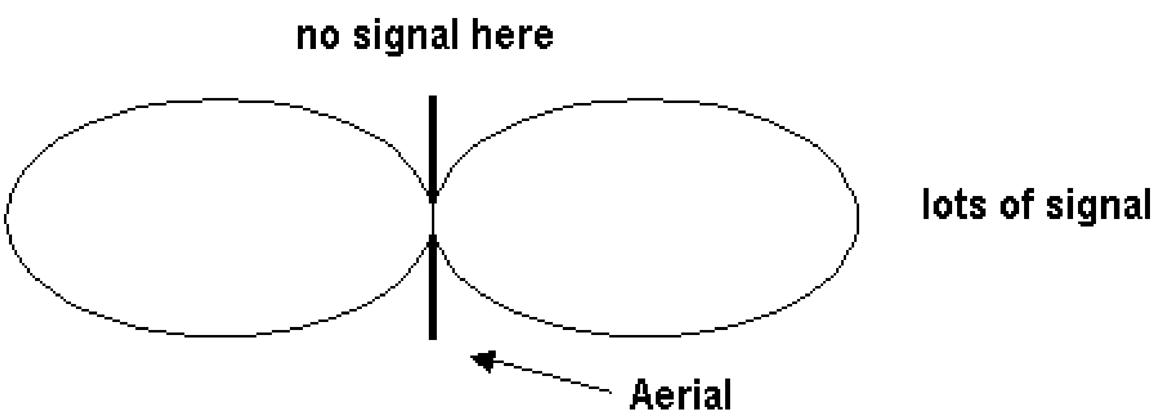
full computer simulation - a snapshot in time

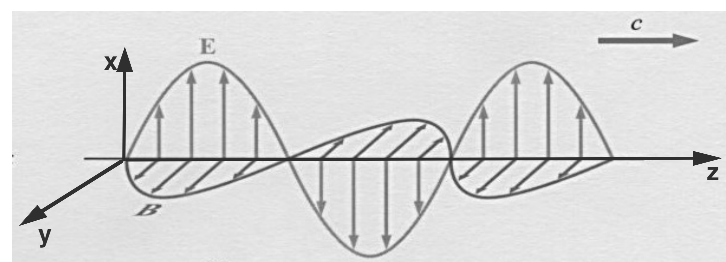


This experiment was done by Hertz with radio waves in the 19th century

Dipole radiation pattern

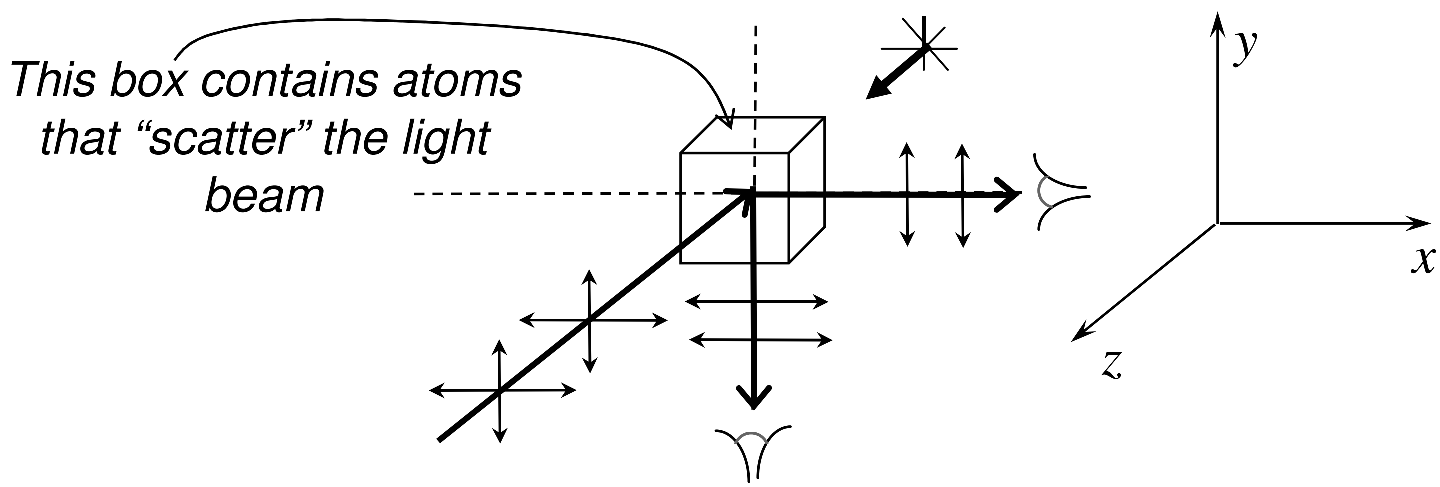
- Oscillating electric dipole generates e-m radiation that is **linearly polarized** in the direction of the dipole
- Radiation pattern is doughnut shaped & outward traveling
  - zero amplitude above and below dipole ($\sin^2\Theta$)
  - maximum amplitude in-plane





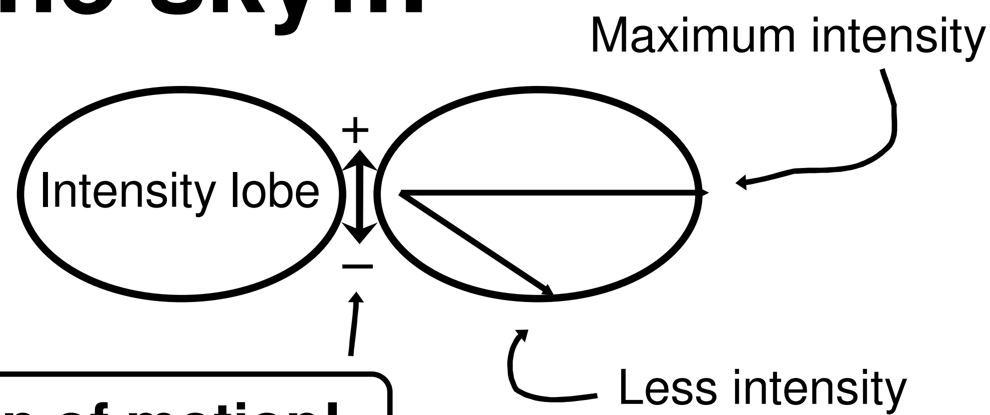
Polarization by Scattering

- Suppose unpolarized light encounters an atom and scatters (energy absorbed & reradiated).
  - What happens to the polarization of the scattered light?
  - The scattered light is preferentially polarized perpendicular to the plane of the scattering.
    - » For example, assume the incident unpolarized light is moving in the z-direction.
    - » Scattered light observed along the x-direction (scattering plane = x-z) will be polarized along the y-direction.
    - » Scattered light observed along the y-direction (scattering plane = y-z) will be polarized along the x-direction.



Polarization of the sky…

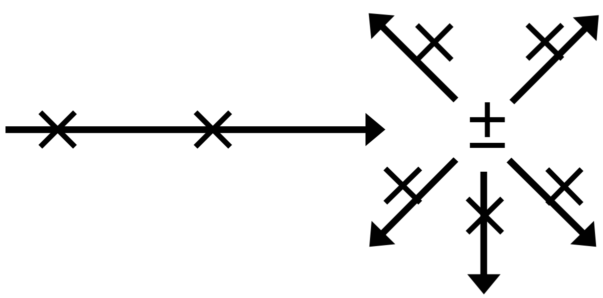
In the atmosphere light is radiated/scattered by atoms – oscillating electric dipoles.



***No* radiation along direction of motion!**

Start with sunlight with all polarizations & randomly oriented dipoles.

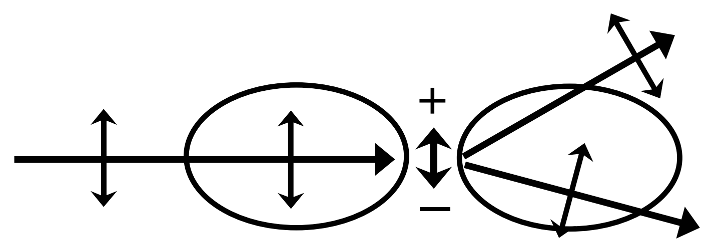
<u>2 cases</u>:



Dipole oscillates *into* the paper.

Horizontal dipoles reradiate H-polarized light downward.

(Do not respond to incident V-light.)



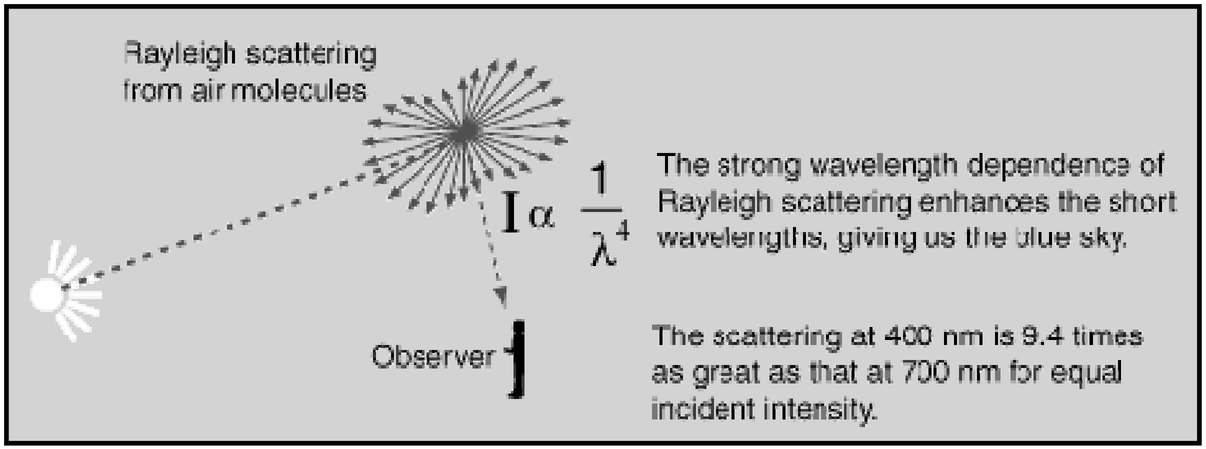
Dipole oscillates vertically.

Vertical dipoles reradiate V-polarized light to the sides (*not* downward).

(Do not respond to incident H-light.)

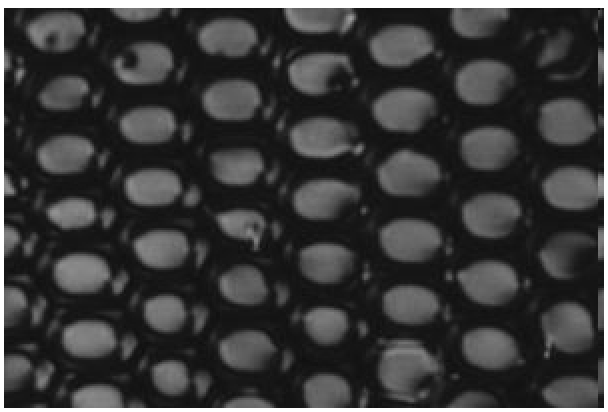
Why is the sky blue?

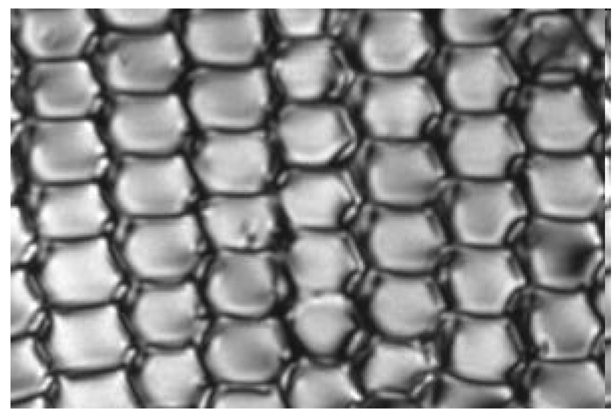
- Light from Sun scatters off of air particles–"Rayleigh scattering"
  - – Rayleigh scattering is wavelength-dependent.
  - – Shorter wavelengths (blue end of the visible spectrum) scatter more.



- This is also why sunsets are red!
  - – At sunset, the light has to travel through more of the atmosphere.
  - – If longer wavelengths (red and orange) scatter *less*…
  - – The more air sunlight travels through, the redder it will appear!
  - – This effect is more pronounced if there are more particles in the atmosphere (e.g., sulfur aerosols from industrial pollution).

Pixels in a LCD (Liquid Crystal Display)



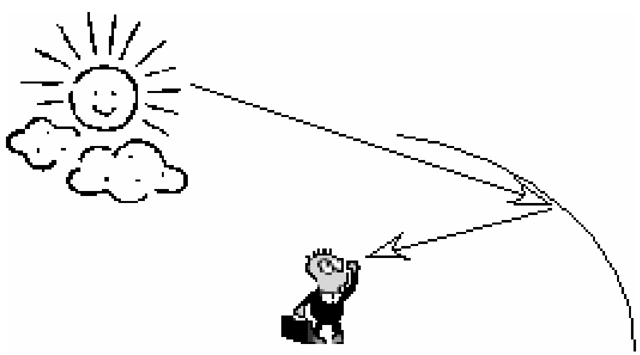


Apply an electric field to change the orientation of pixels and block or transmit polarized light

Applications

- **Sunglasses**
  - **The reflection off a horizontal surface (e.g., water, the hood of a car, etc.) is strongly polarized. Which way?**
  - **A perpendicular polarizer can preferentially reduce this glare.**

- **Polarized sky**
  - **The same argument applies to light scattered off the sky:**



Polarizing filters important in photography!

More Polarizations

General linear polarization state: $\vec{E} = \underbrace{(\cos\theta\,\hat{x} + \sin\theta\,\hat{y})E_0}_{\equiv \vec{E}_0}\ \sin(kz - \omega t)$

if $\theta = 45°$ $\quad \vec{E}_0 = \dfrac{\hat{x} + \hat{y}}{\sqrt{2}}\ E_0$

What if instead we had $\vec{E}_0 = \dfrac{\hat{x} - \hat{y}}{\sqrt{2}}\ E_0$ **?** ⇨ **Polarized at** $-45°$**.**

Another way to write these:

$$\vec{E}_{45} = E_0\,\hat{x}\sin(kz - \omega t) + E_0\,\hat{y}\sin(kz - \omega t)$$

$$\vec{E}_{-45} = E_0\,\hat{x}\sin(kz - \omega t) + E_0\,\hat{y}\sin(kz - \omega t + \pi)$$

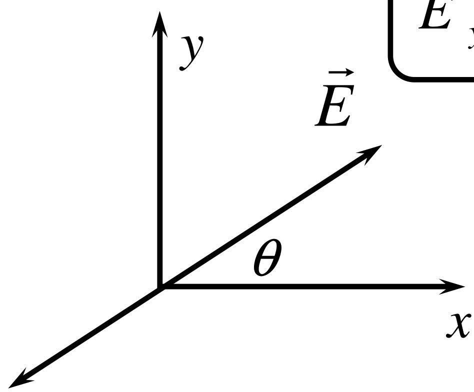
So the only difference is a phase shift of π **between** E_{0x} **and** E_{0y} **.**

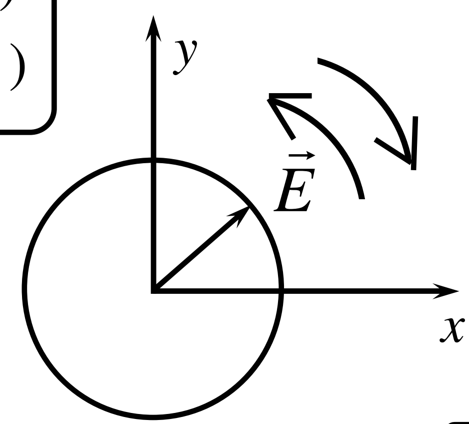
In general, this phase shift can take other values!

Other Polarization States?

- **Are there polarizations other than linear?**
  - **Sure!!**
  - **The general harmonic solution for a plane wave traveling in the +z-direction is:**

$$E_x = E_{x0}\sin(kz - \omega t + \phi_x)$$
$$E_y = E_{y0}\sin(kz - \omega t + \phi_y)$$





Linear Polarization ⇨

$$\phi \equiv \phi_x - \phi_y = 0$$

$$\frac{E_{yo}}{E_{xo}} = \tan\theta$$

Circular Polarization ⇨

$$\phi \equiv \phi_x - \phi_y = \pm\frac{\pi}{2}$$

$$E_{yo} = E_{xo}$$

(E_{0x} and E_{0y} are $\pm 90°$ out of phase.)

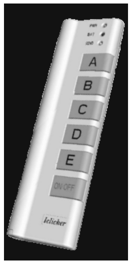
Clicker

- **What is the polarization of an electromagnetic wave whose *E* vector is described as:**

$$E_x = -E_0 \cos(kz - \omega t)$$

$$E_y = E_0 \sin(kz - \omega t)$$

(a) linear **(b) circular** **(c) elliptical**



Clicker

- **What is the polarization of an electromagnetic wave whose *E* vector is described as:**

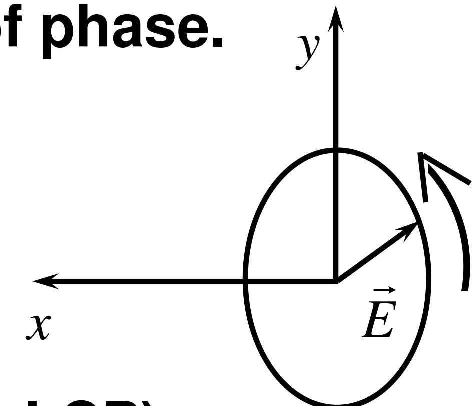
$$E_x = -E_0 \cos(kz - \omega t)$$

$$E_y = E_0 \sin(kz - \omega t)$$

(a) linear **(b) circular** **(c) elliptical**

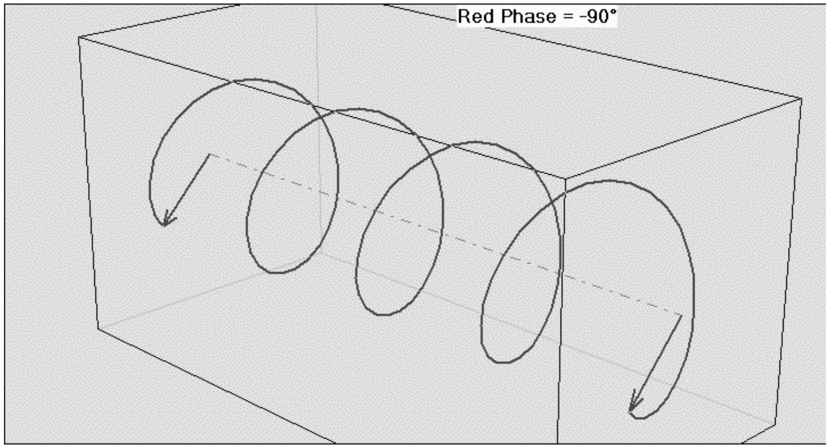
Not linear because and are out of phase.

It would be "elliptical" if :



The correct answer is circular (actually LCP).

Visualization



- Why do we call this circular polarization?

•Basis for space vehicle antenna designs.

There is no vertical or horizontal in space (polarization), but there is direction of travel and helicity.

Direction is well defined.

Helicity – right-handed or left-handed circular polarization. That is a well-defined polarization in space.

Example: Wave Plates

- **Birefringent crystals with precise thicknesses**

  Ex.: Crystal which produces a phase change of π/2 → "quarter wave plate" (a "full wave plate" produces a relative shift of $4\times\frac{\pi}{2}=2\pi$ → no effect).

Light polarized along the fast or slow axis merely travels through at the appropriate speed → polarization is unchanged.

Light linearly polarized at $45°$ to the fast or slow axis will acquire a relative phase shift between these two components → alter the state of polarization.

The phase of the component along the fast axis is π/2 out of phase with the component along the slow axis. E.g.,

Before QWP ⇨

$$E_x = E_0 \sin(kz - \omega t)$$
$$E_y = E_0 \sin(kz - \omega t)$$

After QWP ⇨

$$E_x = E_0 \sin(kz - \omega t)$$
$$E_y = E_0 \sin(kz - \omega t - \frac{\pi}{2})$$

Quarter Wave Plate summary:
- linear along fast axis → linear
- linear at $45°$ to fast axis → circular
- circular → linear at $45°$ to fast axis

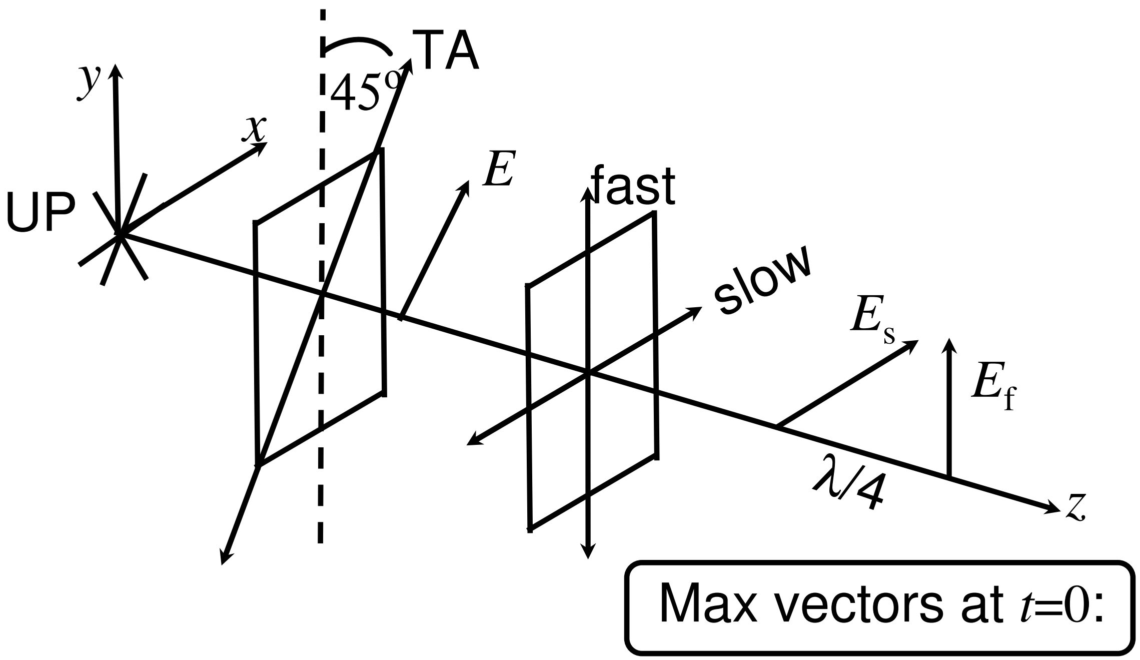
Quarter Wave Plates

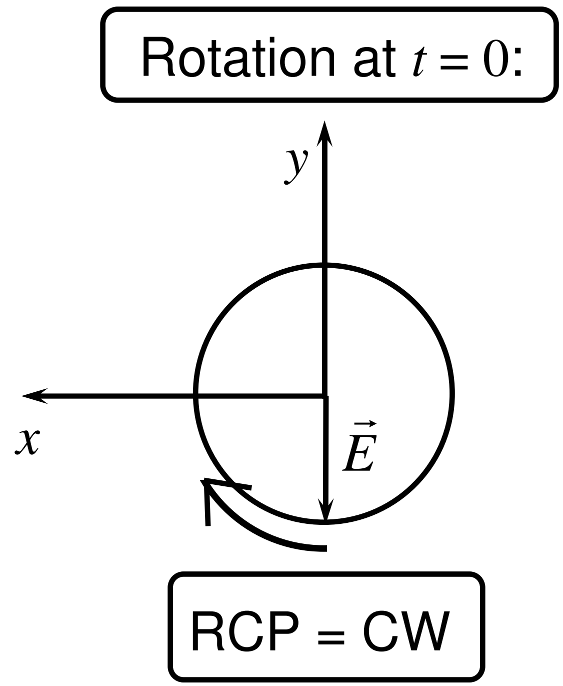
- **Light linearly polarized at 45° incident on a quarter wave plate produces the following wave after the quarter wave plate:**

Fast axis: $E_y = -E_0 \cos(kz - \omega t)$

Slow axis: $E_x = E_0 \sin(kz - \omega t)$

RCP

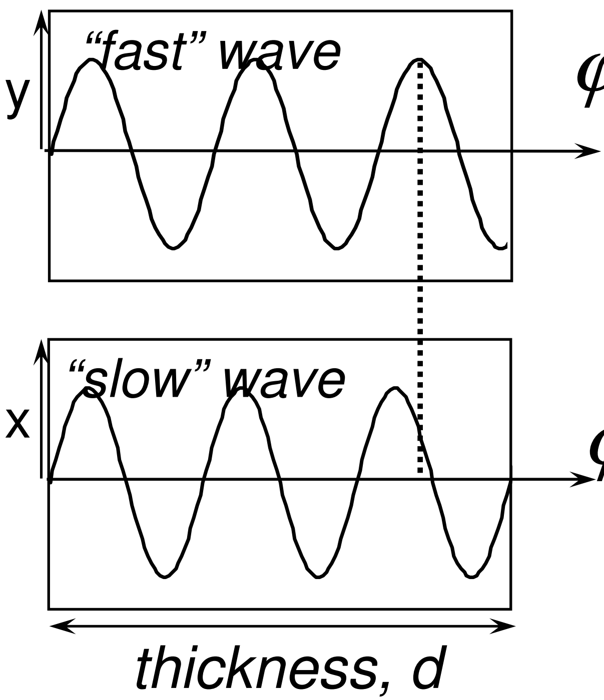




QWP: fast ahead of slow by λ/4)

Birefringence

- How can we make polarizations other than linear, e.g., circular?
  - Birefringence!
    - » Birefringent materials (e.g., crystals or stressed plastics) have the property that the speed of light is different for light polarized in the two transverse dimensions (polarization-dependent speed), i.e.,
      - light polarized along the "fast axis" propagates at speed v_{fast}
      - light polarized along the "slow axis" propagates at speed v_{slow}
    - » Thus, if the "fast" and "slow" polarizations start out in phase, inside the birefringent material the "fast" polarization will 'pull ahead':



$$\varphi_{fast} = \omega t_{fast} = \omega \, d/v_{fast}$$

$$\varphi_{slow} = \omega t_{slow} = \omega \, d/v_{slow}$$

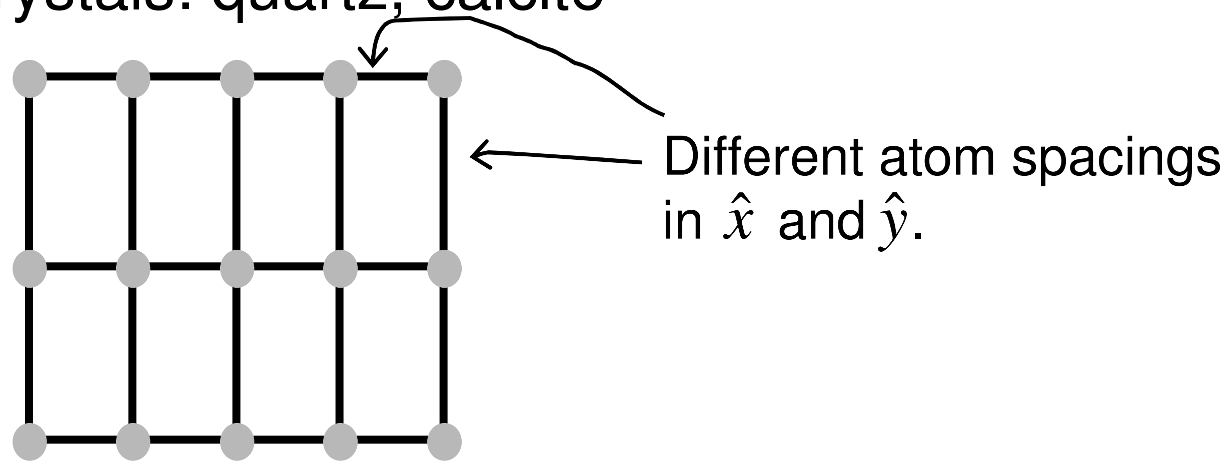
$$\varphi_{fast} - \varphi_{slow} = \omega d \left(\frac{1}{v_{fast}} - \frac{1}{v_{slow}} \right)$$

For a given birefringent material, the relative phase is determined by the thickness d, and frequency ω.

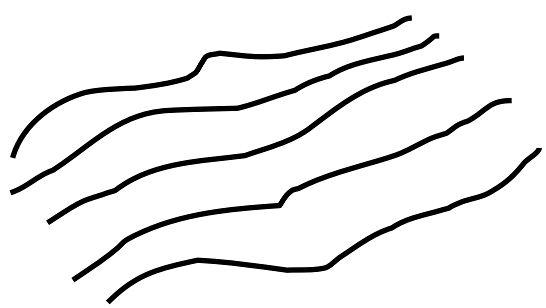
What Causes Birefringence?

Birefringence can occur in any material that possesses some asymmetry in its structure, so that the material is more “springy” in one direction than another.

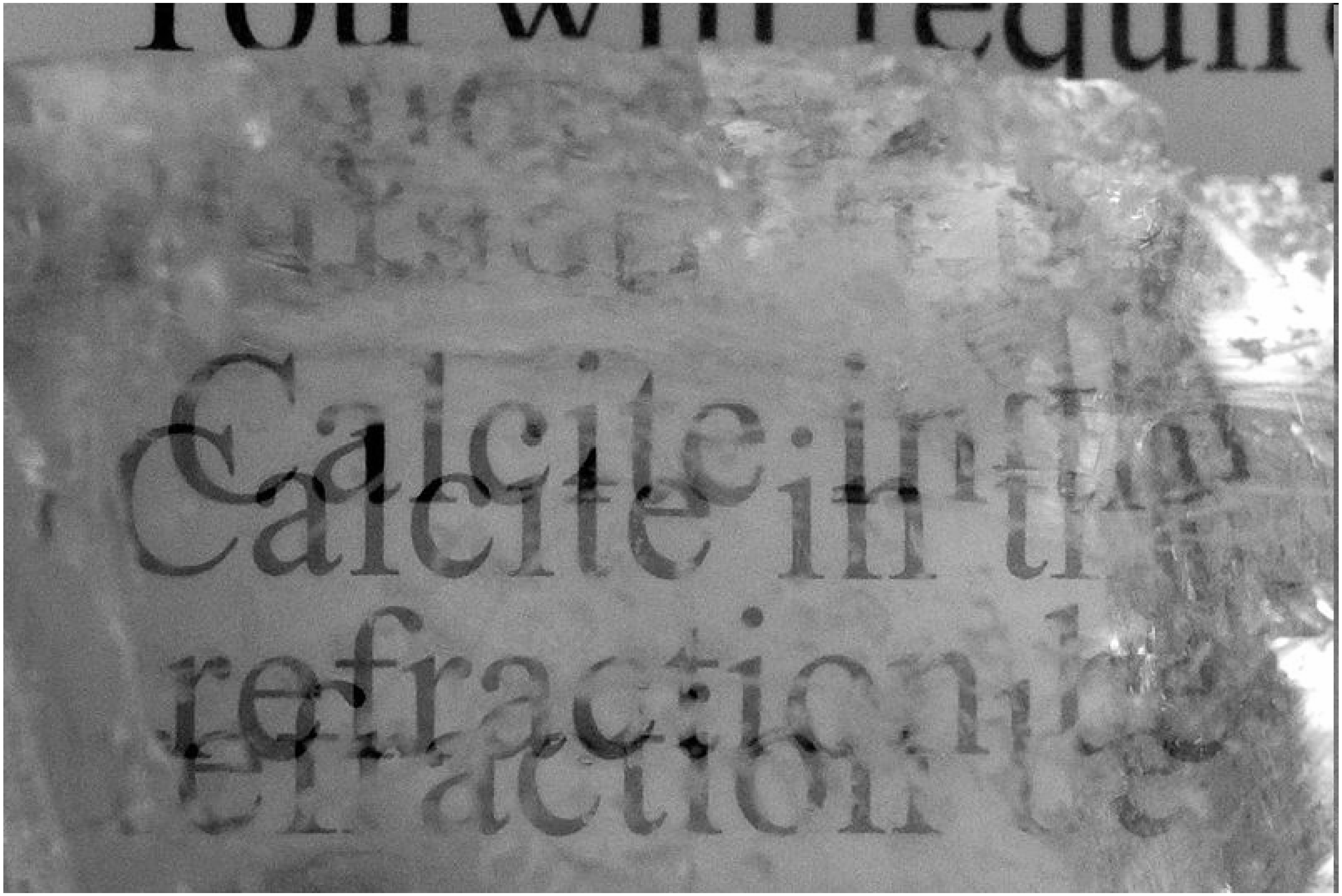
Examples: Crystals: quartz, calcite



Long stretched molecular chains: saran wrap, cellophane tape

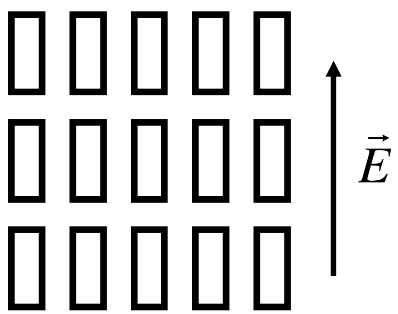


Birefringence in Calcite (double refraction)



Birefringence, cont.

Oblong molecules: “liquid crystals” *(the killer app)*

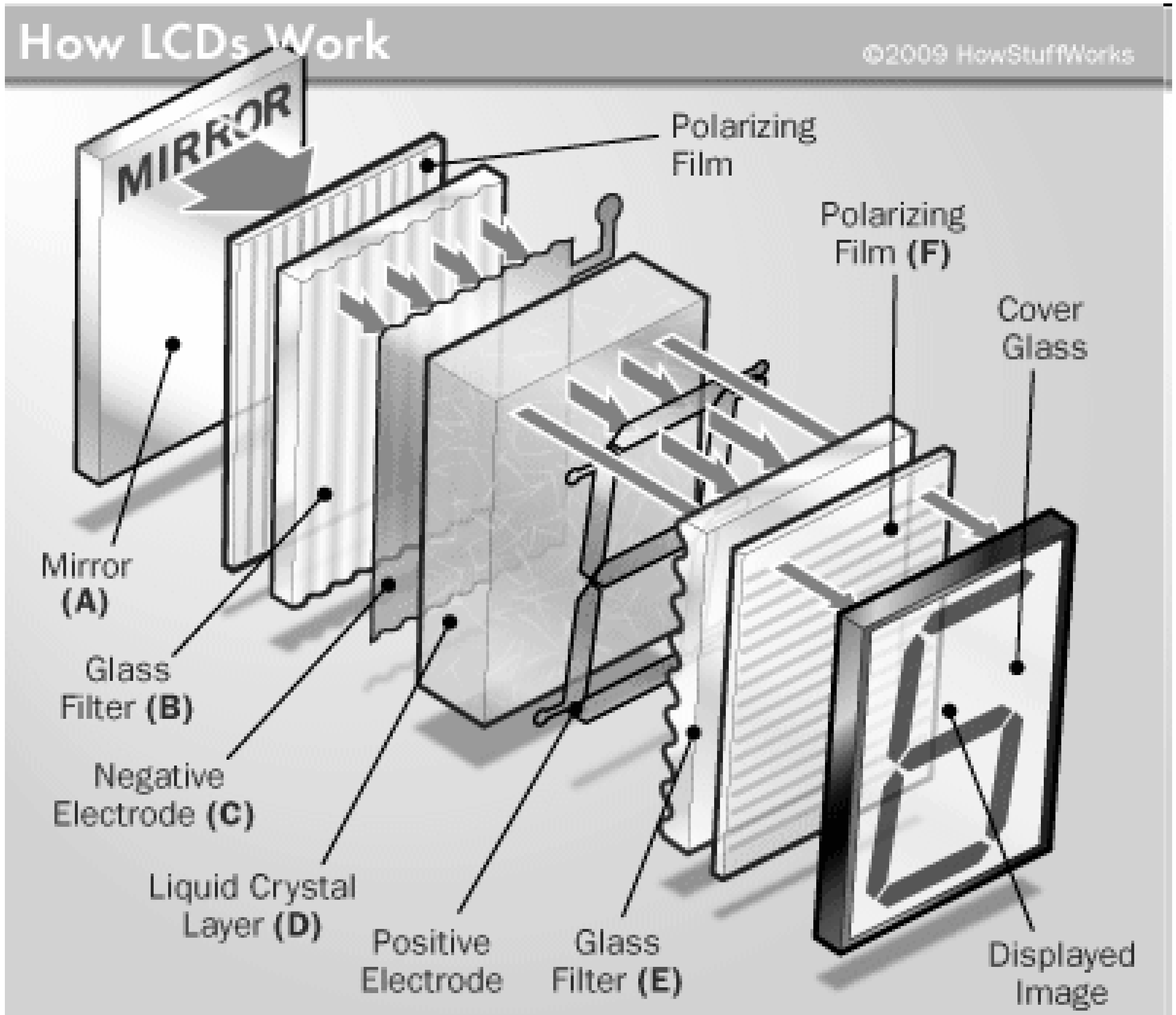


Dipoles of the molecules orient along an externally applied electric field. Change the field → change the birefringence → change the polarization of transmitted light → pass through polarization analyzer to change the intensity

→ Digital displays, LCD monitors, etc.

Stress-induced birefringence:

Applying a mechanical stress to a material will often produce an asymmetry → birefringence.
This is commonly used to measure stress.


How LCDs Work
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MIRROR
Polarizing Film
Polarizing Film (F)
Cover Glass
Mirror (A)
Glass Filter (B)
Negative Electrode (C)
Liquid Crystal Layer (D)
Positive Electrode
Glass Filter (E)
Displayed Image