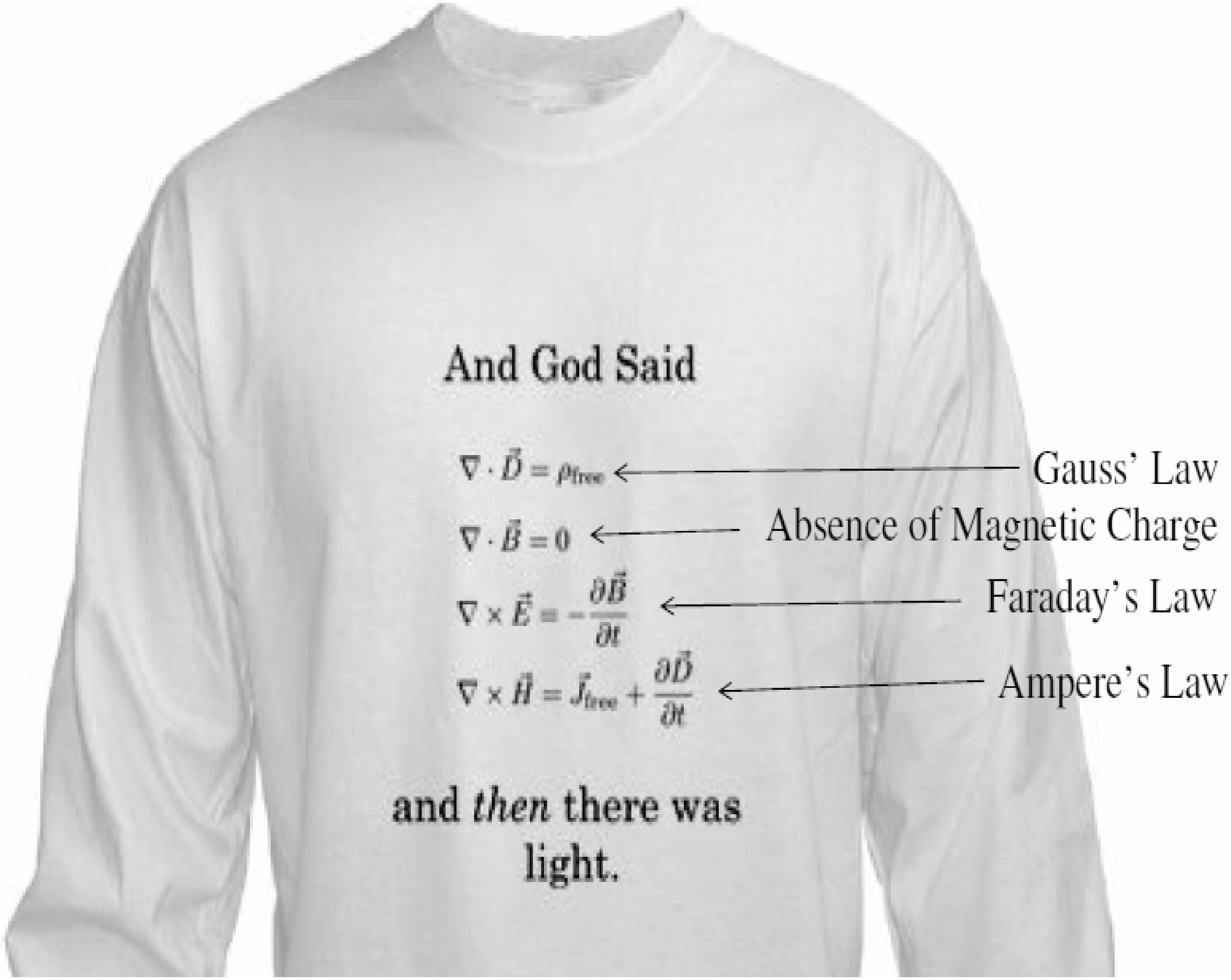
Physics-272 Lecture 21

Description of Wave Motion
Electromagnetic Waves
Maxwell's Equations Again!

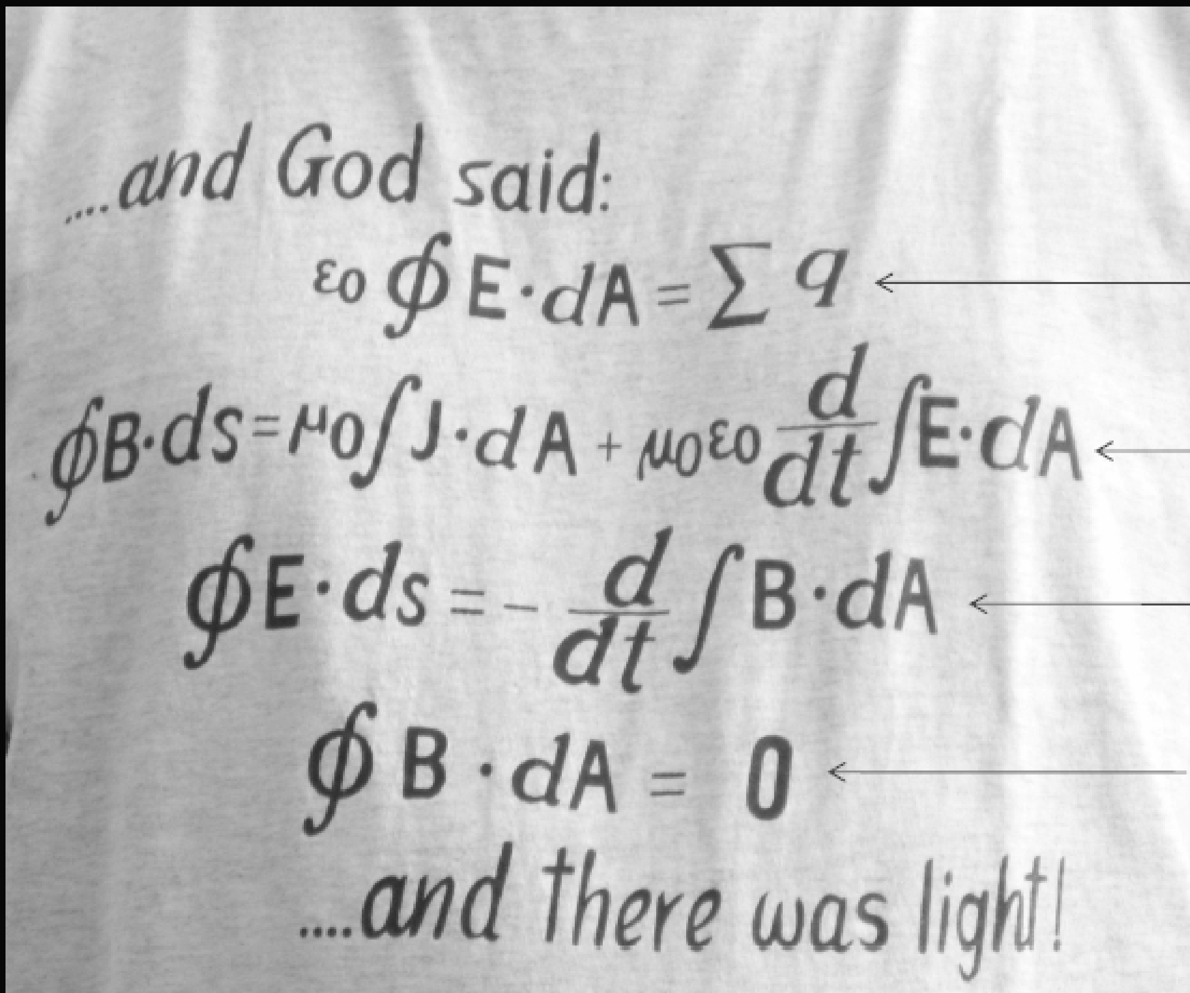
Maxwell's Equations

James Clerk Maxwell (1831-1879)

- **Generalized Ampere's Law**
- **And made equations symmetric:**
  - a changing magnetic field produces an electric field
  - a changing electric field produces a magnetic field
- **Showed that Maxwell's equations predicted electromagnetic waves and $c = 1/\sqrt{\varepsilon_0\mu_0}$**
- **Unified electricity and magnetism and light.**


And God Said
$\nabla \cdot \vec{D} = \rho_{\text{free}}$
$\nabla \cdot \vec{B} = 0$
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
$\nabla \times \vec{H} = \vec{J}_{\text{free}} + \frac{\partial \vec{D}}{\partial t}$
and *then* there was light.
Gauss' Law
Absence of Magnetic Charge
Faraday's Law
Ampere's Law

Maxwell's Equations



Gauss's Law

Ampere's Law

Faraday's Law

Non-existence of Magnetic Charge

Maxwell's Equations (integral form)

Name	Equation	Description
Gauss' Law for Electricity	$\int \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$	Charge and electric fields
Gauss' Law for Magnetism	$\int \vec{B} \cdot d\vec{A} = 0$	Magnetic fields
Faraday's Law	$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$	Electrical effects from changing B field
Ampere's Law (modified by Maxwell)	$\oint \vec{B} \cdot dl = \mu_0 \left(i_c + \varepsilon_0 \frac{d\Phi_E}{dt} \right)$	Magnetic effects from current and Changing E field

All of electricity and magnetism is summarized by Maxwell's Equations.

On to Waves!!

- **Note the symmetry now of Maxwell's Equations in free space, meaning when no charges or currents are present**

$$\int \vec{E} \cdot d\vec{A} = 0 \qquad \int \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad \oint \vec{B} \cdot dl = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

- **Combining these equations leads to wave equations for $\boldsymbol{E}$ and $\boldsymbol{B}$, e.g.,**

$$\boxed{\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2}}$$

- **Do you remember the wave equation???**

$$\frac{\partial^2 h}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 h}{\partial t^2}$$

h **is the variable that is changing in space (*x*) and time (*t*). *v* is the velocity of the wave.**

Review of Waves from Physics 170

- **The one-dimensional wave equation:**

$$\frac{\partial^2 h}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 h}{\partial t^2}$$

has a general solution of the form:

$$h(x,t) = h_1(x - vt) + h_2(x + vt)$$

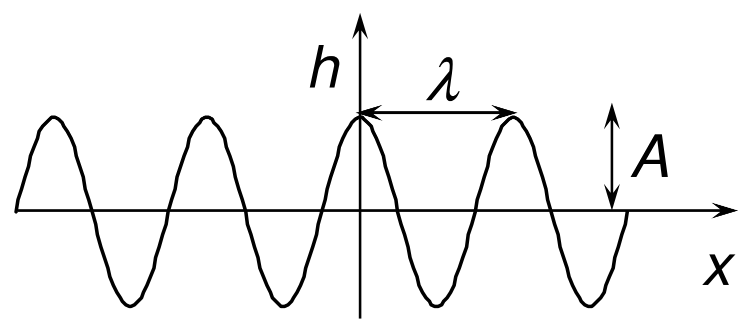
where h_1 represents a wave traveling in the $+x$ direction and h_2 represents a wave traveling in the $-x$ direction.

- **A specific solution for harmonic waves traveling in the $+x$ direction is:**

$$h(x,t) = A\cos(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda} \quad \omega = 2\pi f = \frac{2\pi}{T}$$

$$v = \lambda f = \frac{\omega}{k}$$



A = amplitude
λ = wavelength
f = frequency
v = speed
k = wave number

Clicker problem

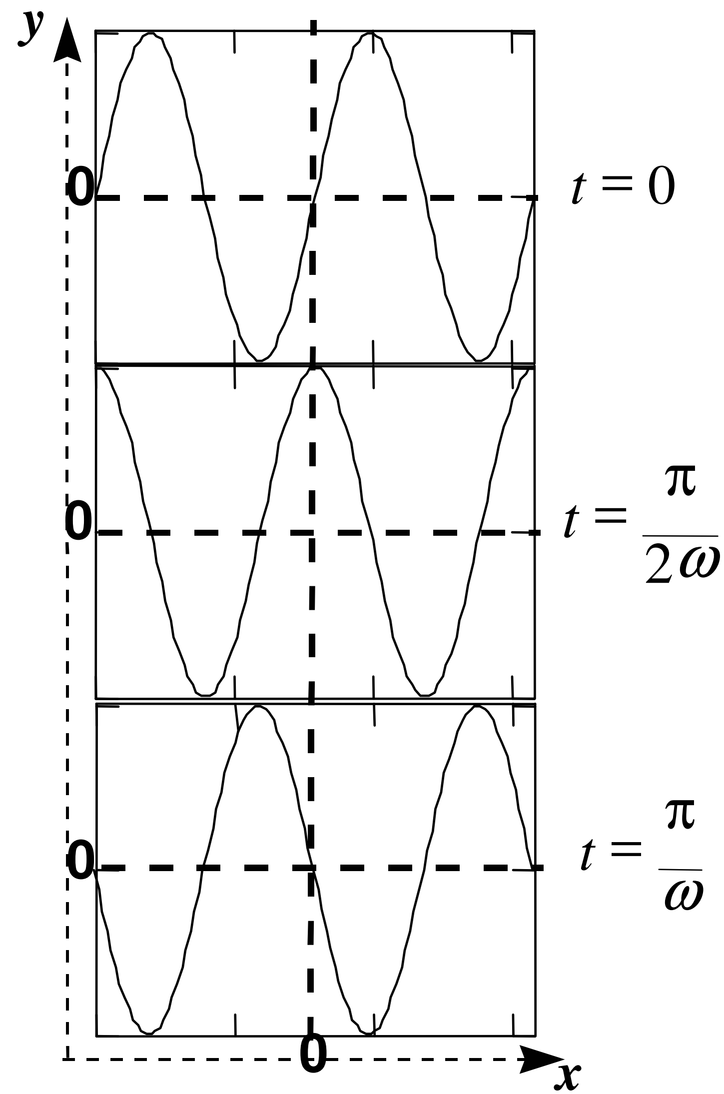
- **Snapshots of a wave with angular frequency ω are shown at 3 times:**

2A – **Which of the following expressions describes this wave?**

(a) $y = \sin(kx - \omega t)$

(b) $y = \sin(kx + \omega t)$

(c) $y = \cos(kx + \omega t)$



2B • **In what direction is this wave traveling?**

(a) $+x$ direction **(b) $-x$ direction**

Clicker problem

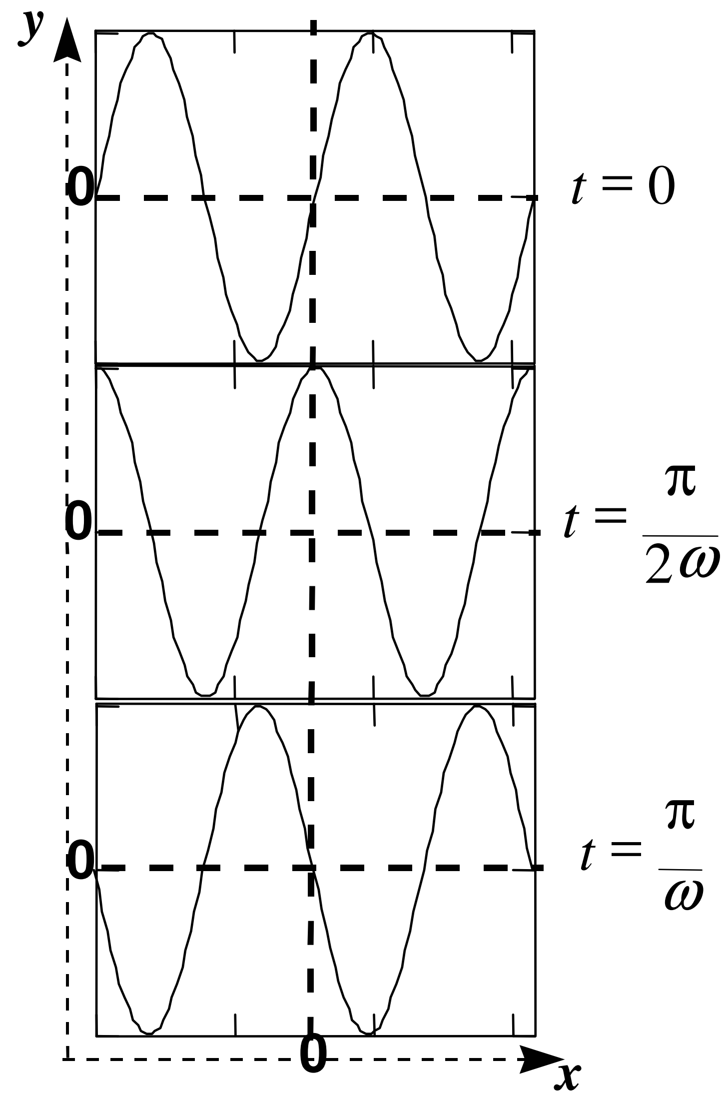
- **Snapshots of a wave with angular frequency ω are shown at 3 times:**

2A – **Which of the following expressions describes this wave?**

(a) $y = \sin(kx - \omega t)$

$\boxed{\textbf{(b)}\ y = \sin(kx + \omega t)}$

(c) $y = \cos(kx + \omega t)$



- **The $t = 0$ snapshot ⇒ at $t = 0$, $y = \sin kx$**
- **At $t = \pi/2\omega$ and $x = 0$, (a) ⇒ $y = \sin(-\pi/2) = -1$**
- **At $t = \pi/2\omega$ and $x = 0$, (b) ⇒ $y = \sin(+\pi/2) = +1$**

Clicker problem

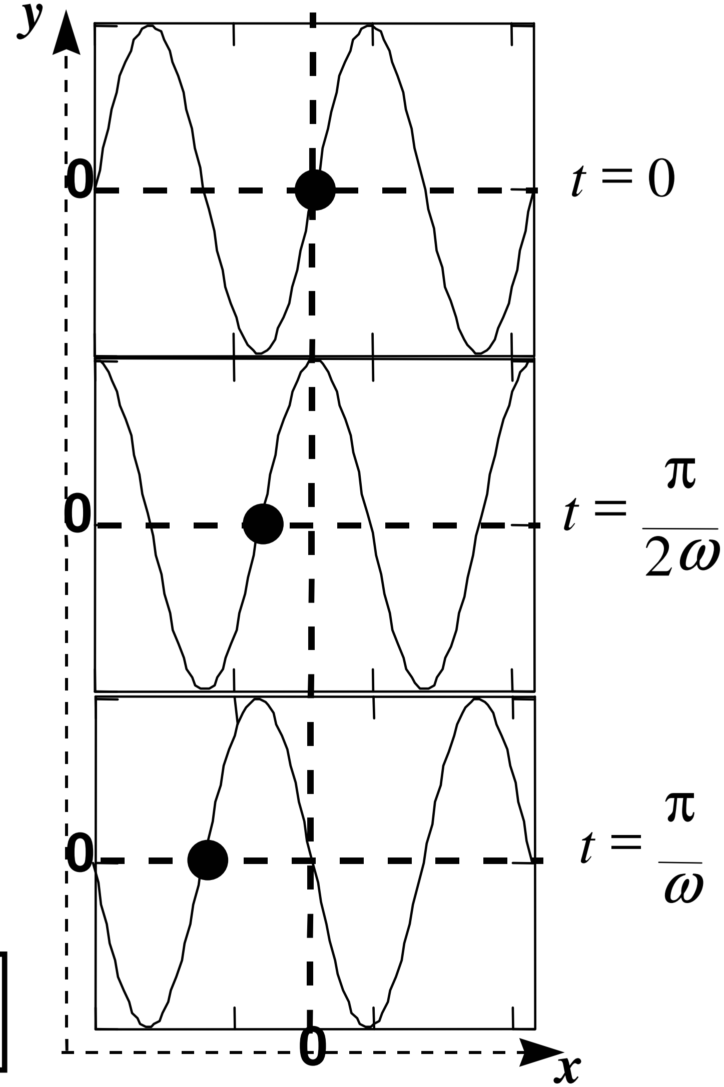
- Snapshots of a wave with angular frequency ω are shown at 3 times:

2A – Which of the following expressions describes this wave?

(a) $y = \sin(kx - \omega t)$

$\boxed{\text{(b) } y = \sin(kx + \omega t)}$

(c) $y = \cos(kx + \omega t)$



2B • In what direction is this wave traveling?

(a) $+x$ direction

$\boxed{\text{(b) } -x \text{ direction}}$

- We claim this wave moves in the $-x$ direction.
- The orange dot marks a point of constant phase.
- It clearly moves in the $-x$ direction as time increases!!

Velocity of Electromagnetic Waves

- **The wave equation for E_x (Maxwell derived it in ~1865!):**

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

- **Comparing to the general wave equation:**

$$\frac{\partial^2 h}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 h}{\partial t^2}$$

we have the velocity of electromagnetic waves in free space:

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3.00 \times 10^8 \ \mathrm{m/s} \equiv c$$

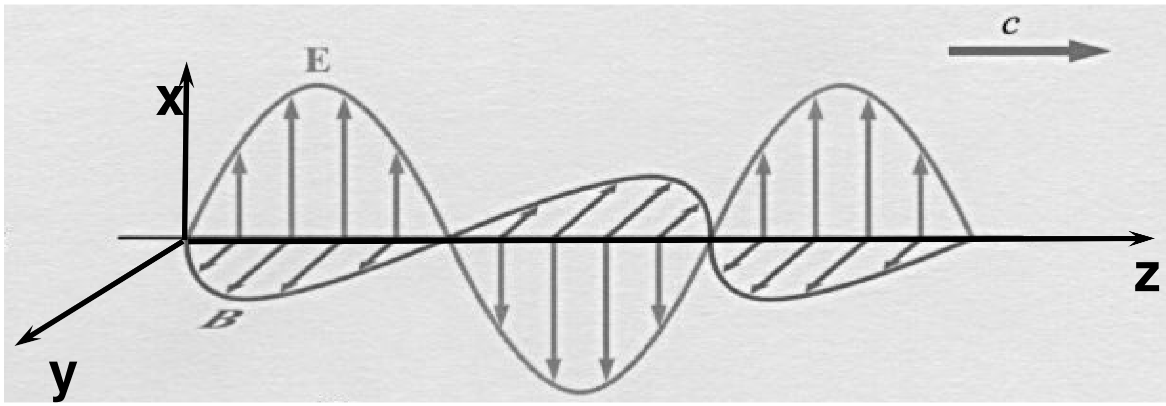
- **This value is essentially identical to the speed of light measured by Foucault in 1860!**
  - **Maxwell identified light as an electromagnetic wave.**

E & *B* in Electromagnetic Wave

- **Plane Harmonic Wave:**

$$E_x = E_0 \sin(kz - \omega t)$$
$$B_y = B_0 \sin(kz - \omega t)$$

where $\omega = kc$



➢ $\boxed{B_y \text{ is in phase with } E_x}$ ➢ $\boxed{B_0 = E_0 / c}$

➢**The direction of propagation $\hat{s}$ is given by the cross product**

$$\boxed{\hat{s} = \hat{e} \times \hat{b}}$$

where $(\hat{e}, \hat{b})$ are the unit vectors in the (E,B) directions.

Nothing special about (E_x, B_y); e.g., could have (E_y, -B_x)

Clicker Problem

- Suppose the electric field in an e-m wave is given by:

3A In what direction is this wave traveling ?

(a) $+z$ direction (b) $-z$ direction

3B Which of the following expressions describes the magnetic field associated with this wave?

(a) $B_x = -(E_o/c)\cos(kz + \omega t)$

(b) $B_x = +(E_o/c)\cos(kz - \omega t)$

(c) $B_x = +(E_o/c)\sin(kz - \omega t)$

Clicker Problem

- **Suppose the electric field in an e-m wave is given by:**

$$\vec{E} = -\hat{y}\ E_0 \cos(-kz + \omega t)$$

3A – In what direction is this wave traveling ?

(a) + z direction **(b) - z direction**

- **To determine the direction, set phase = 0:** $-kz + \omega t = 0 \Rightarrow z = +\dfrac{\omega}{k}t$
- **Therefore wave moves in + z direction!**
- **Another way: Relative signs opposite means + direction**

Clicker Problem

- **Suppose the electric field in an e-m wave is given by:**

$$\vec{E} = -\hat{y}\ E_0 \cos(-kz + \omega t)$$

3A – **In what direction is this wave traveling ?**

$\boxed{\textbf{(a) } +z \textbf{ direction}}$ **(b) -** z **direction**

3B • **Which of the following expressions describes the magnetic field associated with this wave?**

(a) $B_x = -(E_o/c)\cos(kz + \omega t)$

$\boxed{\textbf{(b) } B_x = +(E_o/c)\cos(kz - \omega t)\ )}$

(c) $B_x = +(E_o/c)\sin(kz - \omega t)$

- B **is in phase with** E **and has direction determined from:** $\hat{b} = \hat{s} \times \hat{e}$
- **At** $t=0$, $z=0$, $E_y = -E_o$
- **Therefore at** $t=0$, $z=0$, $\hat{b} = \hat{s} \times \hat{e} = \hat{k} \times (-\hat{j}) = \hat{i}$

$$\Rightarrow \boxed{\vec{B} = +\hat{i}\ \frac{E_0}{c} \cos(kz - \omega t)}$$

10) An electromagnetic wave is travelling along the x-axis, with its electric field oscillating along the y-axis. In what direction does the magnetic field oscillate?

a) along the x-axis
b) along the z-axis
c) along the y-axis

Note: the direction of propagation $\hat{s}$ is given by the cross product

$$\boxed{\hat{s} = \hat{e} \times \hat{b}}$$

where $\left(\hat{e}, \hat{b}\right)$ are the unit vectors in the (E,B) directions.

In this case, direction of $\hat{s}$ is x and direction of $\hat{e}$ is y

$$\boxed{\hat{x} = \hat{y} \times \hat{z}}$$

The fields must be perpendicular to each other and to the direction of propagation.

Leads to the possibility of "polarization" of light to be discussed in next chapter.

Electromagnetic (EM) Waves in free space

Maxwell's eqns predict electric & magnetic fields can propagate in vacuum.
Examples of EM waves include; radio/TV waves, light, x-rays, and microwaves.

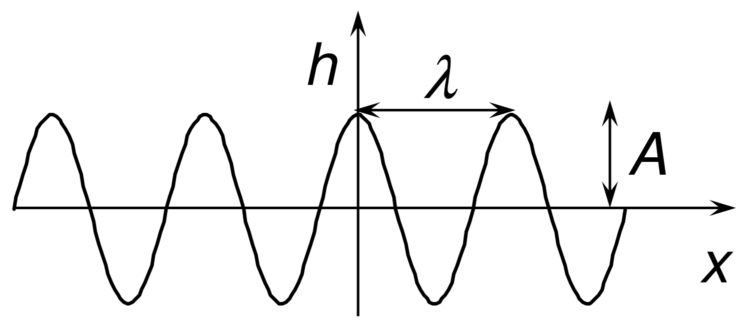
$$\boxed{\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2}}$$

- **A specific solution for harmonic waves traveling in the +*x* direction is:**

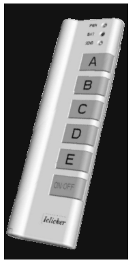
$$h(x,t) = A\cos(kx - \omega t)$$

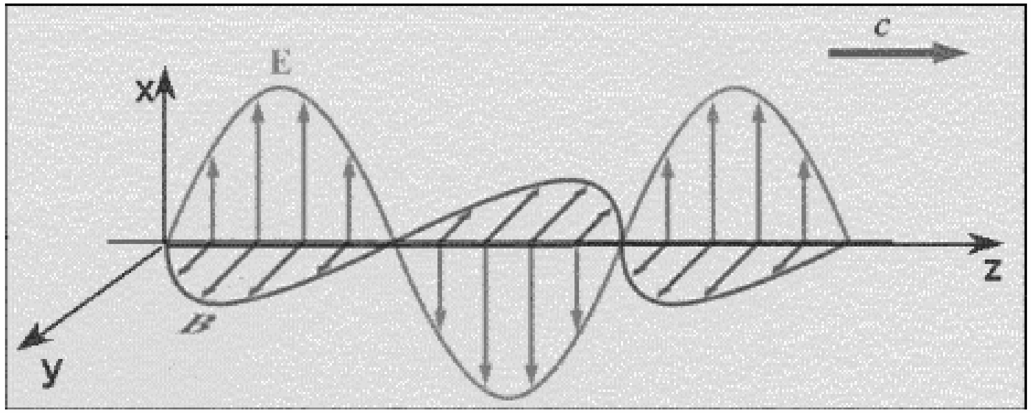
$$k = \frac{2\pi}{\lambda} \quad \omega = 2\pi f = \frac{2\pi}{T}$$

$$v = \lambda f = \frac{\omega}{k}$$



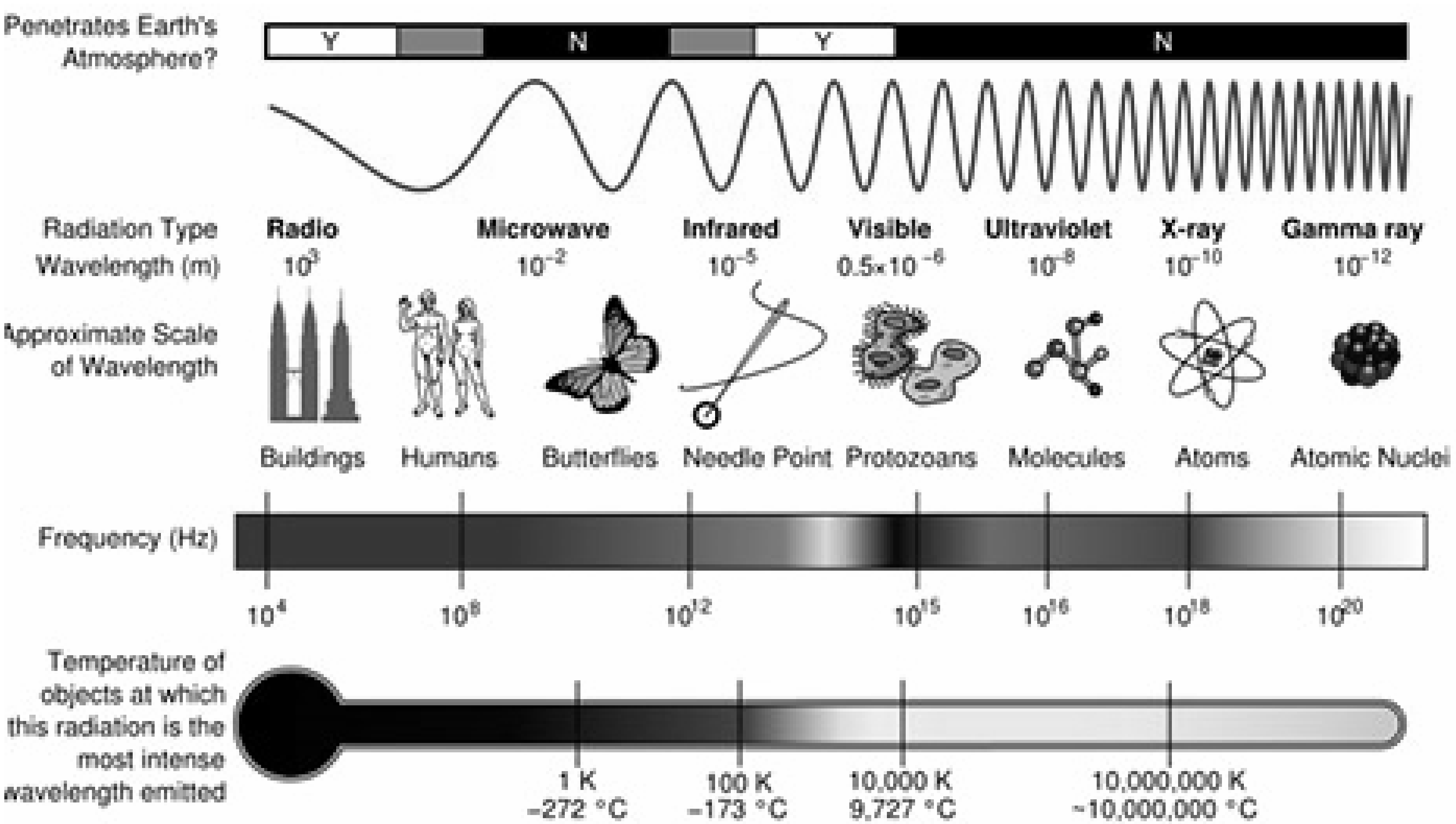
A **= amplitude**
λ **= wavelength**
f **= frequency**
v **= speed**
k **= wave number**



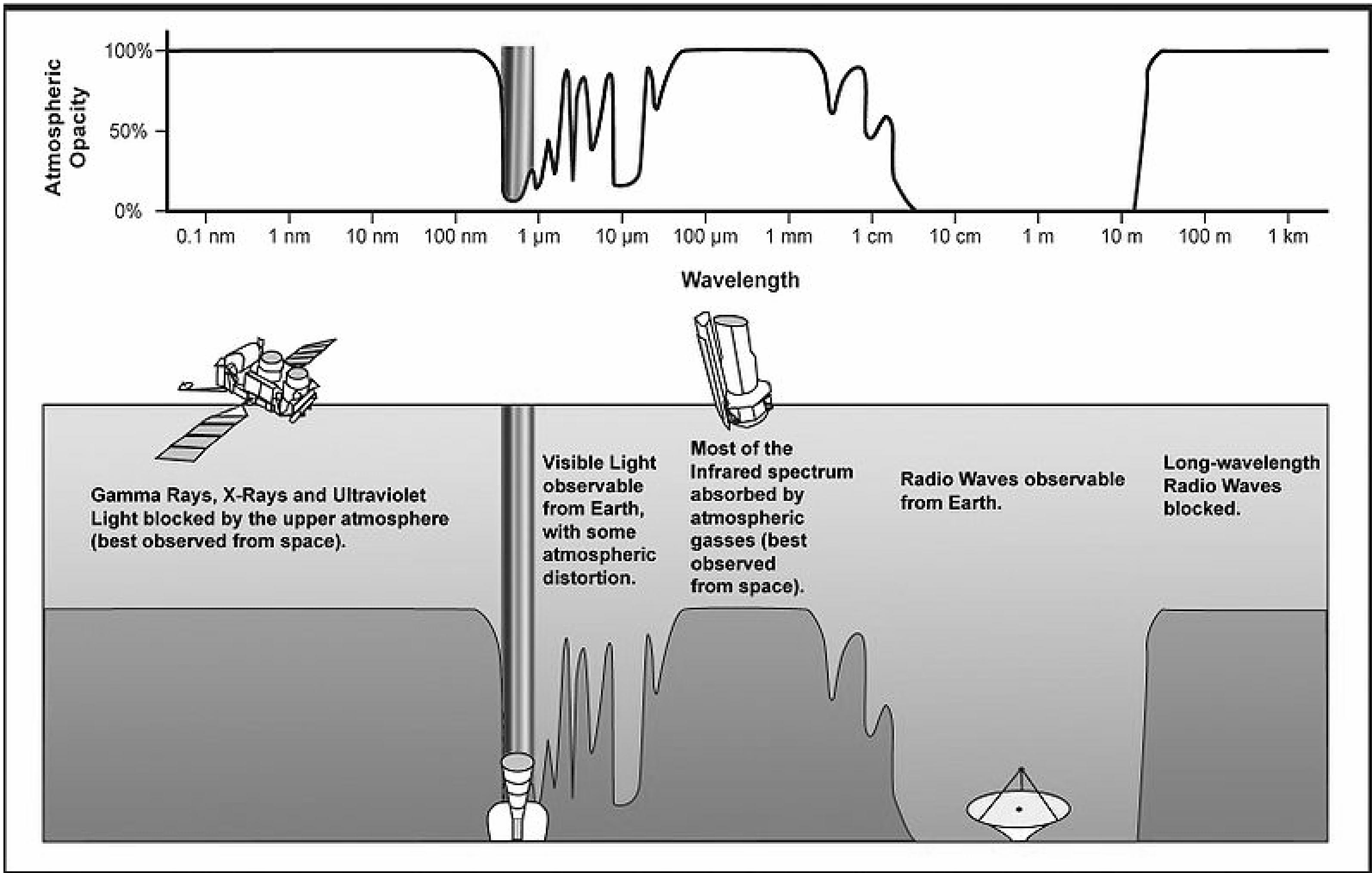


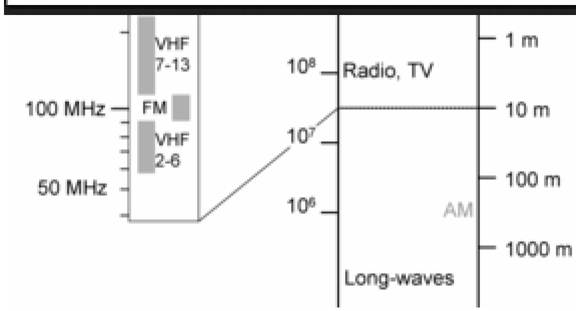
Which equation correctly describes this electromagnetic wave?

- ○ $\mathbf{E}_x = \mathbf{E}_o \sin(kz + \omega t)$ No – moving in the minus z direction
- ○ $\mathbf{E}_y = \mathbf{E}_o \sin(kz - \omega t)$ No – has Ey rather than Ex
- $\boxed{○\ \mathbf{B}_y = \mathbf{B}_o \sin(kz - \omega t)}$

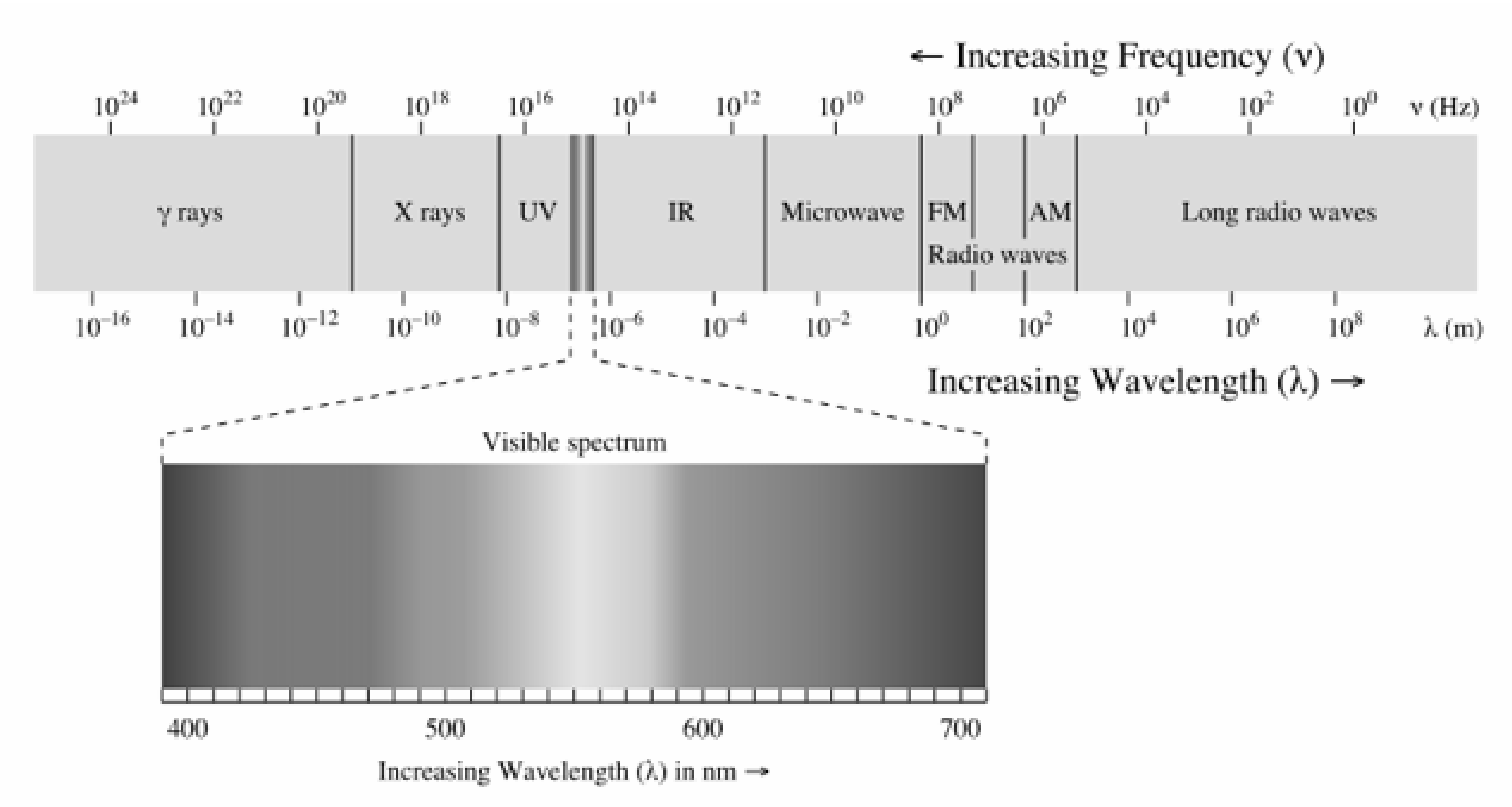


Penetrates Earth's Atmosphere?
Y
N
Y
N
Radiation Type
Wavelength (m)
Radio
10^3
Microwave
10^{-2}
Infrared
10^{-5}
Visible
0.5×10^{-6}
Ultraviolet
10^{-8}
X-ray
10^{-10}
Gamma ray
10^{-12}
Approximate Scale of Wavelength
Buildings
Humans
Butterflies
Needle Point
Protozoans
Molecules
Atoms
Atomic Nuclei
Frequency (Hz)
10^4
10^8
10^{12}
10^{15}
10^{16}
10^{18}
10^{20}
Temperature of objects at which this radiation is the most intense wavelength emitted
1 K
−272 °C
100 K
−173 °C
10,000 K
9,727 °C
10,000,000 K
~10,000,000 °C

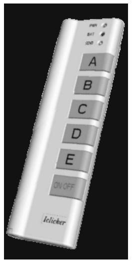




The earth's atmosphere blocks out much of the EM spectrum (e.g. much of UV and X-ray band)



Students must become familiar with the wavelengths of common types of EM radiation

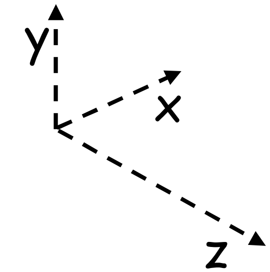


5) Your iclicker operates at a frequency of approximately 900 MHz (900x10^6 Hz). What is the approximately wavelength of the EM wave produced by your iclicker?

- ○ 0.03 meters
- ○ 0.3 meters
- ○ 3.0 meters
- ○ 30. meters

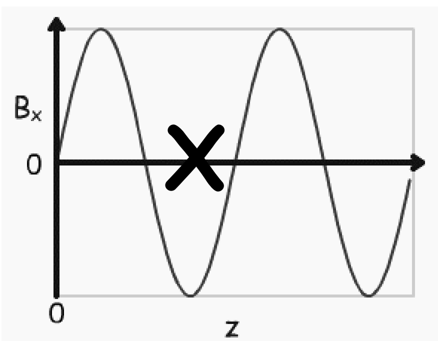
Wavelength is equal to the speed of light divided by the frequency.

$$\lambda = \frac{c}{f} = \frac{300,000,000}{900,000,000} = \frac{1}{3}$$

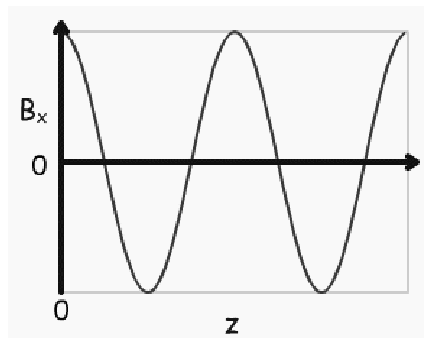


An electromagnetic wave is described by: $\vec{E} = \hat{j}E_0\cos(kz-\omega t)$
where $\hat{j}$ is the unit vector in the +y direction.

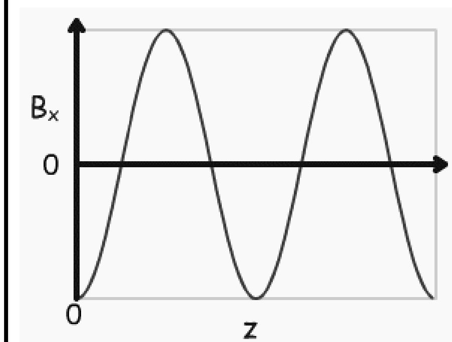
Which of the following graphs represents the z-dependence of B_x at $t = 0$?



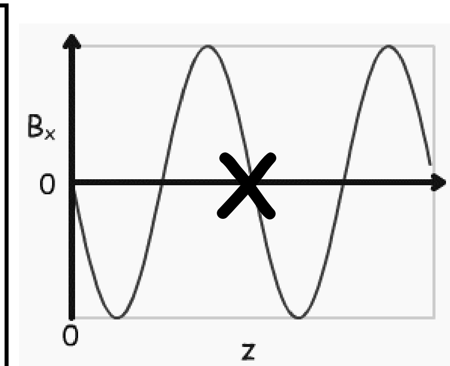
(A)



(B)



(C)

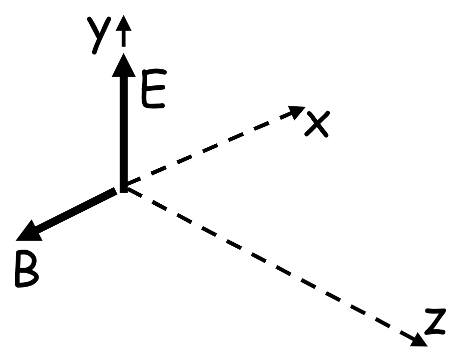


(D)

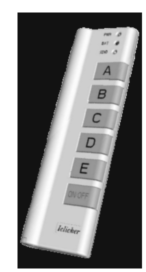
E and B are "in phase" (or 180° out of phase)

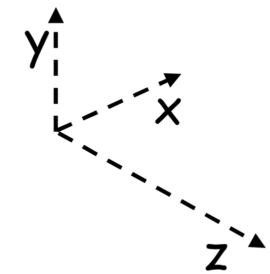
$\vec{E} = \hat{j}E_0\cos(kz-\omega t)$ ⇨ Wave moves in +z direction

$\vec{E}\times\vec{B}$ points in direction of propagation



$\vec{B} = -\hat{i}B_0\cos(kz-\omega t)$





An electromagnetic wave is described by: $\vec{E} = \frac{\hat{i}+\hat{j}}{\sqrt{2}} E_0 \cos(kz + \omega t)$

What is the form of B for this wave?

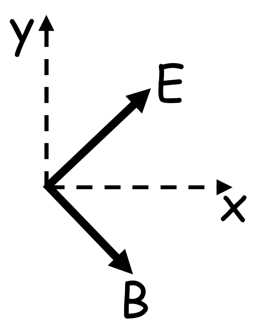
A) $\vec{B} = \frac{\hat{i}+\hat{j}}{\sqrt{2}} (E_0 / c) \cos(kz + \omega t)$

B) $\vec{B} = \frac{-\hat{i}-\hat{j}}{\sqrt{2}} (E_0 / c) \cos(kz + \omega t)$

C) $\vec{B} = \frac{\hat{i}-\hat{j}}{\sqrt{2}} (E_0 / c) \cos(kz + \omega t)$

D) $\vec{B} = \frac{-\hat{i}-\hat{j}}{\sqrt{2}} (E_0 / c) \cos(kz + \omega t)$

$\vec{E} = \frac{\hat{i}+\hat{j}}{\sqrt{2}} E_0 \cos(kz + \omega t)$ $\Rightarrow$ Wave moves in –z direction

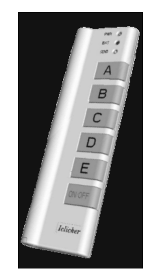


+z points out of screen

-z points into screen

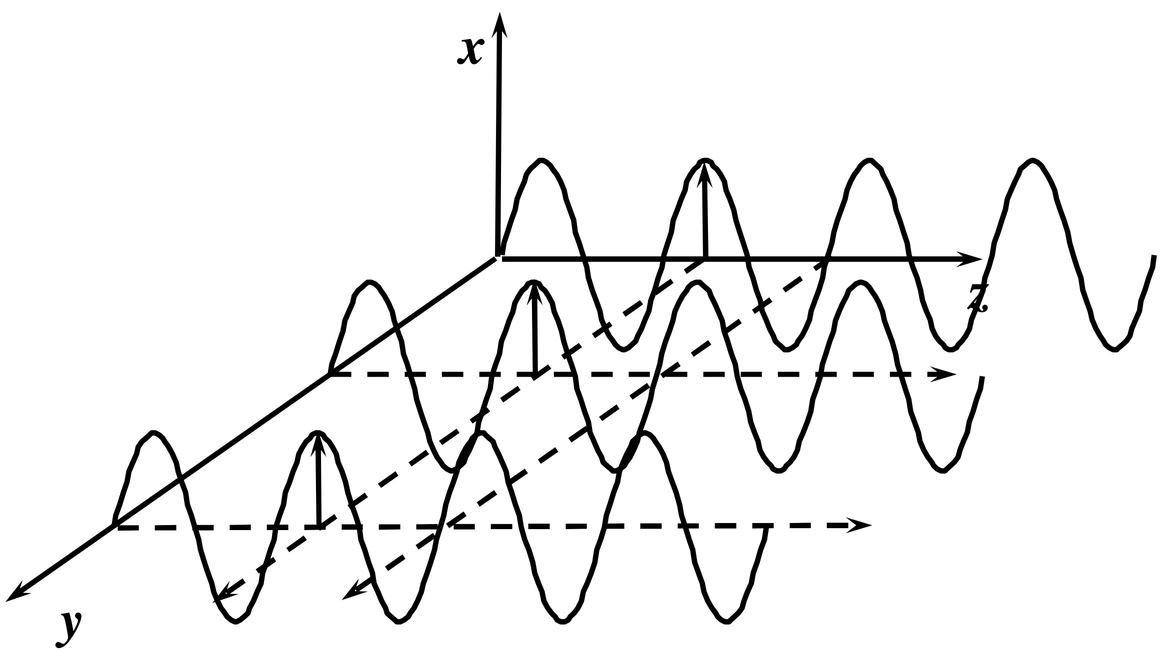
$\vec{E} \times \vec{B}$ points in negative z-direction

B must be perp to E

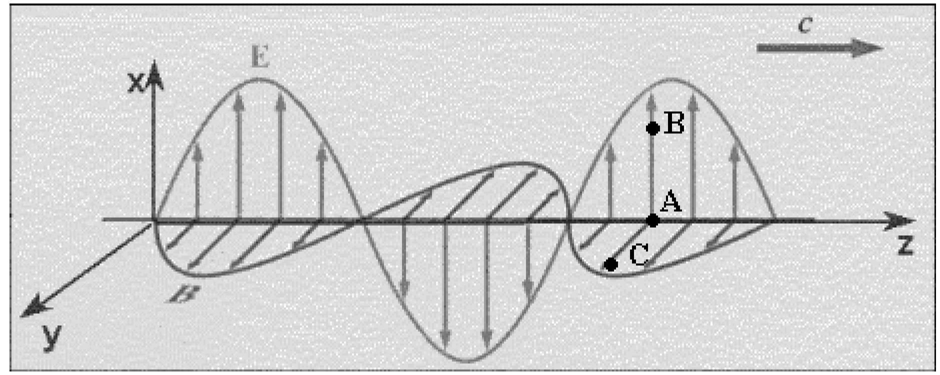


Plane Waves

- For any given value of z, the magnitude of the electric field is uniform everywhere in the x-y plane with that z value.



Shown is an EM wave at an instant in time. Points A, B, and C lie in the same **x-y** plane.



3) Compare the magnitudes of the electric field at points A and B.

a) $\mathbf{E_a < E_b}$

b) $\mathbf{E_a = E_b}$

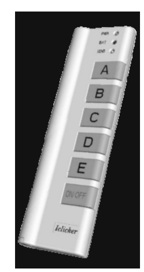
c) $\mathbf{E_a > E_b}$

4) Compare the magnitudes of the electric field at points A and C.

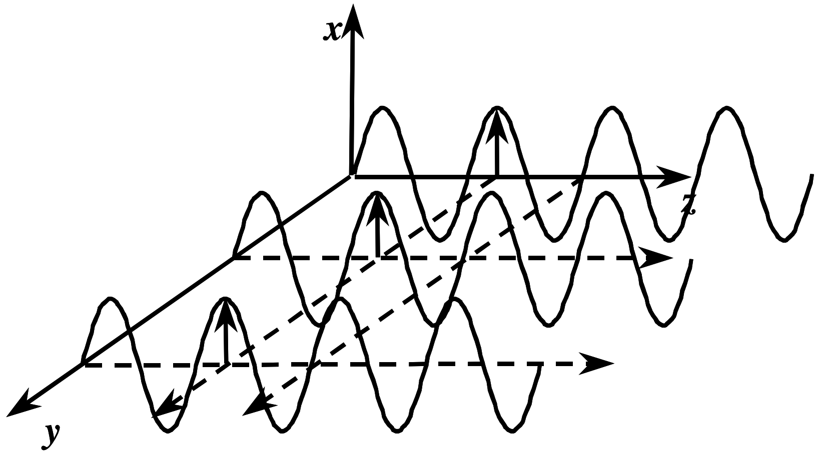
a) $\mathbf{E_a < E_c}$

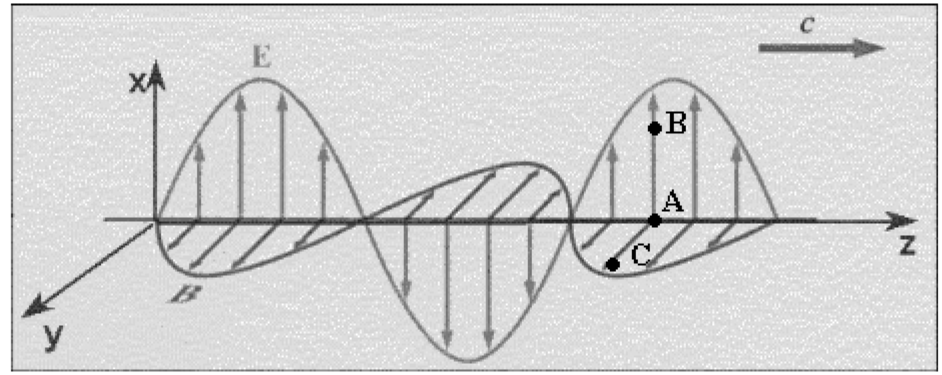
b) $\mathbf{E_a = E_c}$

c) $\mathbf{E_a > E_c}$



Students said …





Magnitude of field is determined only by value of z (and t) !!!

Right:
• for any given value of z, the magnitude of the electric field is uniform everywhere in the x-y plane with that z value

Wrong:
• The point A is where the electric field and magnetic field are zero. So the points c and b have a greater field.

(a common misconception)

EM wave; energy momentum, angular momentum

Previously, we demonstrated the energy density existed in E fields in capacitors and in B fields in inductors. We can sum these energies,

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2$$

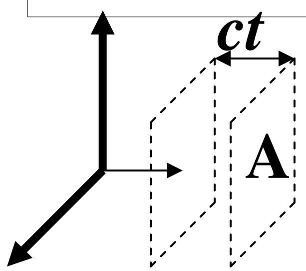
Since, *E=cB* and $c = 1/\sqrt{\mu_0 \varepsilon_0}$ **then *u* in terms of E,**

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\mu_0 \left(\frac{E}{c}\right)^2 = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\varepsilon_0 E^2 = \varepsilon_0 E^2$$

=> EM waves have energy and can transport energy at speed c

Almost all energy resources on the earth come directly or indirectly via sunlight. What doesn't?

EM wave; energy flow & Poynting Vector



Suppose we have transverse E and B fields moving at velocity c, to the right. If the energy density is $u = \varepsilon_0 E^2$ **and the field is moving through area A at velocity c covers volume** ***Act*****, that has energy,** ***uAct*****. Hence the** **<u>amount of energy flow per unit time per unit surface area</u>** **is,**

$$\frac{1}{A}\frac{dU}{dt} = \frac{1}{At}\left(uAct\right) = uc = \varepsilon_0 cE^2$$

This quantity is called, ***S*****. In terms of E&B field magnitudes,**

$$S = \varepsilon_0 cE^2 = \varepsilon_0 cEcE / c = \varepsilon_0 cEcB = EB / \mu_0$$

We also define the energy flow with direction, called the Poynting vector, $\boxed{\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}}$ **, which is in the direction of propagation**

EM wave; Intensity

The intensity, I, is the time average of the magnitude of the Poynting vector and represents the <u>average energy flow per unit area</u>.

$$I = \left\langle \vec{S}(x,y,z,t) \right\rangle = \frac{1}{\mu_0} \left\langle \vec{E}(x,y,z,t) \times \vec{B}(x,y,z,t) \right\rangle$$

In Y&F, section 32.4, the average S for a sinusoidal plane wave (squared) is worked out,

$$I = \left\langle \vec{S}(x,y,z,t) \right\rangle = \frac{E_{\max} B_{\max}}{2\mu_0} \left\langle 1 + \cos(2kx - 2\omega t) \right\rangle = \frac{E_{\max} B_{\max}}{2\mu_0}$$

The can be rewritten as,

$$\boxed{I = \left\langle \vec{S}(x,y,z,t) \right\rangle = \frac{E_{\max} B_{\max}}{2\mu_0} = \frac{\varepsilon_0 c}{2} E_{\max}^2 = \frac{\varepsilon_0}{2c} B_{\max}^2}$$

Cell Phone Problem

32.22) A sinusoidal electromagnetic wave emitted by a cell phone has a wavelength of 35.4 cm and an electric field amplitude of 5.4×10^{-2} V/m at a distance of 250 m from the antenna. (a) What is the magnetic field amplitude? (b) The intensity? (C) The total average power?

a.) $E = cB$; $B = E/c = 5.4 \times 10^{-2}\ (V/m)/3 \times 10^{8}\ m/s$

$B = 1.8 \times 10^{-10}$ T

b.)
$$I = \frac{1}{2}\varepsilon_0 c E^2;$$
$$I = \frac{1}{2}(8.85\times10^{-12} C^2/Nm^2)(3\times10^8 m/s)(5.4\times10^{-2} V/m)$$
$$I = 3.87\times10^{-6} W/m^2$$

c.) Assume isotropic: $I=P/A$; $P=4\pi r^2 I = 4\pi(250m)^2 I$

$P = 3$ W

Waves Carry Energy

Total Energy Density

$$u = \varepsilon_o E^2$$

Average Energy Density

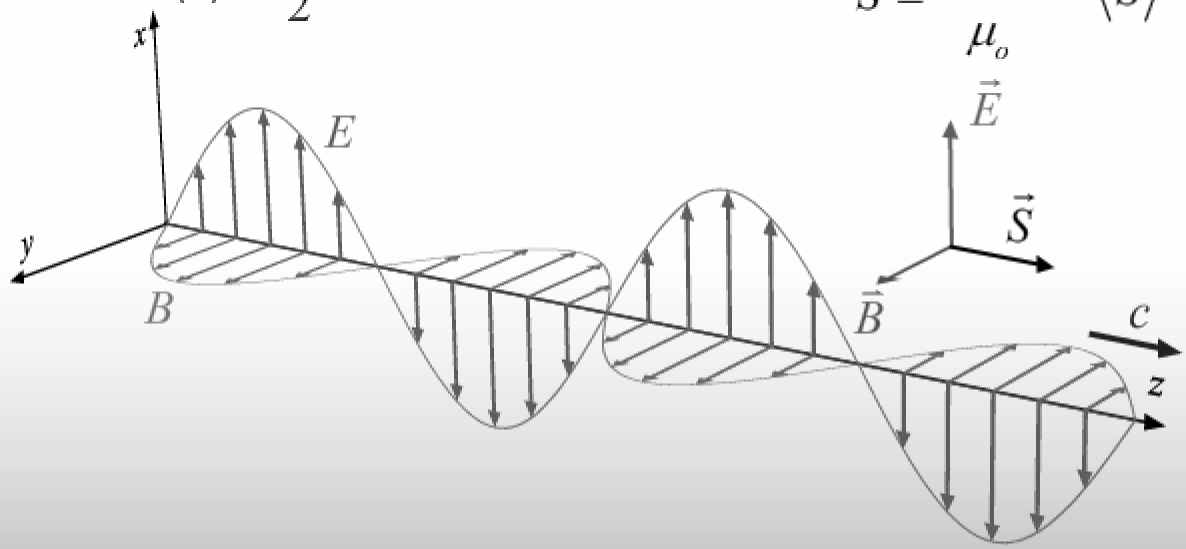
$$\langle u \rangle = \frac{1}{2}\varepsilon_o {E_o}^2$$

Intensity

$$I = \frac{1}{2} c\,\varepsilon_o {E_o}^2 = c\,\langle u \rangle$$

Poynting Vector

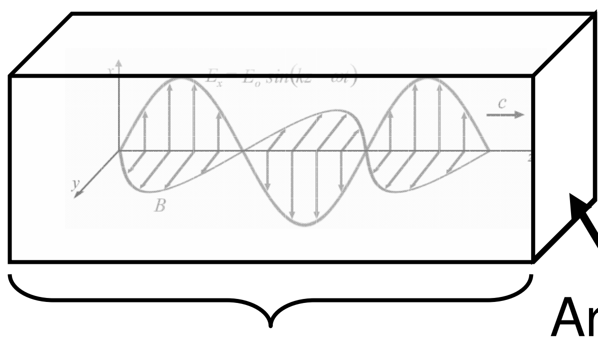
$$\vec{S} \equiv \frac{\vec{E} \times \vec{B}}{\mu_o} \qquad \langle S \rangle = I$$



Intensity

Intensity = energy delivered per unit time, per unit area

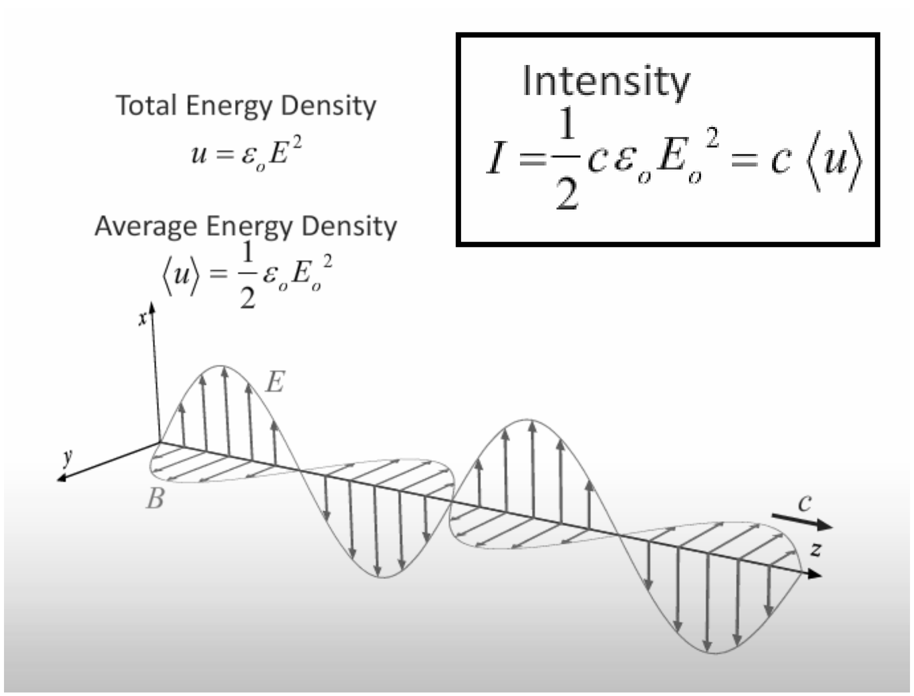
$$I = \frac{1}{A}\frac{dU}{dt}$$ (U = energy)



Area = A

Length = $c\,dt$

$$dU = \langle u \rangle \cdot volume = \langle u \rangle A c dt$$



$$I = c\langle u \rangle$$

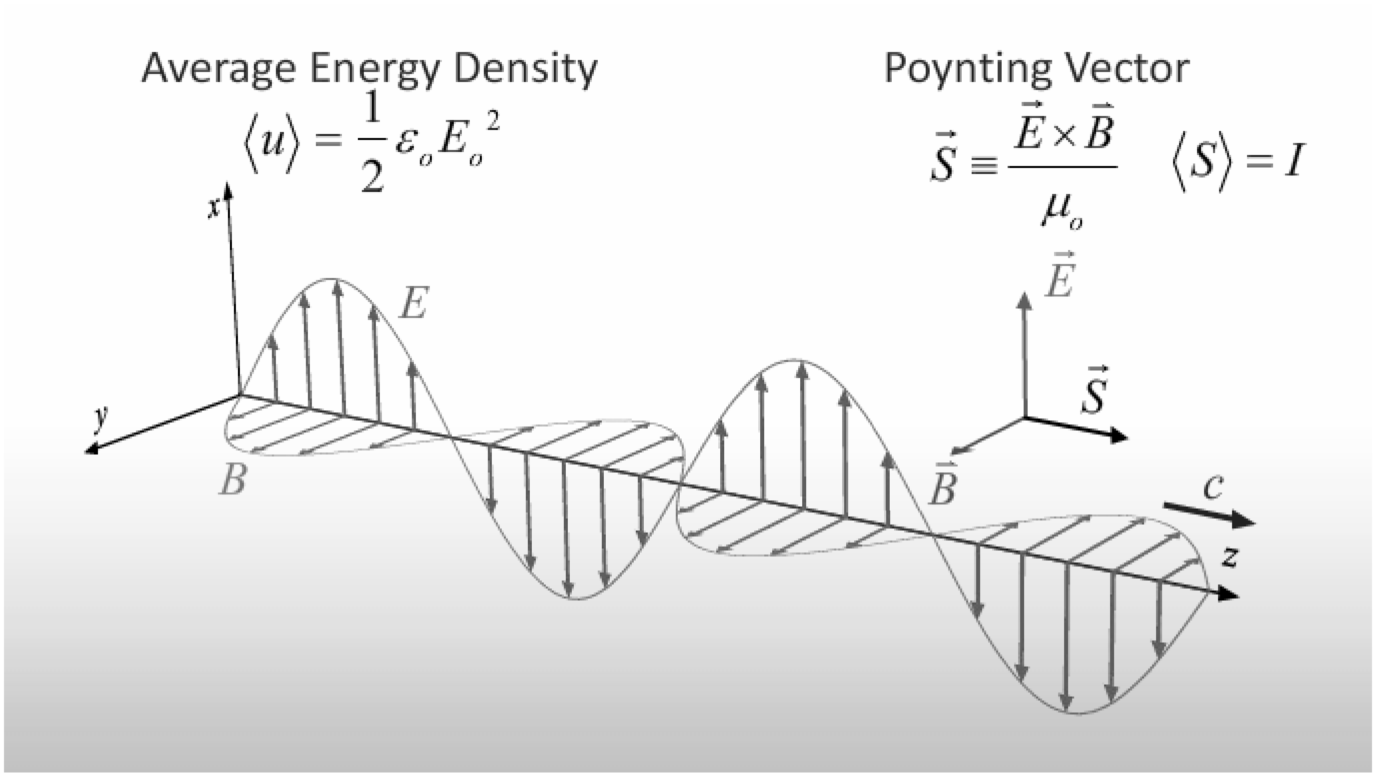
Sunlight on Earth:

I ~ 1000 J/s/m²

~ 1 kW/m²

Comment on the Poynting Vector

Just another way to keep track of all this
- It's the same thing as I but it has a direction



Light has Momentum!

If it has energy and it's moving, then it also has momentum:

Analogy from mechanics:

$$E = \frac{p^2}{2m}$$

$$\frac{dE}{dt} = \frac{\cancel{2}p}{\cancel{2}m}\frac{dp}{dt} = \frac{\cancel{m}v}{\cancel{m}}\frac{dp}{dt} = vF$$

For E-M waves: $\frac{dE}{dt} \rightarrow \frac{dU}{dt} = IA$ $\qquad v \rightarrow c$

$$IA = cF$$

$$\frac{I}{c} = \frac{F}{A}$$ pressure

$$P = \frac{I}{c}$$

Radiation pressure

CLASS	FREQUENCY	WAVELENGTH	ENERGY
Y	300 EHz	1 pm	1.24 MeV
HX	30 EHz	10 pm	124 keV
	3 EHz	100 pm	12.4 keV
SX	300 PHz	1 nm	1.24 keV
EUV	30 PHz	10 nm	124 eV
NUV	3 PHz	100 nm	12.4 eV
NIR	300 THz	1 μm	1.24 eV
MIR	30 THz	10 μm	124 meV
FIR	3 THz	100 μm	12.4 meV
EHF	300 GHz	1 mm	1.24 meV
SHF	30 GHz	1 cm	124 μeV
UHF	3 GHz	1 dm	12.4 μeV
VHF	300 MHz	1 m	1.24 μeV
HF	30 MHz	10 m	124 neV
MF	3 MHz	100 m	12.4 neV
LF	300 kHz	1 km	1.24 neV
VLF	30 kHz	10 km	124 peV
VF/ULF	3 kHz	100 km	12.4 peV
SLF	300 Hz	1 Mm	1.24 peV
ELF	30 Hz	10 Mm	124 feV
	3 Hz	100 Mm	12.4 feV

3 G cell phone networks in the US use 1710-1756 MHz frequencies.

What is the corresponding wavelength range ?

$$3\times10^{8}\, m/s/(1710\times10^{6}\, Hz) = 0.175m$$

$$3\times10^{8}\, m/s/(1756\times10^{6}\, Hz) = 0.170m$$