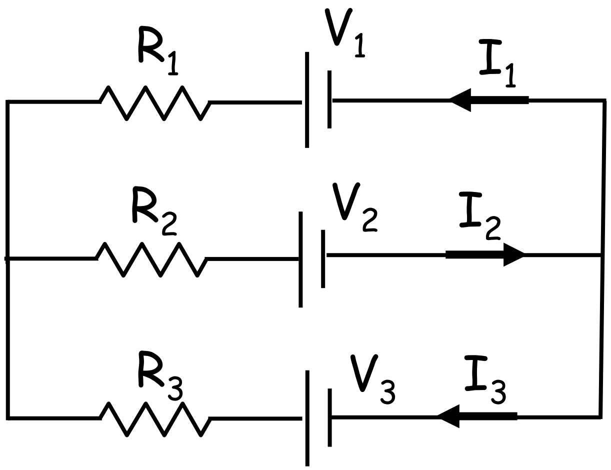
# Lecture 11

Kirchoff Circuit Rules

RC- Circuits

# Review




In this circuit, assume $V_i$ and $R_i$ are known.

What is $I_2$ ??

- We have the following 4 equations:
  1. $I_2 = I_1 + I_3$
  2. $-V_1 + I_1R_1 - I_3R_3 + V_3 = 0$
  3. $-V_3 + I_3R_3 + I_2R_2 + V_2 = 0$
  4. $-V_2 - I_2R_2 - I_1R_1 + V_1 = 0$

**<u>We need 3 equations: which 3 should we use?</u>**

A) Any 3 will do

B) 1, 2, and 4

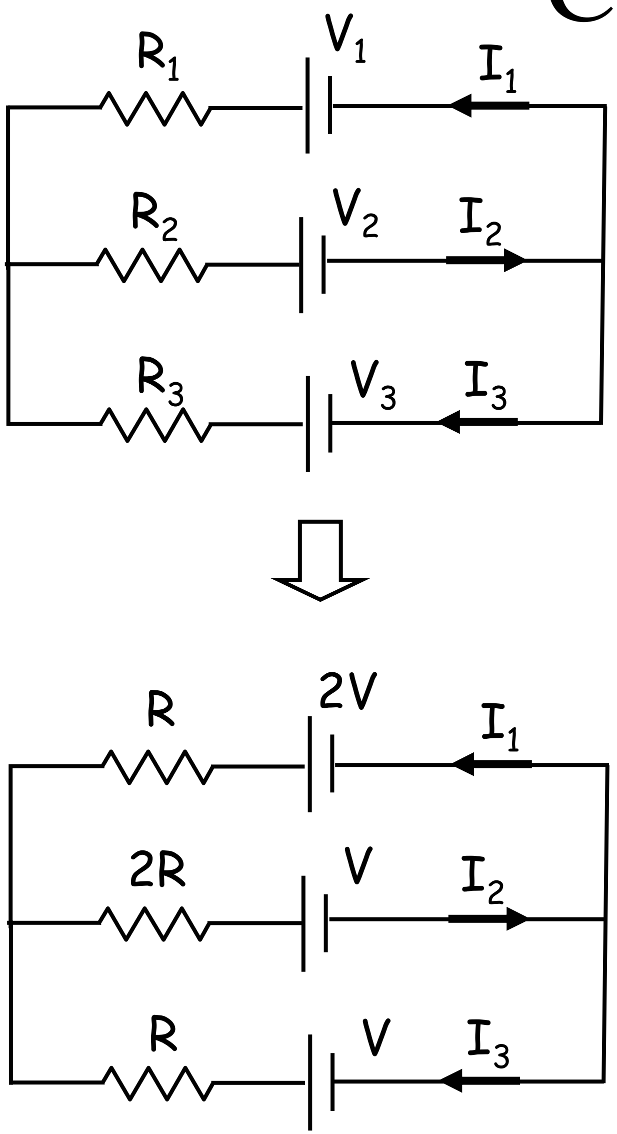
C) 2, 3, and 4

EQ2 + EQ3

$-V_1 + I_1R_1 + I_2R_2 + V_2 = 0$

- Why??
  - We need 3 INDEPENDENT equations
  - We must choose Equation 1 and any two of the remaining ( 2, 3, and 4)
  - Equations 2, 3, and 4 are NOT INDEPENDENT
    - Eqn 2 + Eqn 3 = - Eqn 4

# Calculation




In this circuit, assume $V_i$ and $R_i$ are known.

What is $I_2$ ??

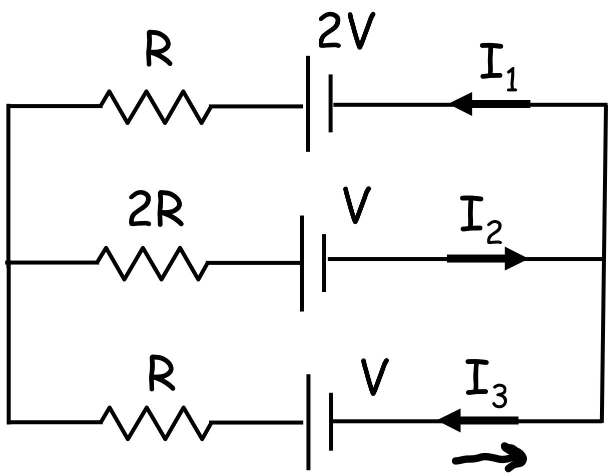
- We have 3 equations and 3 unknowns.

$$I_2 = I_1 + I_3$$
$$V_1 + I_1R_1 - I_3R_3 + V_3 = 0$$
$$V_2 - I_2R_2 - I_1R_1 + V_1 = 0$$

- The solution will get very messy in general.
- <u>Instead do a simpler problem</u>:
- assume $V_2 = V_3 = V$; $V_1 = 2V$; $R_1 = R_3 = R$ with $R_2 = 2R$

# Calculation: Simplify

In this circuit, assume V and R are known. What is $I_2$ ??




- We have 3 equations and 3 unknowns.

$$I_2 = I_1 + I_3$$

$$-2V + I_1R - I_3R + V = 0$$
(outside)

$$-V – I_2(2R) - I_1R + 2V= 0$$
(top)

- With this simplification, you can verify:

$$\frac{1}{5} = 3/5 + (-2/5) = \frac{1}{5}$$

$$I_2 = (1/5)\ V/R$$

$$I_1 = (3/5)\ V/R$$

$$I_3 = (-2/5)\ V/R$$

→

$$\cancel{V} - \cancel{V} + I_3R + 2I_2R = 0 \quad I_3\cancel{R} + 2I_2\cancel{R} = 0 \rightarrow \boxed{I_2 = -\frac{1}{2}I_3}$$

$$-2V + I_1R - I_3R + V = 0 \rightarrow I_1R - I_3R = V \rightarrow I_1 - I_3 = \frac{V}{R}$$

$$\boxed{I_1 = \frac{V}{R} + I_3}$$

$$I_3 + I_1 - I_2 = 0$$

$$\rightarrow I_3 + \frac{V}{R} + I_3 + \frac{1}{2}I_3 = 0$$

$$I_3(1 + 1 + \tfrac{1}{2}) = -\frac{V}{R} \Rightarrow \frac{5}{2}I_3 = -\frac{V}{R}$$

$$\boxed{I_3 = -\frac{2}{5}\frac{V}{R}}$$

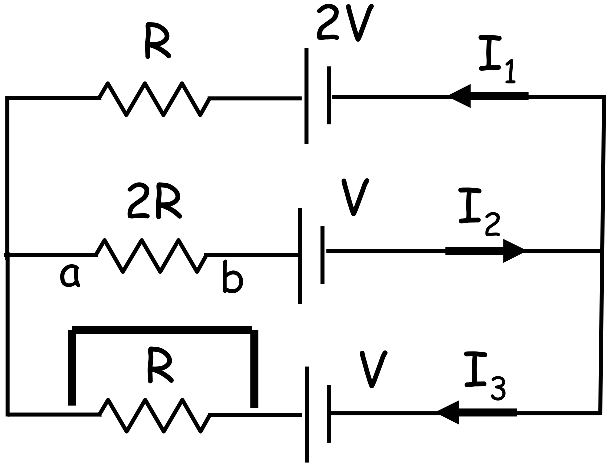
$$\rightarrow \boxed{I_2 = \frac{1}{5}\frac{V}{R}}$$

$$\rightarrow I_1 = I_2 - I_3 = \frac{1}{5}\frac{V}{R} + \frac{2}{5}\frac{V}{R} = \underline{\frac{3}{5}\frac{V}{R}}$$

For reference only

# Follow-Up Clicker

BB




- We know:

$$I_2 = (\ 1/5)\ V/R$$
$$I_1 = \ (\ 3/5)\ V/R$$
$$I_3 = (-2/5)\ V/R$$

- Suppose we short $R_3$: What happens to $V_{ab}$ (voltage across $R_2$) ?

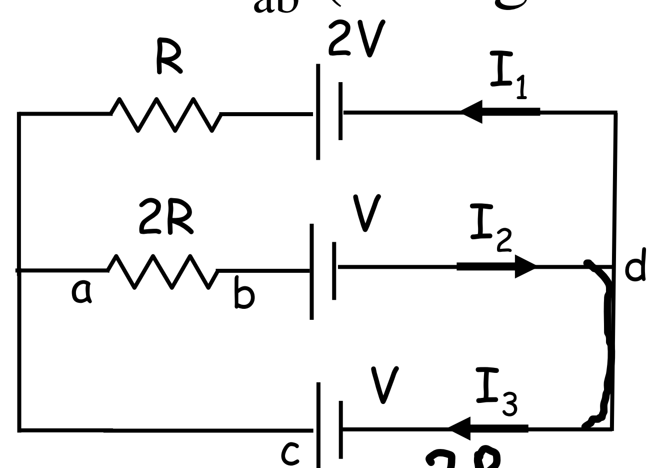
***(A)*** ***$V_{ab}$ remains the same***

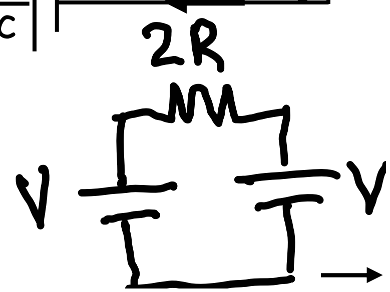
***(B)*** ***$V_{ab}$ changes sign***

***(C)*** ***$V_{ab}$ increases***

$\boxed{\textit{(D) } V_{ab} \textit{ goes to zero}}$

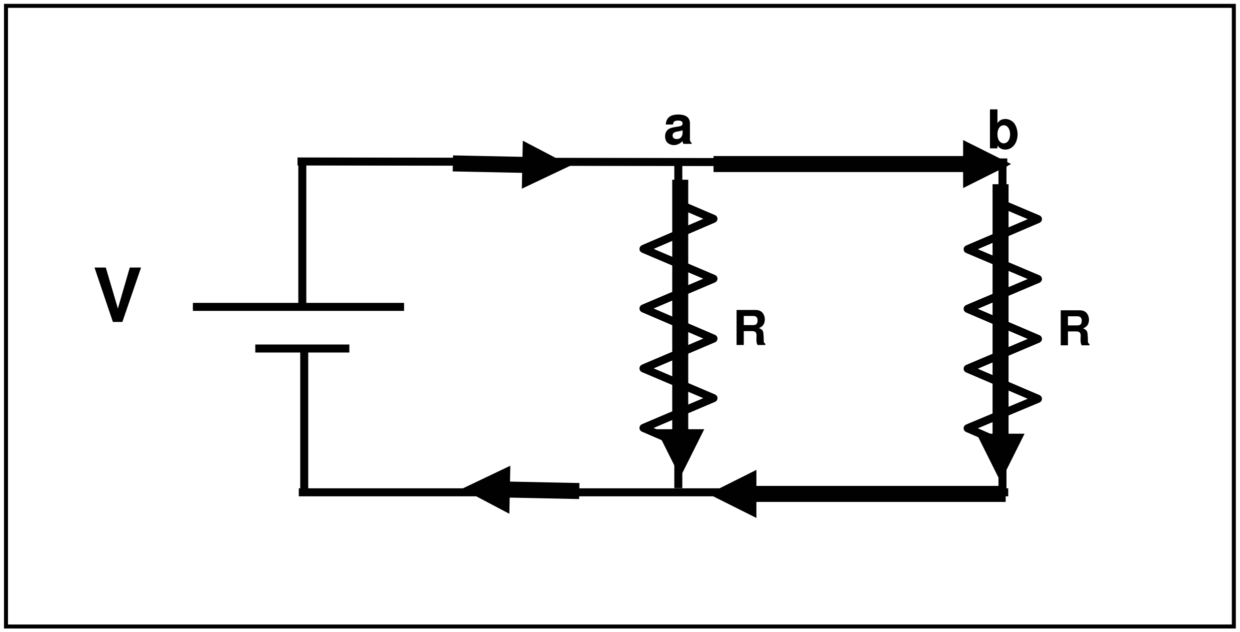
Why?
Redraw:




$V_{cd} = +V$
$V_{bd} = +V$
$V_{ad} = V_{cd} = +V$

$\Rightarrow$ $V_{ab} = V_{ad} - V_{bd} = V - V = 0$





BB

Is there a current flowing between a and b ?

A) Yes
B) No

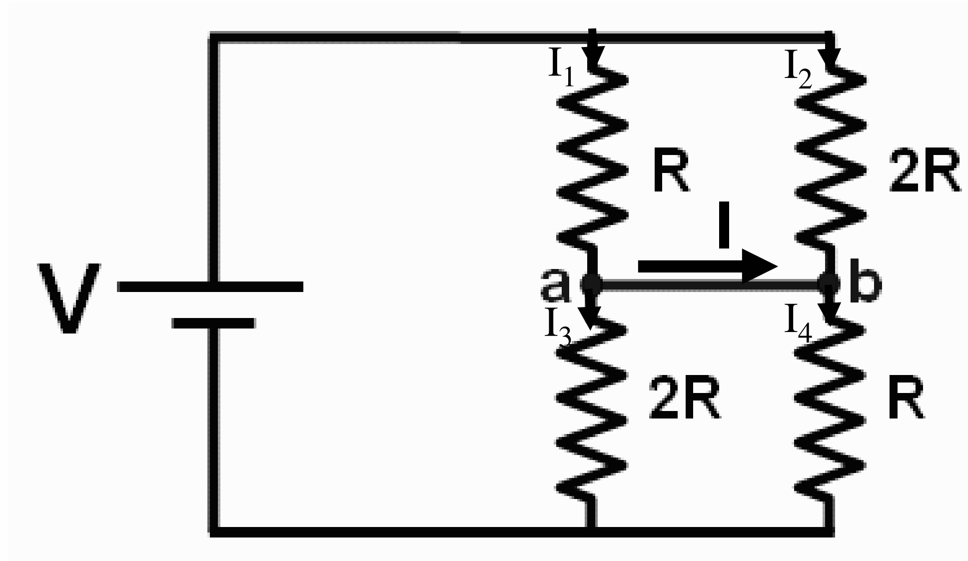
At first guess it might appear that since A & B have the same potential $\not\Rightarrow$ No current flows between A & B

Current flows from battery and splits at A

- Some current flows down
- Some current flows right

Consider the circuit shown below.

## Clicker





BB

Which of the following statements best describes the current flowing in the blue wire connecting points a and b

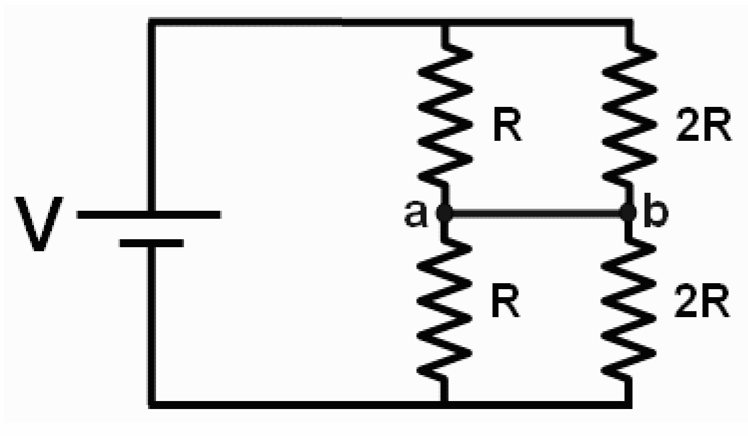
- [x] Positive current flows from a to b
- Positive current flows from b to a
- No current flows between a and b

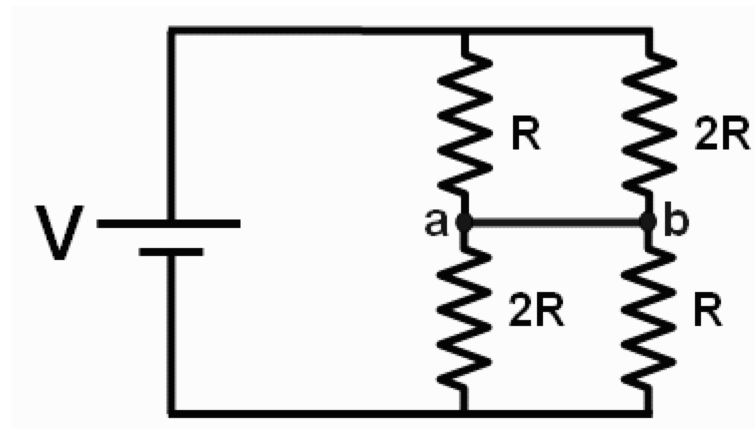
$$I_1R - I_2\,(2R) = 0 \Rightarrow I_1 = 2\,I_2$$

$$I_4R - I_3\,(2R) = 0 \Rightarrow I_4 = 2\,I_3$$

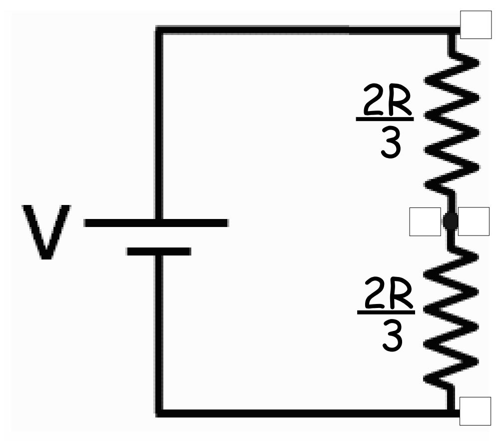
$$I_2 = I_3 \Rightarrow I_1 = 2\,I_3$$

$$\boxed{I = I_1 - I_3 = 2I_3 - I_3 = +I_3}$$

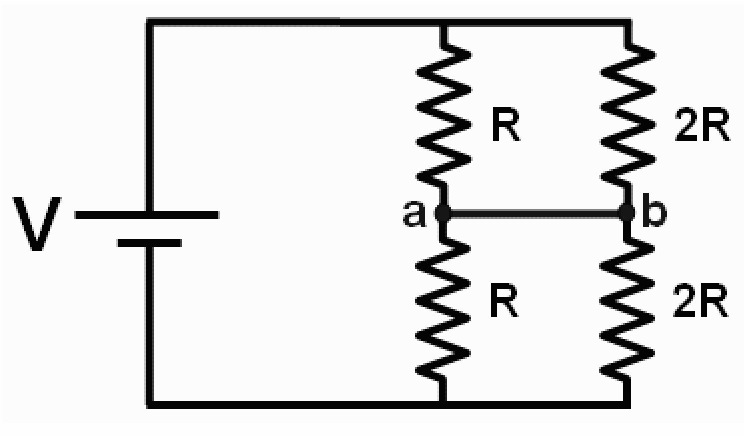


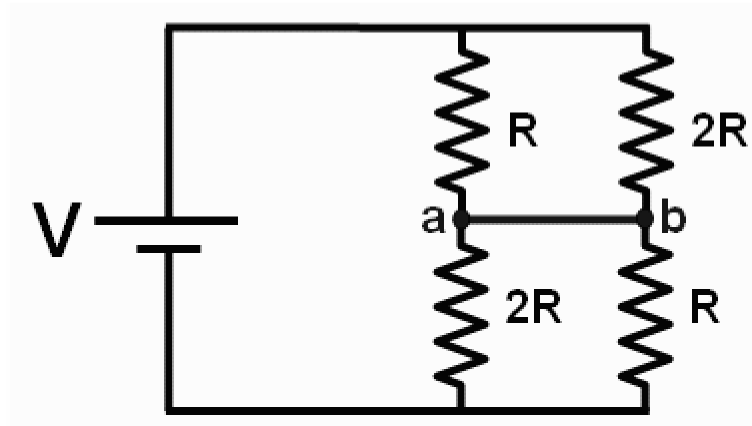




What is the same? Current flowing in and out of the battery

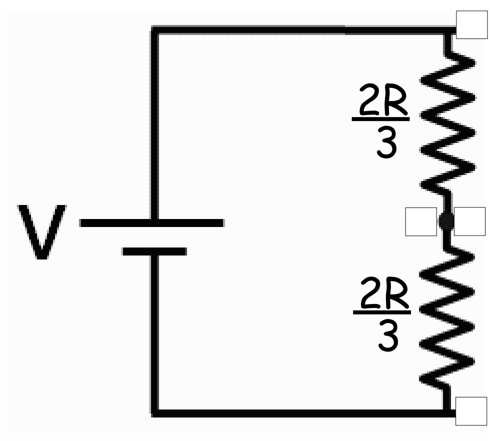



What is different? Current flowing from a to b

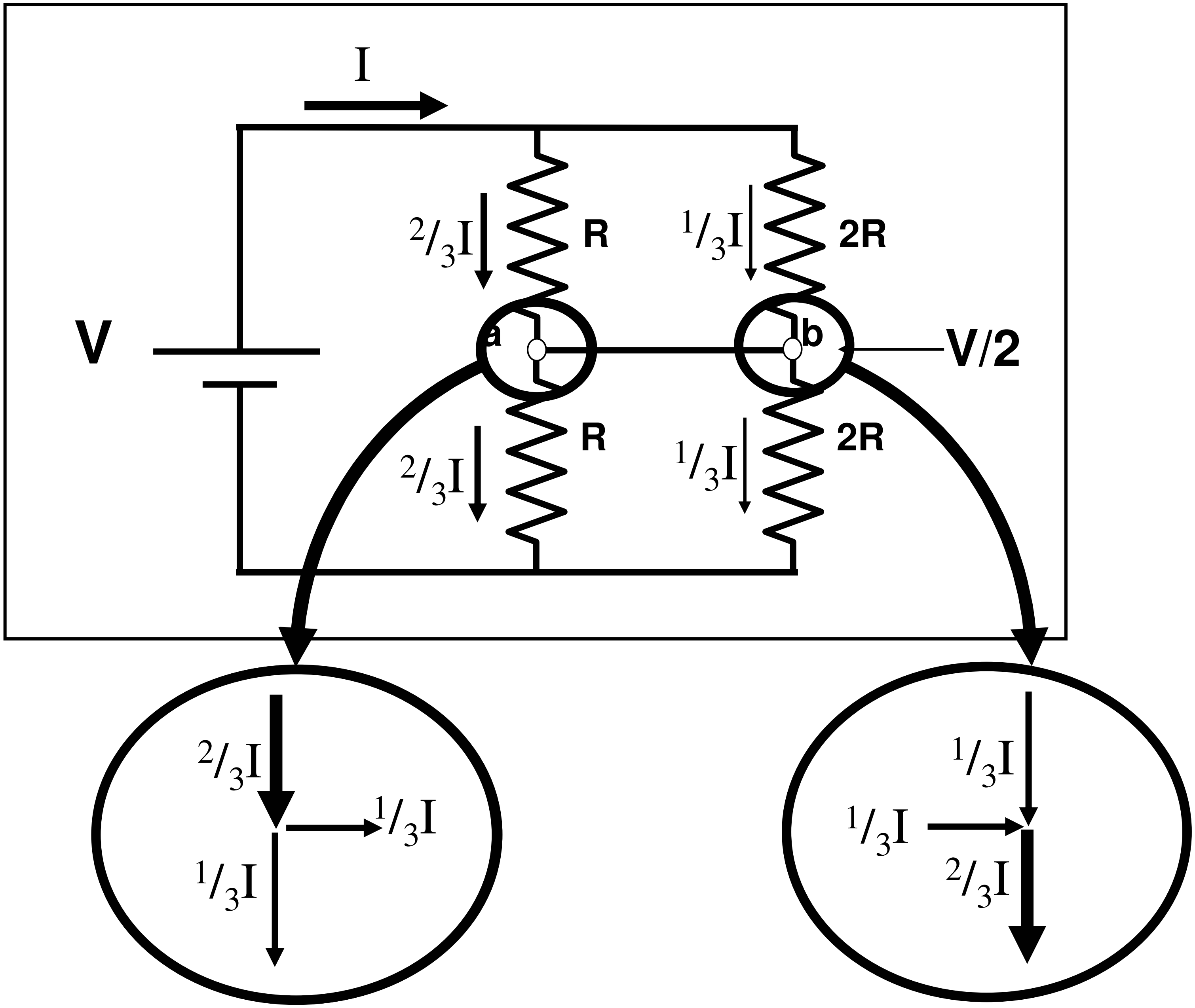



What is the same? Current flowing in and out of the battery



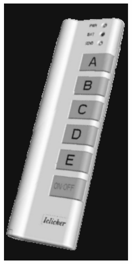

What is different? Current flowing from a to b



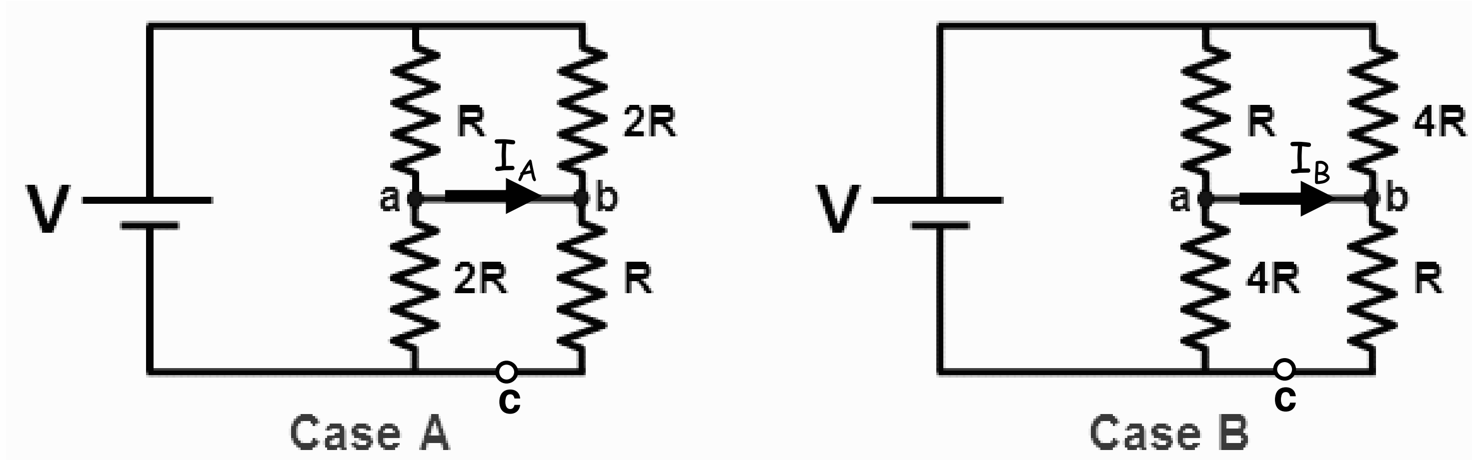
I
V
R
R
2R
2R
a
b
V/2
$^2/_3$I
$^1/_3$I

# Clicker Problem

7) Consider the circuit shown below.



BB




**In which case is the current flowing in the blue wire connecting points a and b biggest**

- ○ Case A
- ○ Case B
- ○ They are the same

Current will flow from left to right in both cases

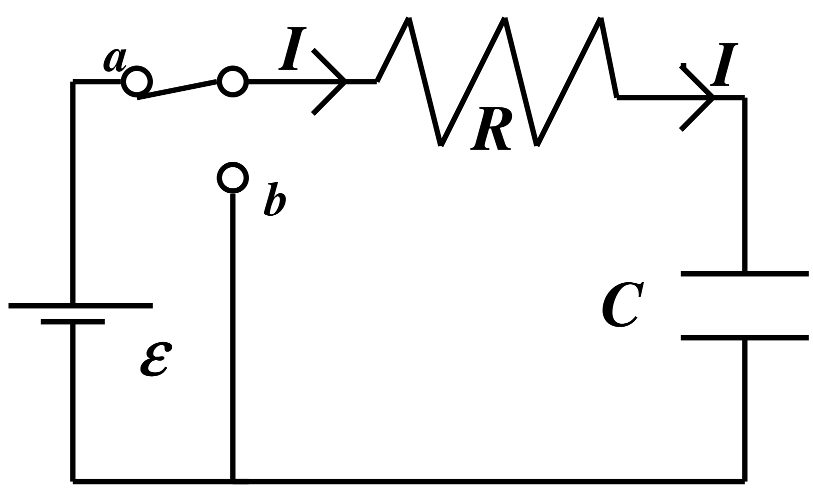
In both cases, $V_{ac} = V/2$

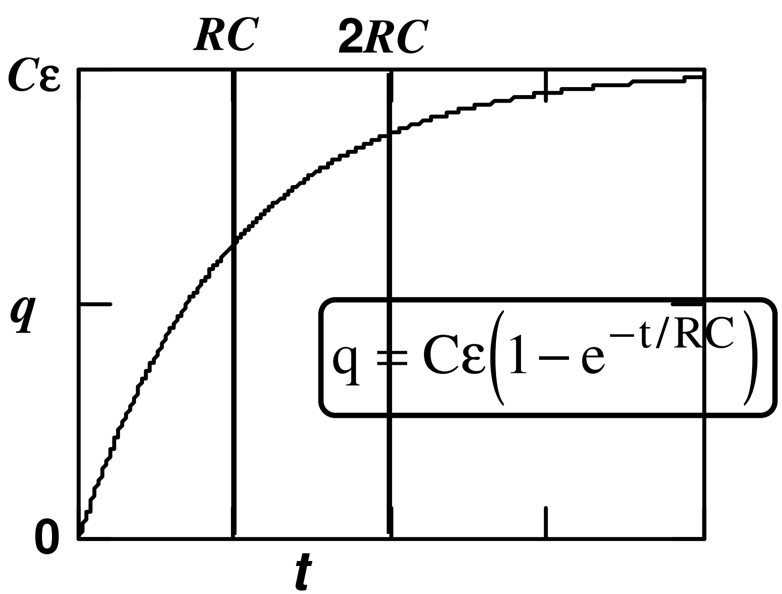
$$I(2R) = 2I(4R)$$

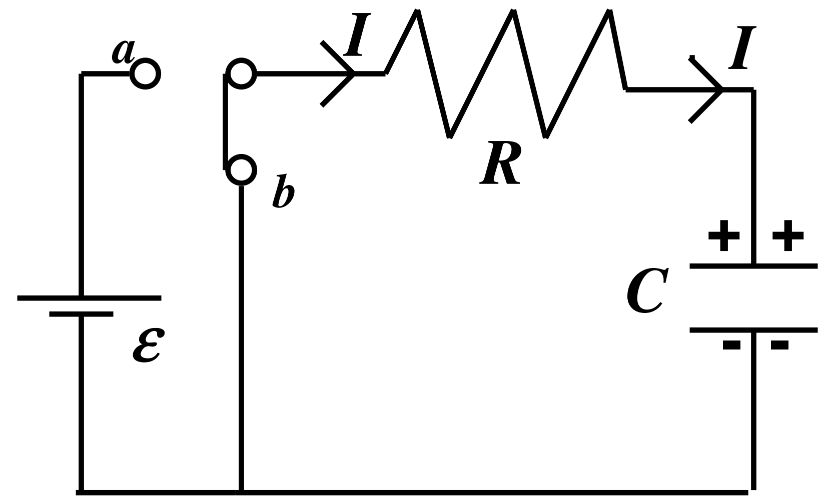
$$I_A = I(R) - I(2R)$$
$$= I(R) - 2I(4R)$$

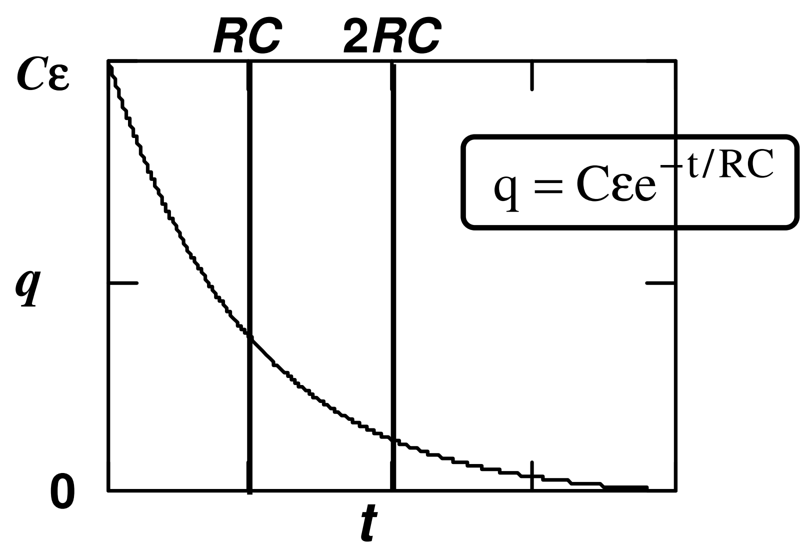
$$I_B = I(R) - I(4R)$$

# RC Circuits


a
b
I
I
R
C
ε


RC
2RC
Cε
q
0
t
$q = C\varepsilon\left(1 - e^{-t/RC}\right)$


a
b
I
I
R
C
ε
+ +
- -

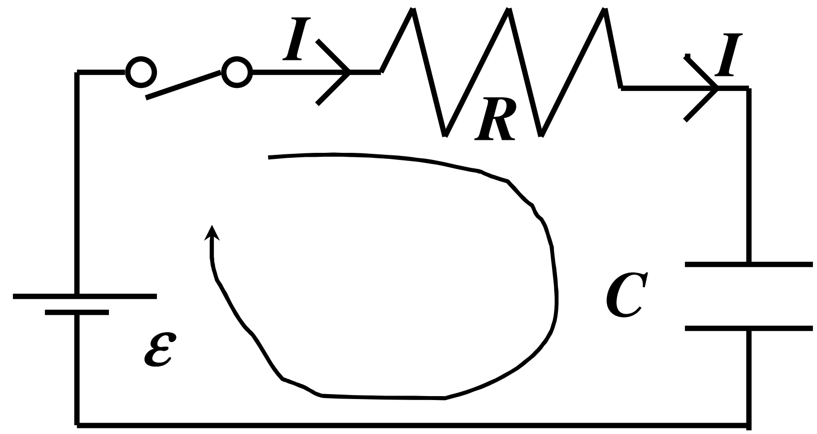

RC
2RC
Cε
q
0
t
$q = C\varepsilon e^{-t/RC}$

# Resistor-capacitor circuits

Let's add a Capacitor to our simple circuit

Recall voltage "drop" on *C*?

$$V = \frac{Q}{C}$$




Write KVL: $\varepsilon - IR - \frac{Q}{C} = 0$

Use $I = \frac{dQ}{dt}$ Now eqn. has only "$Q$":

***KVL gives Differential Equation !*** $$\varepsilon - R\frac{dQ}{dt} - \frac{Q}{C} = 0$$

**We will solve this later. For now, look at qualitative behavior…**

# Capacitors Circuits, Qualitative

**Basic principle: Capacitor resists change in Q →**
**resists changes in V**

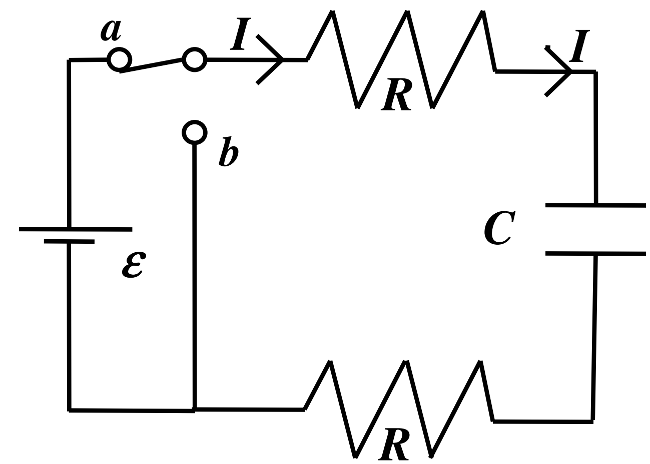
- **Charging (it takes time to put the final charge on)**
  - **Initially, the capacitor behaves like a wire ($\Delta V = 0$, since $Q = 0$).**
  - **As current continues to flow, charge builds up on the capacitor**
    - **→ it then becomes more difficult to add more charge**
    - **→ the current slows down**
  - **After a long time, the capacitor behaves like an open switch.**

- **Discharging**
  - **Initially, the capacitor behaves like a battery.**
  - **After a long time, the capacitor behaves like a wire.**

# Clicker Problem (two parts)

**2A**

- **At $t$=0 the switch is thrown from position $b$ to position $a$ in the circuit shown: The capacitor is initially uncharged.**
  - **What is the value of the current $I_{0+}$ just after the switch is thrown?**




**(a) $I_{0+} = 0$ (b) $I_{0+} = \varepsilon/2R$ (c) $I_{0+} = 2\varepsilon/R$**

---

**2B**

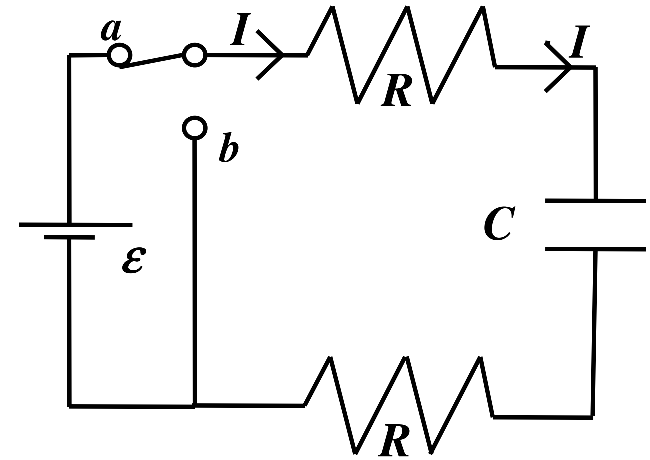
– **What is the value of the current $I_\infty$ after a very long time?**

**(a) $I_\infty = 0$ (b) $I_\infty = \varepsilon/2R$ (c) $I_\infty > 2\varepsilon/R$**

# Clicker problem (2 parts)

**2A**

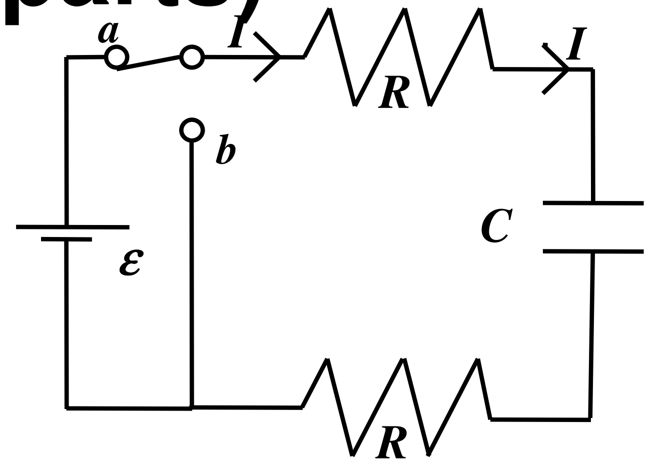
- **At $t$=0 the switch is thrown from position $b$ to position $a$ in the circuit shown: The capacitor is initially uncharged.**
  - **What is the value of the current $I_{0+}$ just after the switch is thrown?**




**(a) $I_{0+} = 0$** **(b) $\boxed{I_{0+} = \varepsilon/2R}$** **(c) $I_{0+} = 2\varepsilon/R$**

- **Just after the switch is thrown, the capacitor still has no charge, therefore the voltage drop across the capacitor = 0!**
- **Applying KVL to the loop at t=0+, $\varepsilon - IR - 0 - IR = 0 \Rightarrow I = \varepsilon/2R$**

# Clicker problem (2 parts)




**2A**

- At $t$=0 the switch is thrown from position $b$ to position $a$ in the circuit shown: The capacitor is initially uncharged.
  – What is the value of the current $I_{0+}$ just after the switch is thrown?

(a) $I_{0+} = 0$ $\boxed{\text{(b) } I_{0+} = \varepsilon/2R}$ (c) $I_{0+} = 2\varepsilon/R$

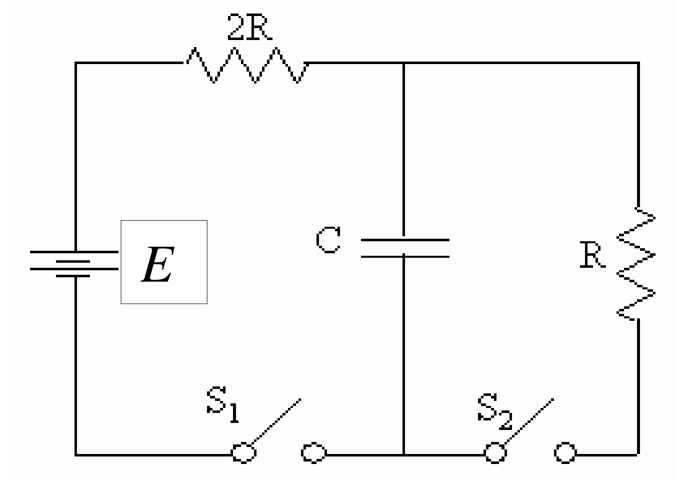
**2B**

– What is the value of the current $I_\infty$ after a very long time?

$\boxed{\text{(a) } I_\infty = 0}$ (b) $I_\infty = \varepsilon/2R$ (c) $I_\infty > 2\varepsilon/R$

- The key here is to realize that as the current continues to flow, the charge on the capacitor continues to grow.
- As the charge on the capacitor continues to grow, the voltage across the capacitor will increase.
- The voltage across the capacitor is limited to $\varepsilon$; the current goes to 0.

**Clicker problem**




**The capacitor is initially uncharged, and the two switches are open.**

3) What is the voltage across the capacitor immediately after switch $S_1$ is closed?

a) $V_c = 0$

b) $V_c = E$

c) $V_c = 1/2\ E$

**Initially:** $\boldsymbol{Q = 0}$

$\boldsymbol{V_C = 0}$ $\boldsymbol{I = E/(2R)}$

4) Find the voltage across the capacitor after the switch has been closed for a very long time.

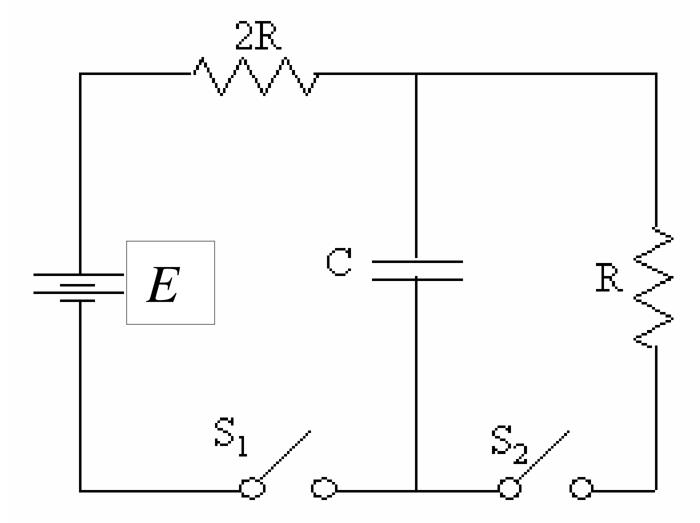
a) $V_c = 0$

b) $V_c = E$

c) $V_c = 1/2\ E$

$\boldsymbol{Q = E\ C}$ $\boldsymbol{I = 0}$

**Clicker problem:**




6) After being closed a long time, switch 1 is opened and switch 2 is closed. What is the current through the right resistor immediately after the switch 2 is closed?

a) $I_R = 0$

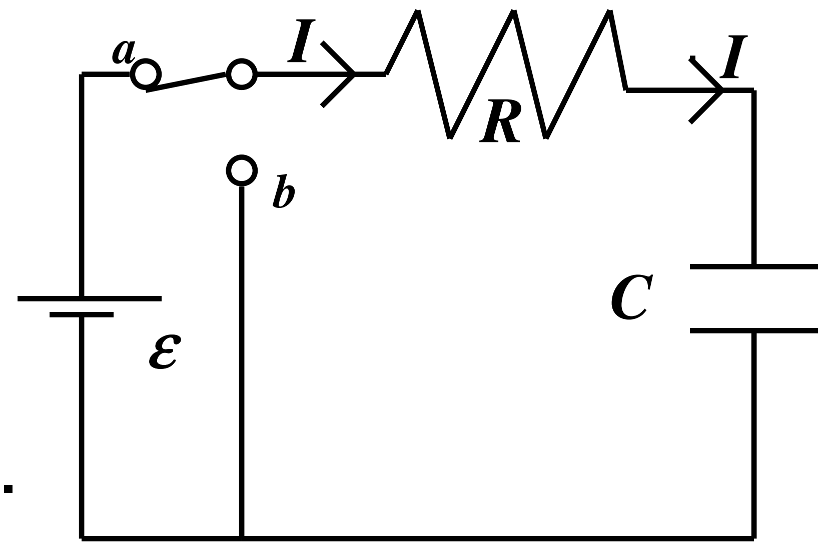
b) $I_R = E/(3R)$

c) $I_R = E/(2R)$

d) $I_R = E/R$

**Now, the battery and the resistor 2*R* are disconnected from the circuit, so we have a different circuit.**
**Since *C* is fully charged, $V_C = E$.**
**Initially, *C* acts like a battery, and $I = V_C/R$.**

# RC Circuits
# (Time-varying currents, charging)




- **Charge capacitor:**

  **$C$ initially uncharged; connect switch to $a$ at $t$=0**

  **Calculate current and**

  **charge as function of time.**

- **Loop theorem** $\Rightarrow$ $\quad \varepsilon - IR - \dfrac{Q}{C} = 0$

**Would it matter where $R$ is placed in the loop??**

**NO!**

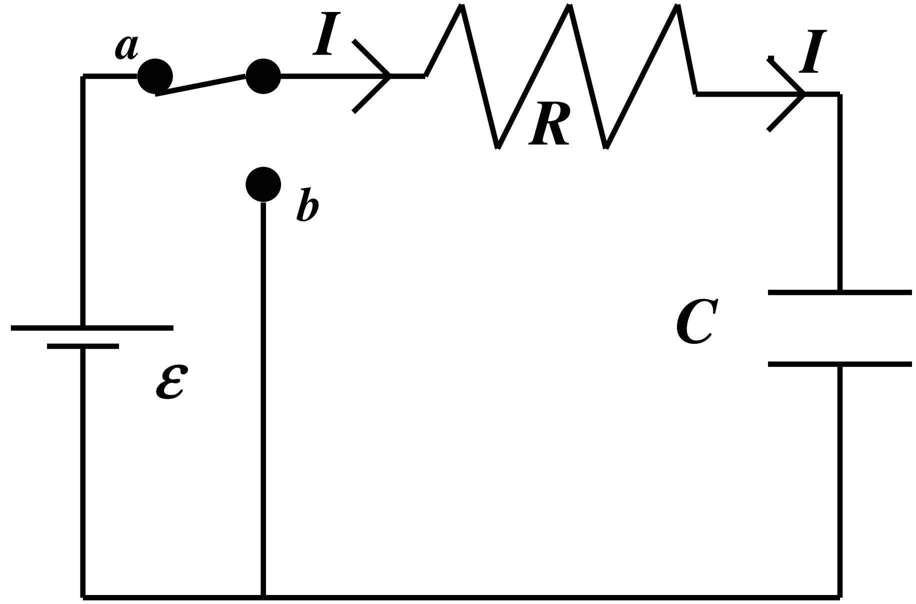
- **Convert to differential equation for $Q$:**

$$I = \frac{dQ}{dt} \quad \Rightarrow \quad \boxed{\varepsilon = R\frac{dQ}{dt} + \frac{Q}{C}}$$

# Charging Capacitor

- **Charge capacitor:**

$$\boxed{\varepsilon = R\frac{dQ}{dt} + \frac{Q}{C}}$$




- **Guess solution:**

$$\boxed{Q = C\varepsilon(1 - e^{-t/RC})}$$

- **Check that it is a solution:**

$$\frac{dQ}{dt} = C\varepsilon e^{-t/RC}\left(\boxed{+}\frac{1}{RC}\right)$$

$$\Rightarrow \quad R\frac{dQ}{dt} + \frac{Q}{C} = \boxed{+}\varepsilon e^{-t/RC} + \varepsilon(1 - e^{-t/RC}) = \varepsilon \quad !$$

**Note that this "guess" fits the boundary conditions:**

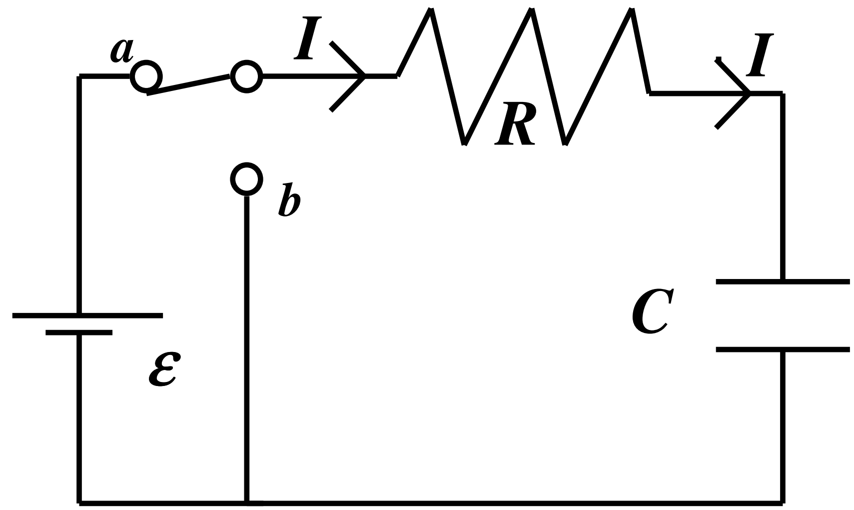
$$t = 0 \Rightarrow Q = 0$$

$$t = \infty \Rightarrow Q = C\varepsilon$$

# Charging Capacitor in a RC circuit

- **Charge capacitor:**

$$\boxed{Q = C\varepsilon\left(1 - e^{-t/RC}\right)}$$

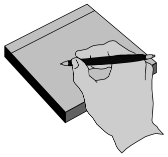



- **Current is found from differentiation:**

$$\boxed{I = \frac{dQ}{dt} = \frac{\varepsilon}{R} e^{-t/RC}} \quad \Rightarrow$$

**Conclusion:**

- **Capacitor reaches its final charge($Q$=$C\varepsilon$ ) exponentially with <u>time constant</u> $\tau$ = $RC$.**
- **Current decays from max (=$\varepsilon/R$) with same time constant.**



# Charging Capacitor

## Charge on $C$

$$Q = C\varepsilon\left(1 - e^{-t/RC}\right)$$

**Max =** $C\varepsilon$

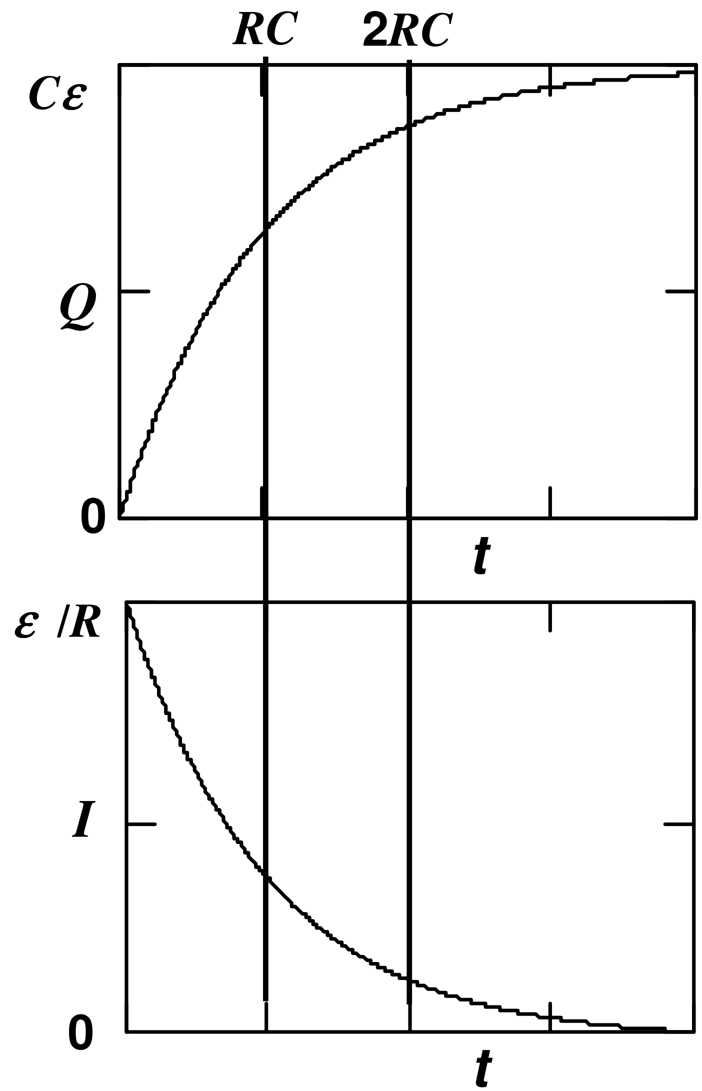
**63% Max at** $t = RC$

## Current

$$I = \frac{dQ}{dt} = \frac{\varepsilon}{R} e^{-t/RC}$$

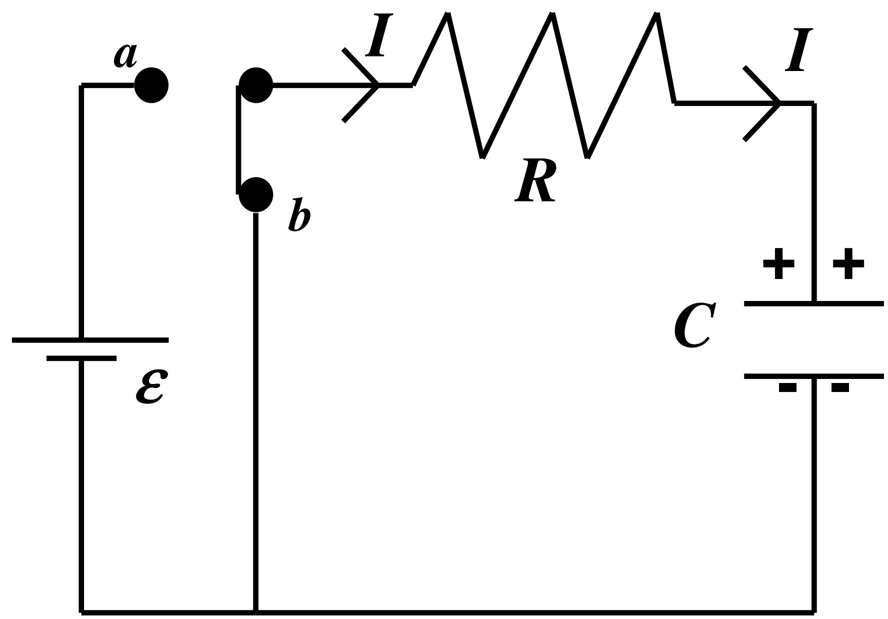
**Max =** $\varepsilon / R$

**37% Max at** $t = RC$



# Discharging Capacitor

$$\boxed{R\frac{dQ}{dt}+\frac{Q}{C}=0}$$




- **Guess solution:**

$$\boxed{Q = Q_0 e^{-t/\tau} = C\varepsilon e^{-t/RC}}$$

- **Check that it is a solution:**

$$\frac{dQ}{dt} = C\varepsilon e^{-t/RC}\left(-\frac{1}{RC}\right)$$

$$\Rightarrow R\frac{dQ}{dt}+\frac{Q}{C} = -\varepsilon e^{-t/RC} + \varepsilon e^{-t/RC} = 0 \ !$$

**Note that this "guess" fits the boundary conditions:**

$$t=0 \Rightarrow Q = C\varepsilon$$

$$t=\infty \Rightarrow Q = 0$$

# Discharging Capacitor

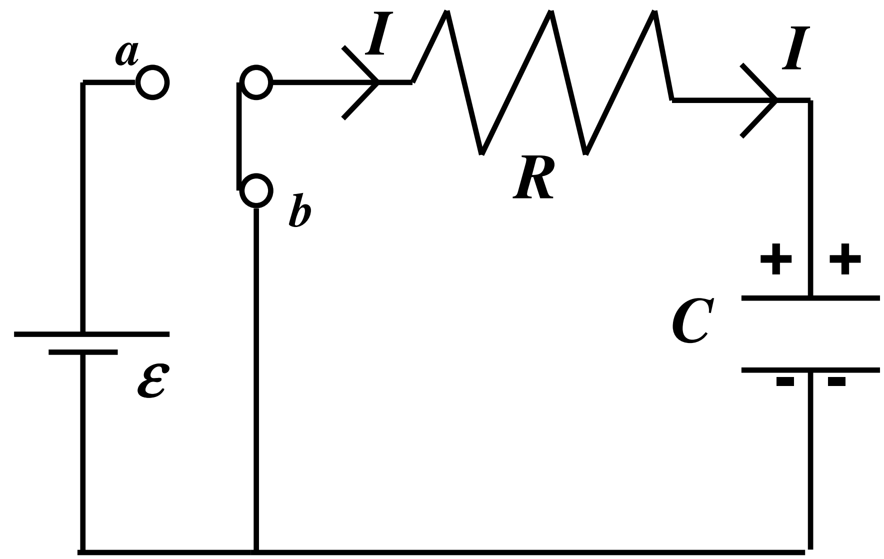
- **Discharge capacitor:**

$$Q = Q_0 e^{-t/\tau} = C\varepsilon e^{-t/RC}$$

- **Current is found from differentiation:**

$$\boxed{I = \frac{dQ}{dt} = -\frac{\varepsilon}{R} e^{-t/RC}}$$

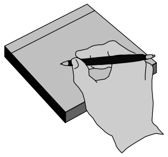
**Minus sign:**
**Current is opposite to original definition, i.e., charges flow *away* from capacitor.**




$\Rightarrow$

**Conclusion:**

- **Capacitor discharges exponentially with time constant $\tau = RC$**
- **Current decays from initial max value (= $-\varepsilon/R$) with same time constant**



# Discharging Capacitor

## Charge on $C$

$$Q = C\varepsilon e^{-t/RC}$$

**Max = $C\varepsilon$**

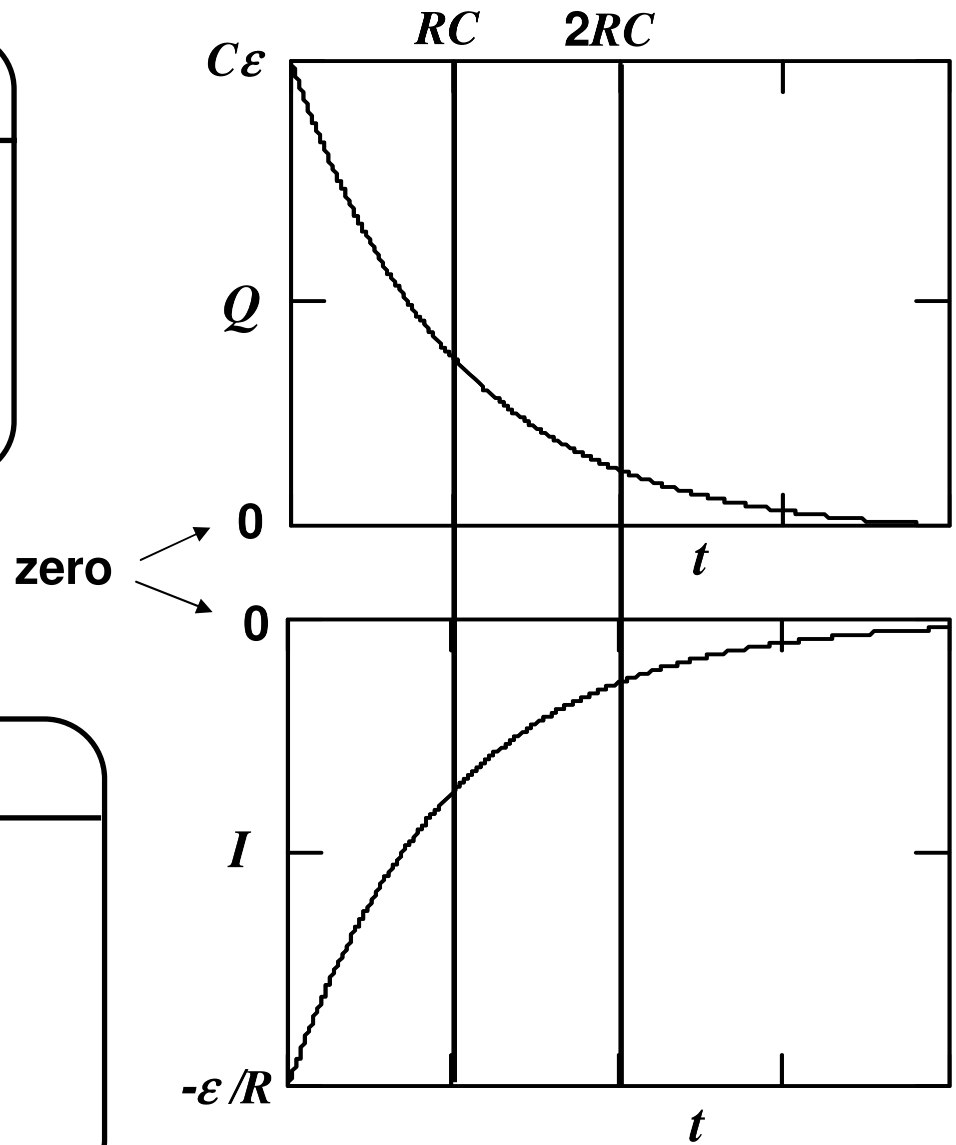
**37% Max at $t = RC$**

## Current

$$I = \frac{dQ}{dt} = -\frac{\varepsilon}{R} e^{-t/RC}$$

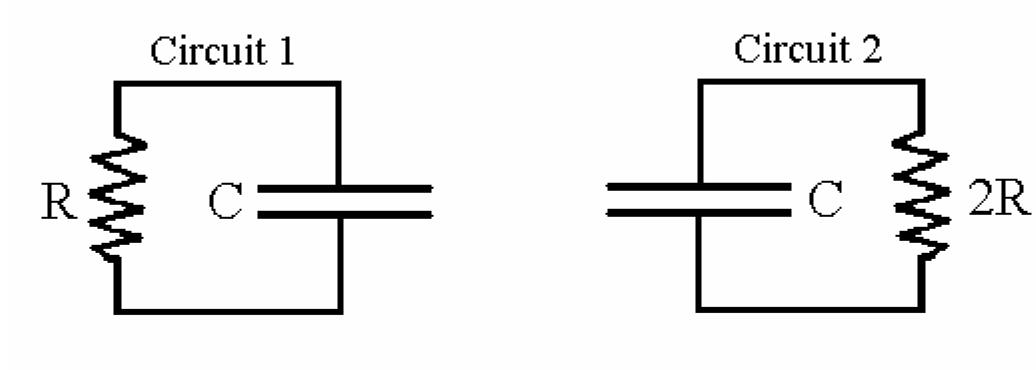
**“Max” = $-\varepsilon/R$**

**37% Max at $t = RC$**



Clicker problem:

The two circuits shown below contain identical fully charged capacitors at $t=0$. Circuit 2 has twice as much resistance as circuit 1.




8) Compare the charge on the two capacitors a short time after $t = 0$

a) $Q_1 > Q_2$

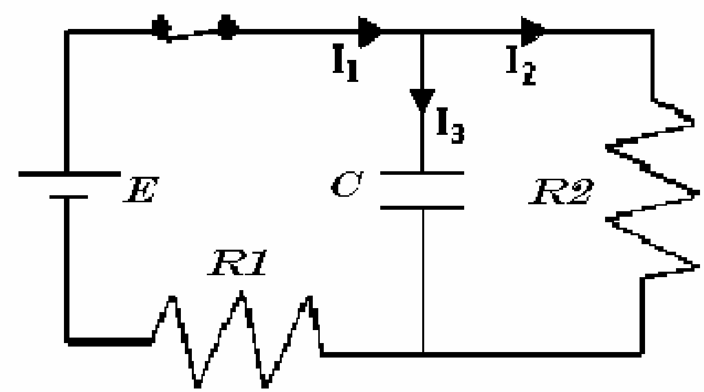
b) $Q_1 = Q_2$

c) $Q_1 < Q_2$

Initially, the charges on the two capacitors are the same. But the two circuits have different time constants: $\tau_1 = RC$ and $\tau_2 = 2RC$. Since $\tau_2 > \tau_1$ it takes circuit 2 longer to discharge its capacitor. Therefore, at any given time, the charge on capacitor 2 is bigger than that on capacitor 1.

Clicker:

The circuit below contains a battery, a switch, a capacitor and two resistors




10) Find the current through $R_1$ after the switch has been closed for a long time.

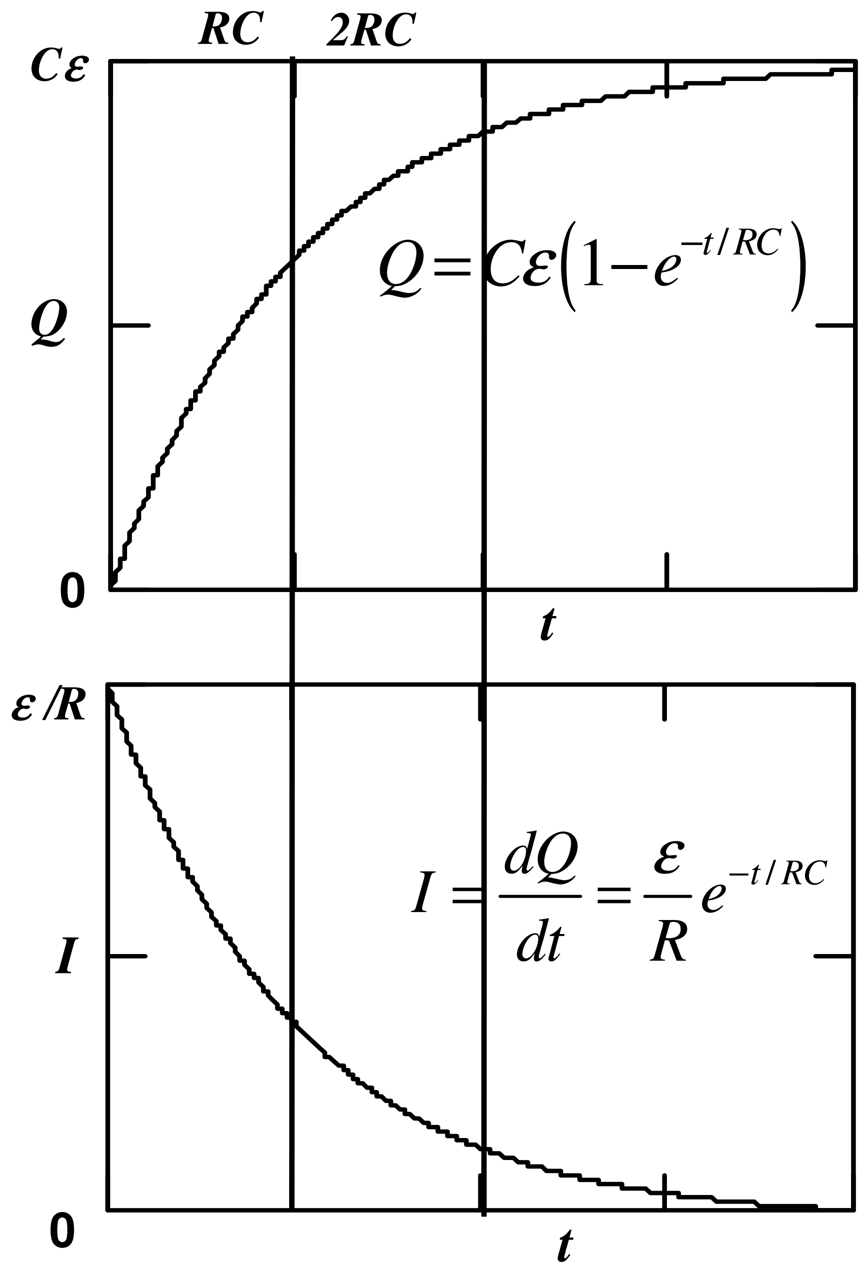
a) $I_1 = 0$ b) $I_1 = E/R_1$ c) $I_1 = \mathrm{E}/(R_1 + R_2)$

**After the switch is closed for a long time …..**

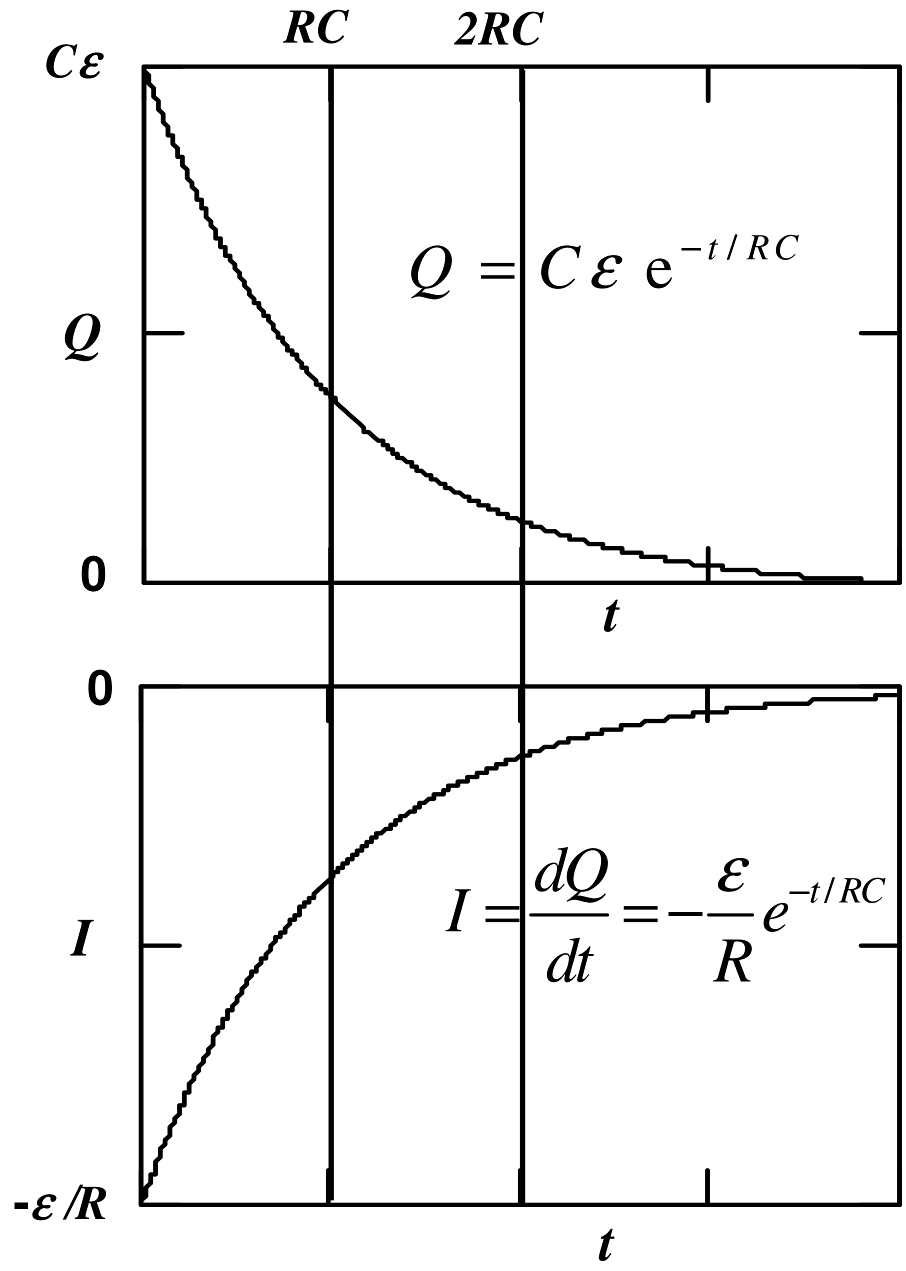
**The capacitor will be fully charged, and $I_3 = 0$.**
**(The capacitor acts like an open switch).**

**So, $I_1 = I_2$, and we have a one-loop circuit with two resistors in series, hence $I_1 = E/(R_1+R_2)$**

# Charging



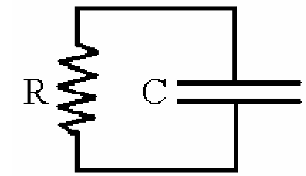

# Discharging



Y&F Problem 26.84 (*<u>Not a clicker problem</u>*)

A resistor with a resistance of 850 Ω is connected to the plates of a charged capacitor with a capacitance of 4.62 μF. Just before the connection is made, the charge on the capacitor is 8.10 mC.

A) What is the energy initially stored in the capacitor?




$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{(8.1x10^{-3})^2}{4.62x10^{-6}} = 7.1\,J$$

B) What is the electrical power dissipated in the resistor just after the connection is made ?

$$P = \frac{V^2}{R} = \frac{(Q/C)^2}{R} = \frac{(8.10x10^{-3}/4.62x10^{-6})^2}{850} = 3600\,W$$

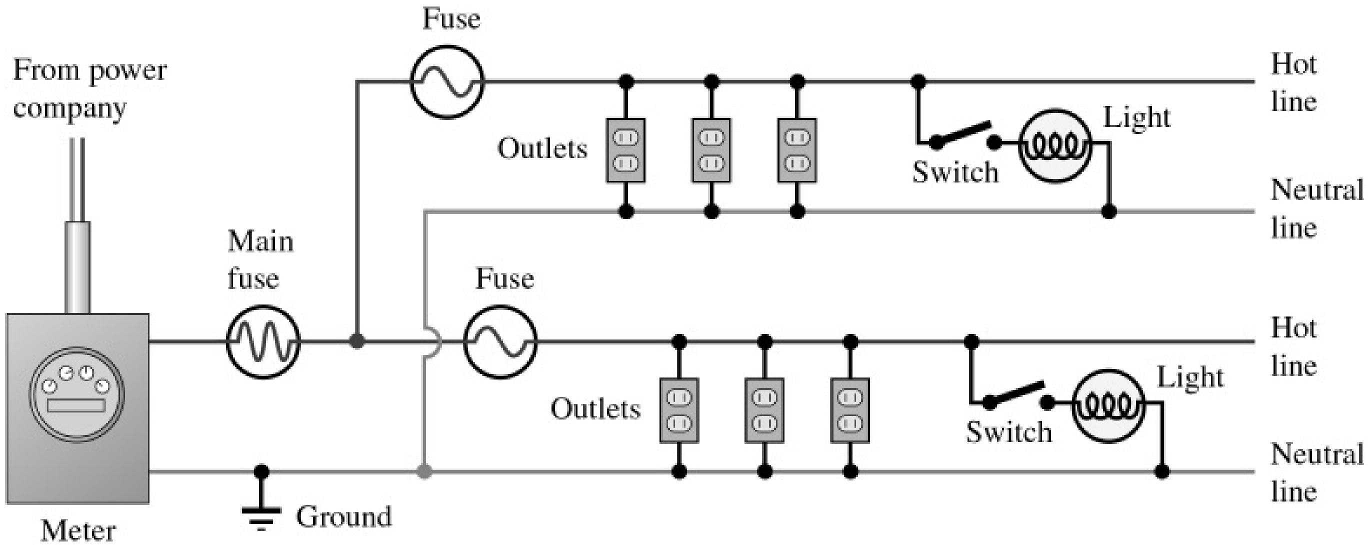
C) At what time has the energy stored in the capacitor has decreased to half the value calculated in part (A)?

$$U_{1/2} = \frac{1}{2}\frac{Q_{1/2}^2}{C} = \frac{1}{2}U_0 = \frac{1}{2}\left(\frac{1}{2}\frac{Q_0^2}{C}\right) \qquad Q_{1/2} = \frac{Q_0}{\sqrt{2}} = Q_0 e^{-t/RC}$$

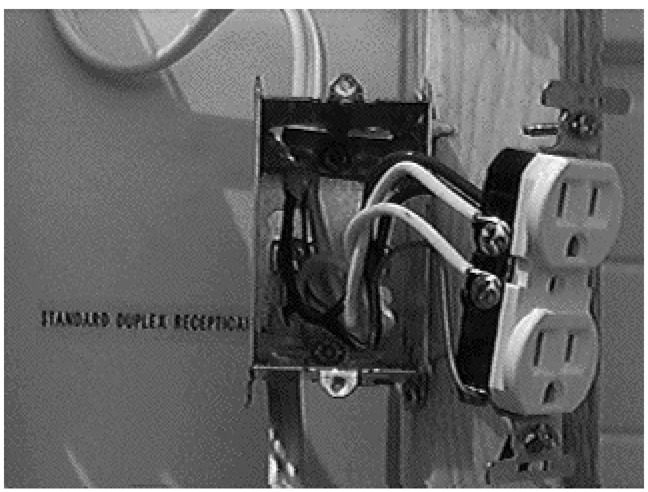
$$-t/RC = \ln(1/\sqrt{2})$$

$$t = -\,(850)(4.62x10^{-6})\ln(1/\sqrt{2}) = 1.36x10^{-3}\,s$$

# Power distribution systems




Conventional House wiring has 240/120V lines



Which prong is hot??

(Hint: Black is Ground)

Remark; houses typically have two single phase, 120V hot lines

# Electricity Costs ($$$$)

Your electric costs are charged in units of energy, Kilowatt-hours

1 Kilowatt-Hour = 1000 watts x 3600 sec
= 3.6 million joules

Oahu residential charges are about 20 cents per K-H. This can be read on your AC Kilowatt-Hour meter outside your house. Electricity costs in Hawaii are high!


Example

100W light bulb run for 10hrs/day for 30 days uses
0.1 KW x 10 x 30 = 30 Kilowatt-Hours and costs $6.00.