From Faraday's Law to Displacement Current

AC generator

Magnetic Levitation Train
Motional Electromotive Force

In Faraday’s Law, we can induce EMF in the loop when the magnetic flux, $\Phi_B$, changes as a function of time. There are two cases when $\Phi_B$ is changing,

1) Change the magnetic field (non-constant over time)
2) Change or move the loop in a constant magnetic field

$$\mathcal{E} = -\int \vec{B} \cdot d\vec{A} = -\frac{d\Phi(B)}{dt}$$
Suppose we move a conducting bar in a constant B field, then a force $\vec{F} = q \vec{v} \times \vec{B}$ moves + charge up and – charge down. The charge distribution produces an electric field and EMF, $\mathcal{E}$, between a & b. This continues until equilibrium is reached.

$$\vec{E} = \frac{\vec{F}}{q} = \frac{q \vec{v} \times \vec{B}}{q} = \vec{v} \times \vec{B} \quad \mathcal{E} = \oint \vec{E} \cdot d\vec{l} = vB l$$

In effect the bar’s motional EMF is an equivalent to a battery EMF.
If the rod is on the U shaped conductor, the charges don't build up at the ends but move through the U shaped portion. They produce an electric field in the circuit. The wire acts as a source of EMF - just like a battery. Called **motional electromotive force**.

\[ \varepsilon = \oint \vec{E} \cdot d\vec{l} = vBl \]
Faraday's Law (continued)

What causes current to flow in wire?
Answer: an $E$ field in the wire.

A changing magnetic flux not only causes a Motional Electromotive Force (EMF) around a loop but an induced electric field. Can write Faraday's Law:

$$
\mathcal{E} = \oint E \cdot d\vec{l} = - \frac{d}{dt} \int \vec{B} \cdot d\vec{A} = - \frac{d\Phi_B}{dt}
$$

Note: For electric fields from static charges, the EMF from a closed path is always zero.
There are two possible sources for electric fields!
Suppose we have electromagnetic that has an increasing magnetic field. Using Faraday’s Law, we predict,

\[ \oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \oint \vec{B} \cdot d\vec{A} = -\frac{d\Phi_B}{dt} \]

If we take a circular path inside and centered on the magnet center axis, the electric field will be tangent to the circle. (E field lines are circles.) NOTE such an E field can never be made by static charges.

E field lines will look like an onion slice.

N.B. there are no wire loops, E fields can appear w/o loops.
If we place a loop there, a current would flow in the loop.
Induced Electric Fields; example

If we have a solenoid coil with changing current there will be circular electric fields created outside the solenoid. It looks very much like the magnetic field around a current carrying wire, but it is an E field and there are no wires or loops.

Note the E fields are predicted by Faraday eqn.

\[ \oint E \cdot d\ell = - \frac{d}{dt} \oint B \cdot dA = - \frac{d\Phi_B}{dt} \]
### Maxwell’s Equations (integral form)

<table>
<thead>
<tr>
<th>Name</th>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauss’ Law for Electricity</td>
<td>[ \Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0} ]</td>
<td>Charge and electric fields</td>
</tr>
<tr>
<td>Gauss’ Law for Magnetism</td>
<td>[ \Phi_B = \oint \vec{B} \cdot d\vec{A} = 0 ]</td>
<td>Magnetic fields</td>
</tr>
<tr>
<td>Faraday’s Law</td>
<td>[ \int \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} ]</td>
<td>Electrical effects from changing B field</td>
</tr>
<tr>
<td>Ampere’s Law</td>
<td>[ \int \vec{B} \cdot d\vec{l} = \mu_0 i ]</td>
<td>Magnetic effects from current</td>
</tr>
</tbody>
</table>

There is a serious asymmetry.

Needs to be modified. + ?
Remarks on Gauss Law's with different closed surfaces

\[ \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\varepsilon_0} \]

\[ \oint \vec{B} \cdot d\vec{A} = 0 \]

Surfaces for integration of E flux

- cylinder
- square
- bagel
- sphere
Remarks on Faraday’s Law with different attached surfaces

\[ \oint \vec{E} \cdot d\vec{l} = - \frac{d}{dt} \oint \vec{B} \cdot d\vec{A} \]

Faraday’s Law works for any closed Loop and any attached surface area.

Line integral defines the Closed loop

Surface area integration for B flux

- disk
- cylinder
- Fish bowl
Ampere’s original law, \( \int \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclose}} \), is incomplete. Consider the parallel plate capacitor and suppose a current \( i_c \) is flowing charging up the plate. If Ampere’s law is applied for the given path in either the plane surface or the bulging surface we should get the same results, but the bulging surface has \( i_c = 0 \), so something is missing.
Generalized Ampere’s Law and displacement current

Maxwell solved dilemma by adding an addition term called displacement current, \( i_D = \varepsilon \frac{d\Phi_E}{dt} \), in analogy to Faraday’s Law.

\[
\int \vec{B} \cdot d\vec{l} = \mu_0 (i_c + i_D) = \mu_0 \left( i_c + \varepsilon_0 \frac{d\Phi_E}{dt} \right)
\]

Current is once more continuous: \( i_D \) between the plates = \( i_c \) in the wire.

\[
q = CV = \frac{\varepsilon A}{d} (Ed) = \varepsilon EA = \varepsilon \Phi_E
\]

\[
\frac{dq}{dt} = i_c = \varepsilon \frac{d\Phi_E}{dt}
\]
Summary of Faraday's Law

\[\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}\]

If we form any closed loop, the line integral of the electric field equals the time rate change of magnetic flux through the surface enclosed by the loop.

If there is a changing magnetic field, then there will be electric fields induced in closed paths. The electric fields direction will tend to reduce the changing B field.

Note; it does not matter if there is a wire loop or an imaginary closed path, an E field will be induced. Potential has no meaning in this non-conservative E field.
What is the displacement current in regions I and III?

A) 2 A B) 1 A C) 0 D) -2 A

Charge is flowing onto this parallel plate capacitor at a rate \( \frac{dQ}{dt} = 2 \, \text{A} \).
What is the displacement current in region II?

A) $-2/3\, \text{A}$  B) $1\, \text{A}$  C) $2\, \text{A}$  D) $0\, \text{A}$

Charge is flowing onto this parallel plate capacitor at a rate $\frac{dQ}{dt}=2\, \text{A}$.
**Summary of Ampere's Generalized Law**

$$\int \overrightarrow{B} \cdot d\overrightarrow{l} = \mu_0 \left( i_c + \varepsilon_0 \frac{d\Phi_E}{dt} \right)$$

If we form any closed loop, the line integral of the B field is nonzero if there is (constant or changing) current through the loop.

If there is a changing electric field through the loop, then there will be magnetic fields induced about a closed loop path.
Maxwell’s Equations

James Clerk Maxwell (1831-1879)

- generalized Ampere’s Law
- made equations symmetric:
  - a changing magnetic field produces an electric field
  - a changing electric field produces a magnetic field
- Showed that Maxwell’s equations predicted electromagnetic waves
- Unified electricity and magnetism and light.

All of electricity and magnetism can be summarized by Maxwell’s Equations.
# Maxwell’s Equations (integral form)

<table>
<thead>
<tr>
<th>Name</th>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauss’ Law for Electricity</td>
<td>( \Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0} )</td>
<td>Charge and electric fields</td>
</tr>
<tr>
<td>Gauss’ Law for Magnetism</td>
<td>( \Phi_B = \oint \vec{B} \cdot d\vec{A} = 0 )</td>
<td>Magnetic fields</td>
</tr>
<tr>
<td>Faraday’s Law</td>
<td>( \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} )</td>
<td>Electrical effects from changing B field</td>
</tr>
<tr>
<td>Ampere’s generalized law</td>
<td>( \int \vec{B} \cdot d\vec{l} = \mu_0 \left( i_c + \varepsilon_0 \frac{d\Phi_E}{dt} \right) )</td>
<td>Magnetic effects from current</td>
</tr>
</tbody>
</table>