General Physics (PHY 170)

Chap. 3

Position, Displacement, Velocity, & Acceleration Vectors

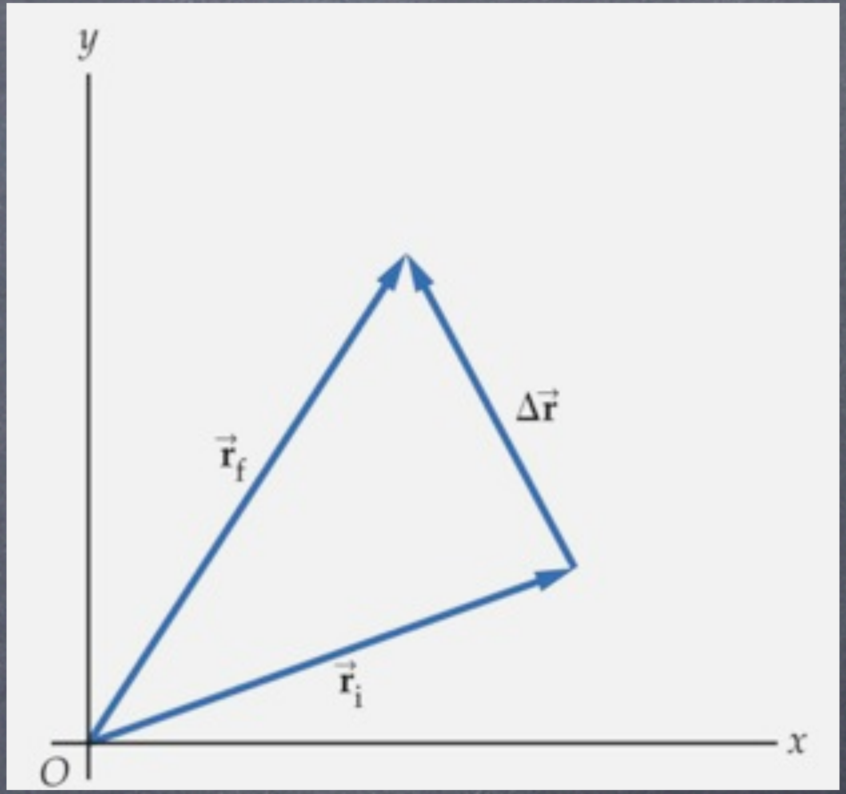
Motion in Two Dimensions

- Using + or – signs is not always sufficient to fully describe motion in more than one dimension
  - Vectors can be used to more fully describe motion: displacement, velocity, and acceleration

Position, Displacement, Velocity, & Acceleration Vectors

The position vector $\vec{r}$ points from the origin to the location in question.

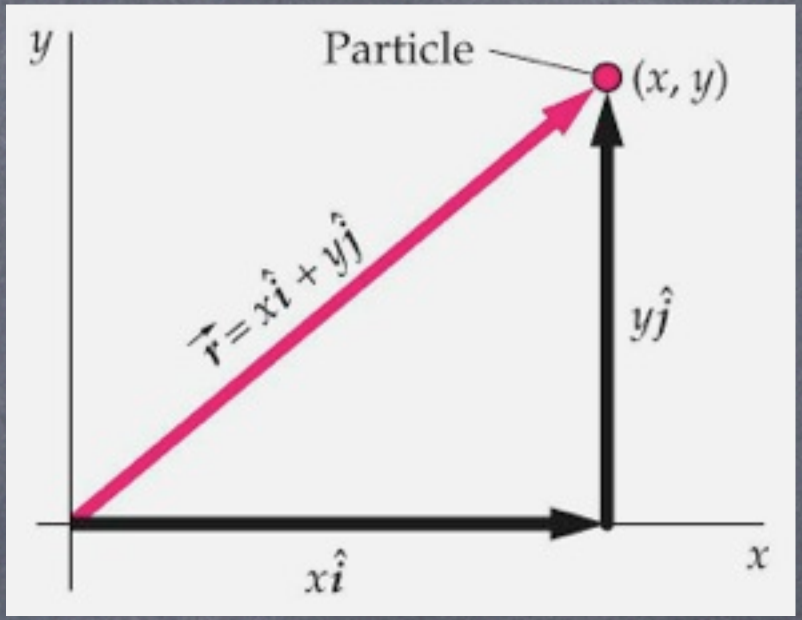
$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$



The displacement vector $\Delta\vec{r}$ points from the original position to the final position.

The Displacement Vector

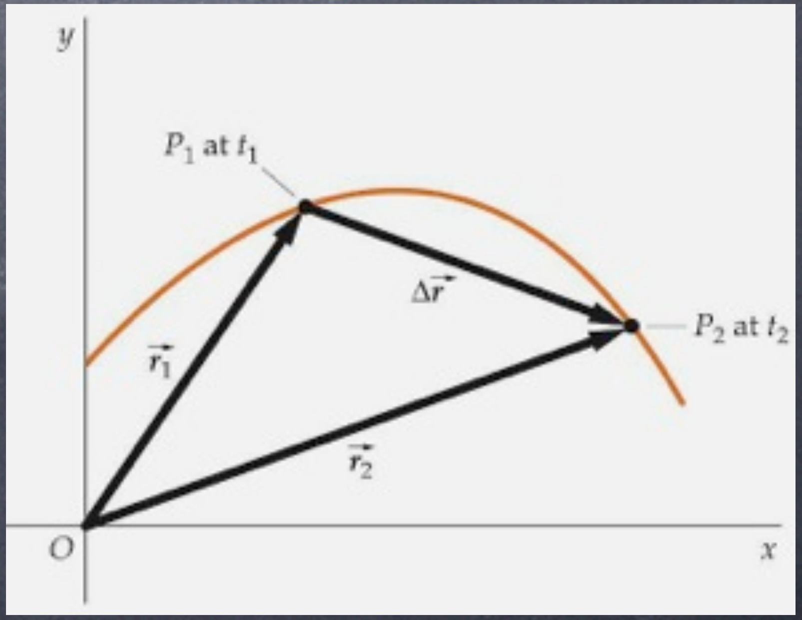
$\vec{r} = X\hat{x} + Y\hat{y}$



$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 =$

$= (X_2\hat{x} + Y_2\hat{y}) - (X_1\hat{x} + Y_1\hat{y})$

$= \Delta x\hat{x} + \Delta y\hat{y}$

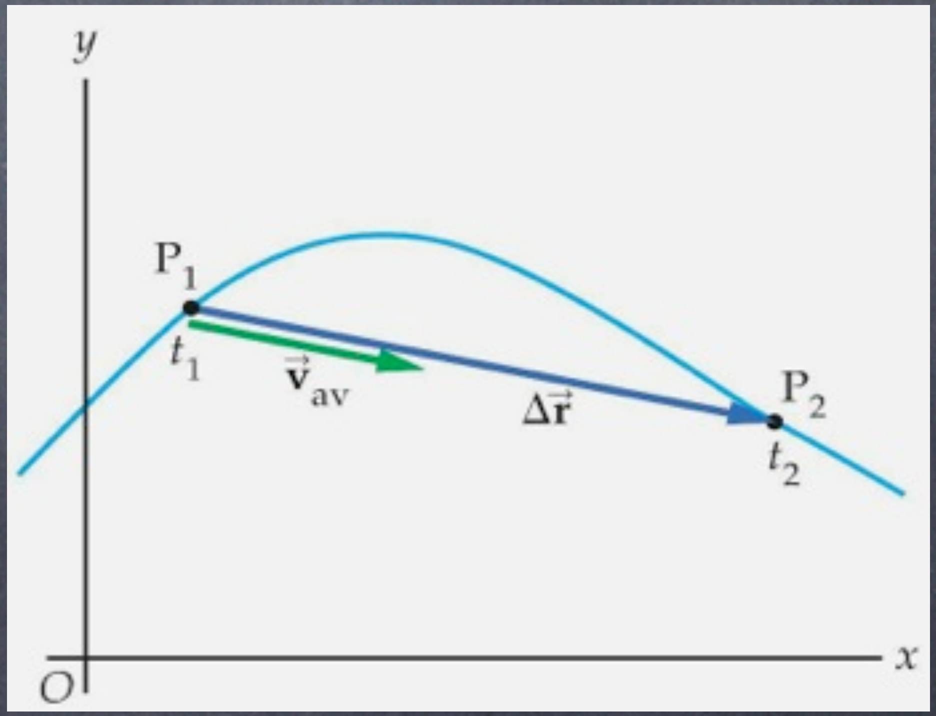


The Average Velocity Vector

Average velocity vector: is the ratio of the displacement to the time interval for the displacement

$$\vec{\mathbf{v}}_{av} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$

$\overrightarrow{V}_{av}$ is in the same direction as $\overrightarrow{\Delta r}$.

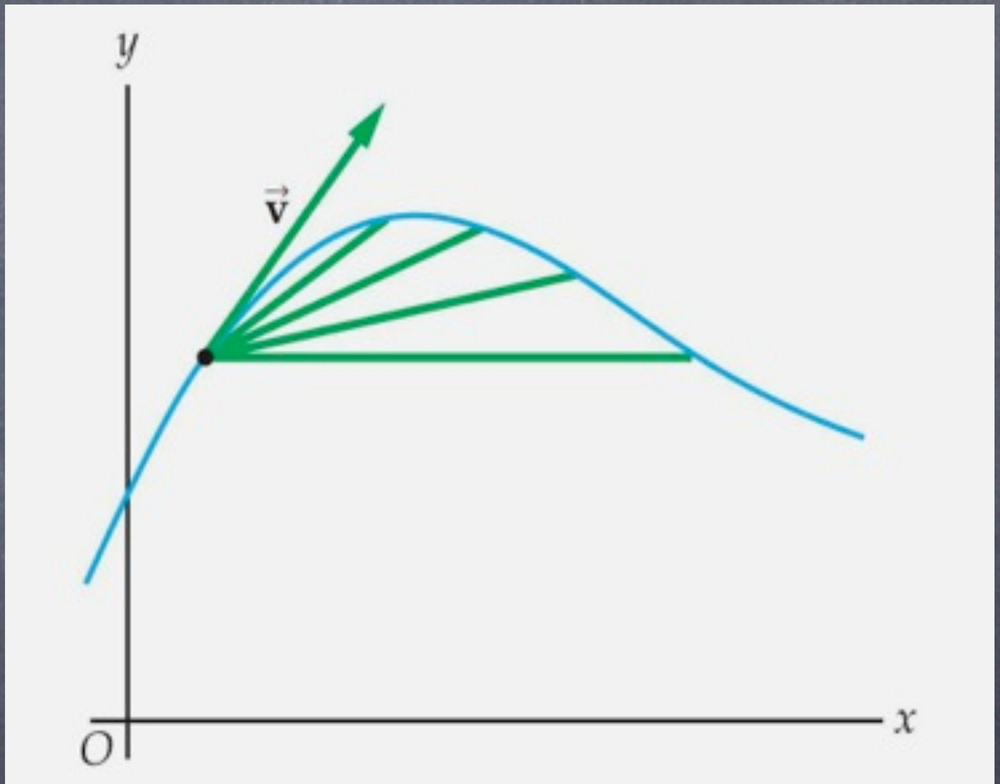


The Instantaneous Velocity Vectors

Instantaneous velocity vector:

is the limit of the average velocity as Δt approaches zero.

Its direction is along a line that is tangent to the path of the particle and in the direction of motion.



$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} \quad \text{(instantaneous velocity vector)} \quad (3.3)$$

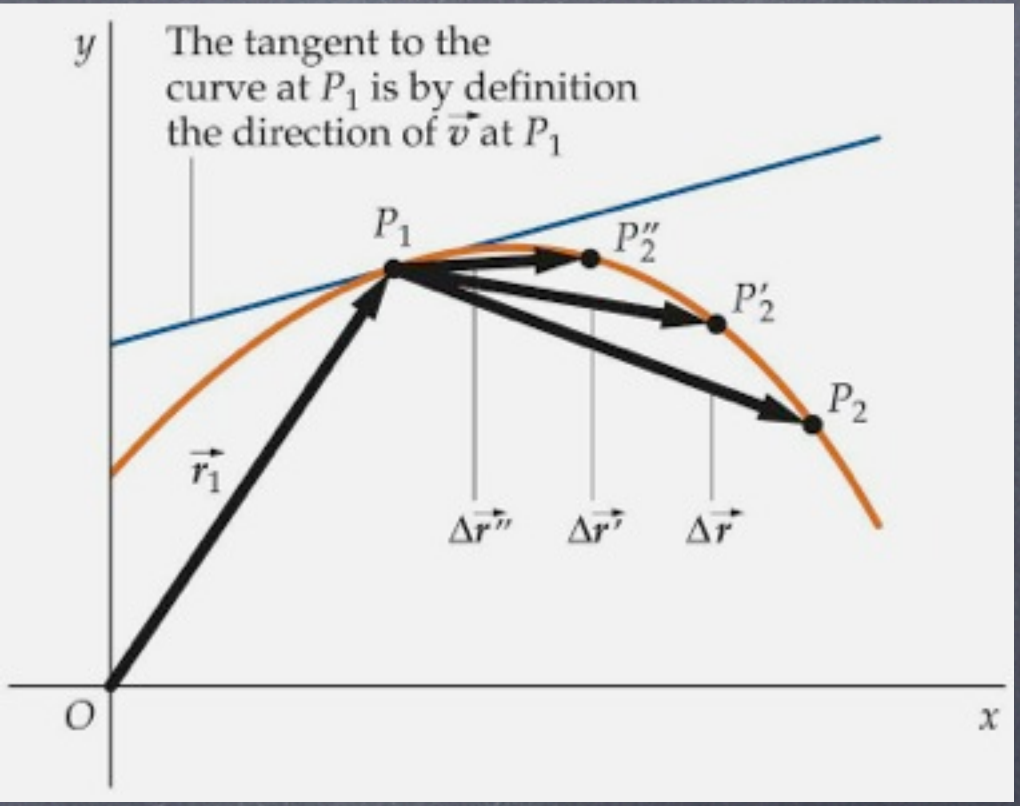
Velocity Vectors

$$\vec{v}_{av} = \frac{\overrightarrow{\Delta r}}{\Delta t} \qquad \vec{v} = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta r}}{\Delta t}$$

$$\begin{aligned}\overrightarrow{\Delta r} &= \vec{r}_2 - \vec{r}_1 \\ &= (x_2\hat{x} + y_2\hat{y}) - (x_1\hat{x} + y_1\hat{y}) \\ &= \Delta x\hat{x} + \Delta y\hat{y}\end{aligned}$$

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta r}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta x\hat{x} + \Delta y\hat{y}}{\Delta t} = \lim_{\Delta t \to 0}\left(\frac{\Delta x}{\Delta t}\right)\hat{x} + \lim_{\Delta t \to 0}\left(\frac{\Delta y}{\Delta t}\right)\hat{y}$$

$$\vec{v} = v_x\hat{x} + v_y\hat{y} \qquad v = \sqrt{v_x^2 + v_y^2}; \quad \theta = \arctan\frac{v_y}{v_x}$$



Example: Velocity of a Sailboat

A sailboat has coordinates (130 m, 205 m) at $t_1=0.0$ s. Two minutes later its position is (110 m, 218 m). (a) Find $\vec{v}_{av}$; (b) Find $|v_{av}|$;

$$\vec{v}_{av} = v_{x\,av}\hat{x} + v_{y\,av}\hat{y}$$

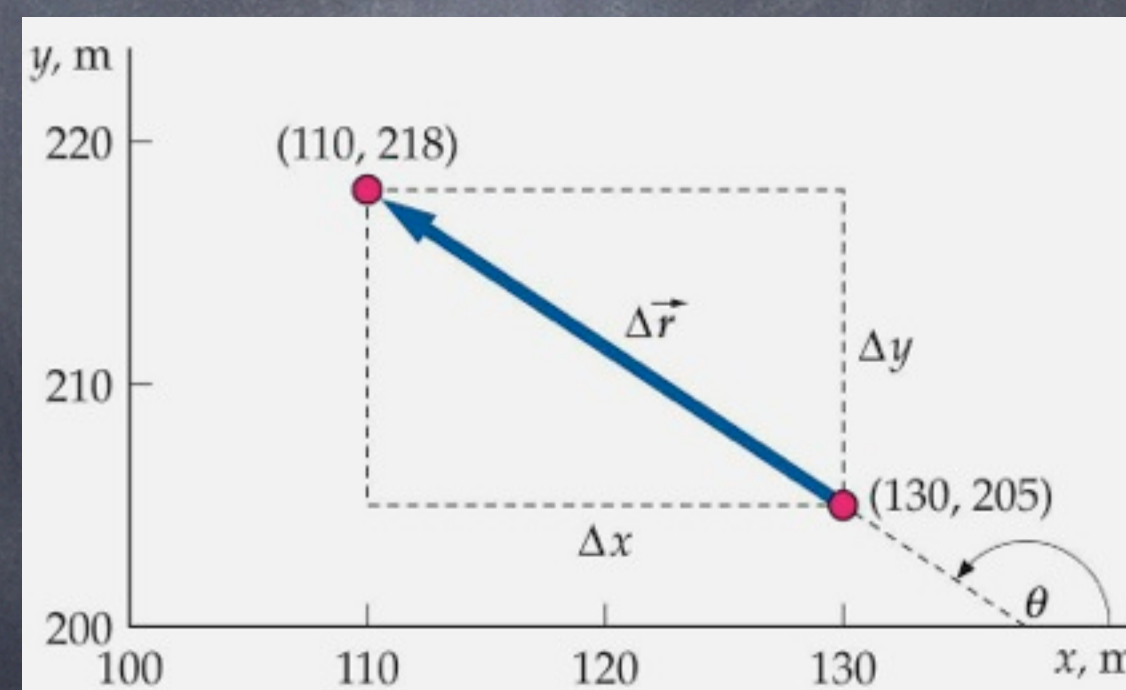
$$v_{x\,av} = \frac{\Delta x}{\Delta t} = \frac{110\text{ m} - 130\text{ m}}{120\text{ s}} = -0.167\text{ m/s}$$

$$v_{y\,av} = \frac{\Delta y}{\Delta t} = \frac{218\text{ m} - 205\text{ m}}{120\text{ s}} = 0.108\text{ m/s}$$

$$\vec{v}_{av} = (-0.167\text{ m/s})\hat{x} + (0.108\text{ m/s})\hat{y}$$

$$v_{av} = \sqrt{(-0.167\text{ m/s})^2 + (0.108\text{ m/s})^2} = 0.199\text{ m/s}$$

$$\theta = \arctan\frac{0.108\text{ m/s}}{-0.167\text{ m/s}} = 147^\circ$$



Example: A Dragonfly

A dragonfly is observed initially at position:

$$\vec{r_1} = (2.00\ \text{m})\hat{x} + (3.50\ \text{m})\hat{y}$$

Three seconds later, it is observed at position:

$$\vec{r_2} = (-3.00\ \text{m})\hat{x} + (5.50\ \text{m})\hat{y}$$

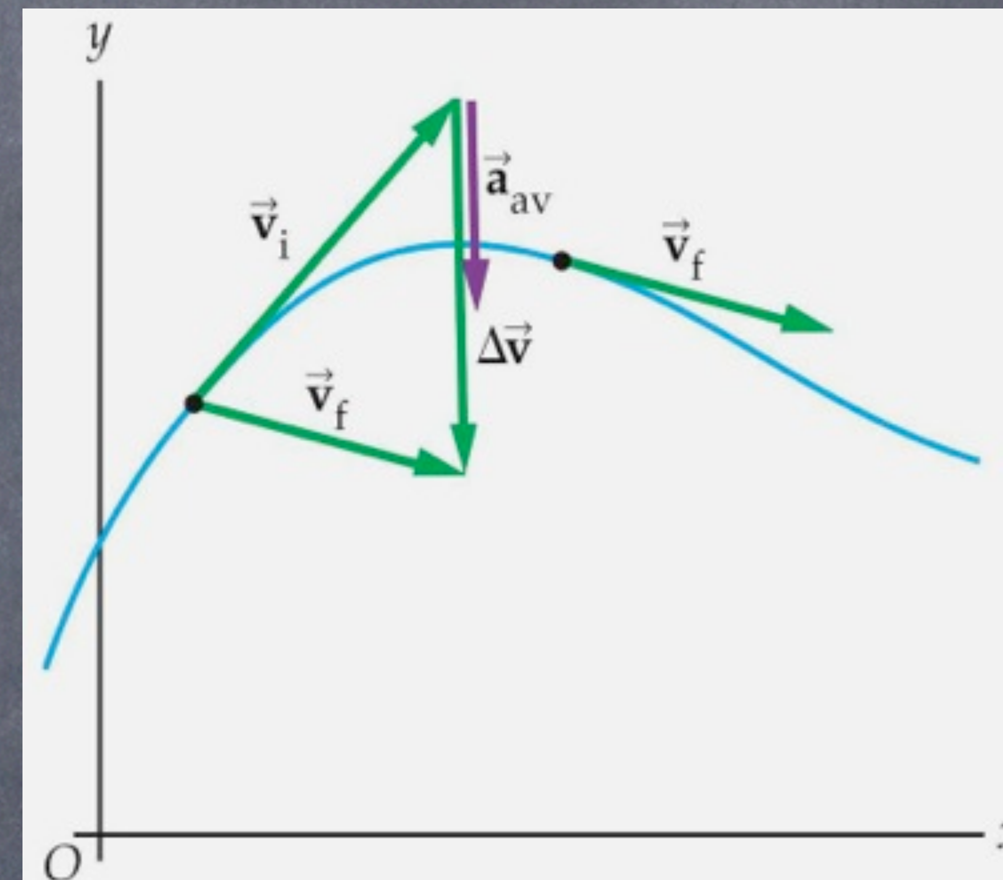
What was the dragonfly's average velocity during this time?

$$\vec{v_{av}} = \frac{\vec{\Delta r}}{\Delta t} = \frac{\vec{r_2} - \vec{r_1}}{\Delta t}$$
$$= \frac{\left[(-3.00\ \text{m}) - (2.00\ \text{m})\right]\hat{x} + \left[(5.50\ \text{m}) - (3.50\ \text{m})\right]\hat{y}}{(3.00\ \text{s})}$$
$$= (-1.67\ \text{m/s})\hat{x} + (0.667\ \text{m/s})\hat{y}$$

Acceleration Vectors

The average acceleration vector: is defined as the rate at which the velocity changes. It is in the direction of the change in velocity $\Delta \vec{v}$.

$$\vec{\mathbf{a}}_{av} = \frac{\Delta \vec{\mathbf{v}}}{\Delta t}$$



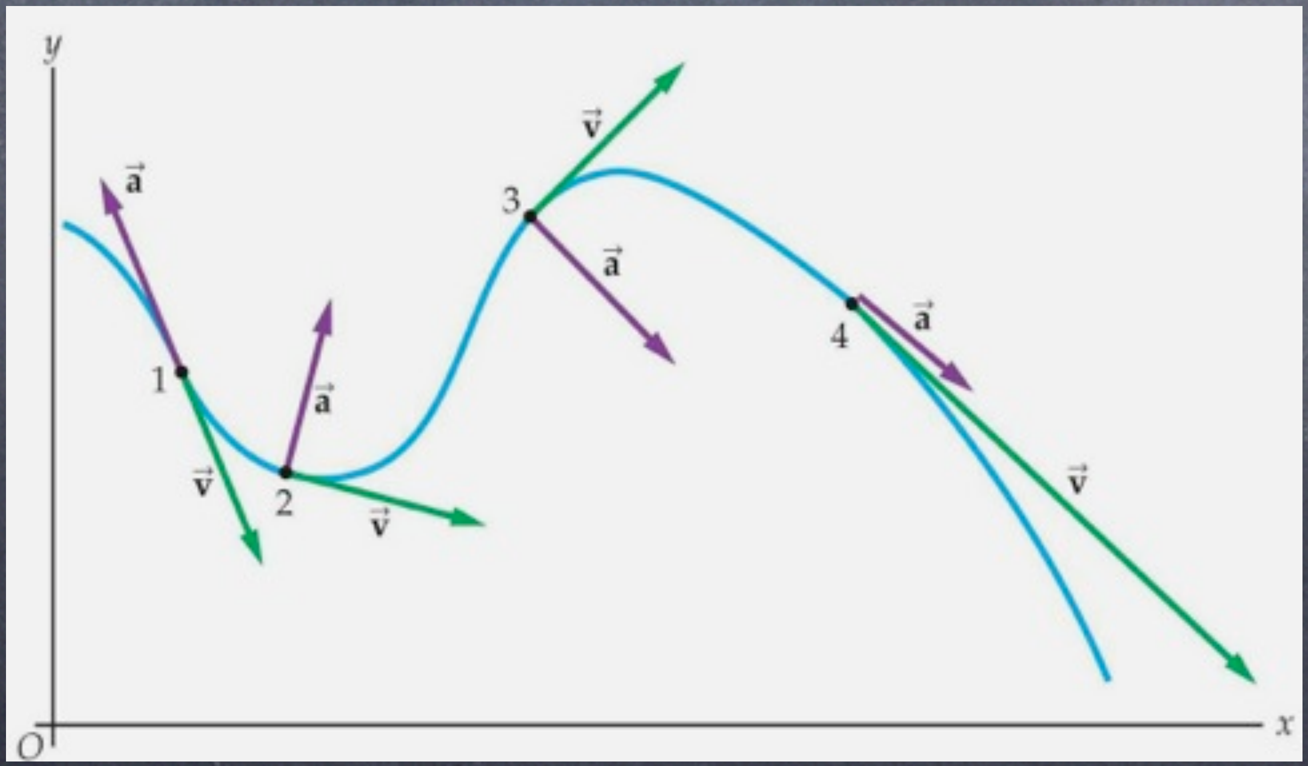
The instantaneous acceleration is the limit of the average acceleration as Δt approaches zero.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} \qquad \text{(instantaneous acceleration vector)} \qquad (3.9)$$

Position, Displacement, Velocity, & Acceleration Vectors

The velocity vector $\vec{v}$ always points in the direction of motion.

The acceleration vector $\vec{a}$ can point anywhere.



Acceleration Vectors

$$\vec{a}_{av} = \frac{\overrightarrow{\Delta v}}{\Delta t}; \qquad \vec{a} = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta v}}{\Delta t}$$

$$\vec{v} = v_x \hat{x} + v_y \hat{y} + v_z \hat{z} = \lim_{\Delta t \to 0} \left(\frac{\Delta x}{\Delta t} \hat{x} + \frac{\Delta y}{\Delta t} \hat{y} + \frac{\Delta z}{\Delta t} \hat{z} \right)$$

$$\vec{a} = \lim_{\Delta t \to 0} \left(\frac{\Delta v_x}{\Delta t} \hat{x} + \frac{\Delta v_y}{\Delta t} \hat{y} + \frac{\Delta v_z}{\Delta t} \hat{z} \right) = a_x \hat{x} + a_y \hat{y} + a_z \hat{z}$$

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}; \quad a_y = \lim_{\Delta t \to 0} \frac{\Delta v_y}{\Delta t}; \quad a_z = \lim_{\Delta t \to 0} \frac{\Delta v_z}{\Delta t}$$

Example: A Thrown Baseball

The position of a thrown baseball is given by:

$$\vec{r} = [1.5\text{ m} + (12\text{ m/s})t]\hat{x} + [(16\text{ m/s})t - (4.9\text{ m/s}^2)t^2]\hat{y}$$

(a) Find the velocity as a function of time.

(b) Find the acceleration as a function of time.

$$x = 1.5\text{ m} + (12\text{ m/s})t; \qquad y = (16\text{ m/s})t - (4.9\text{ m/s}^2)t^2$$

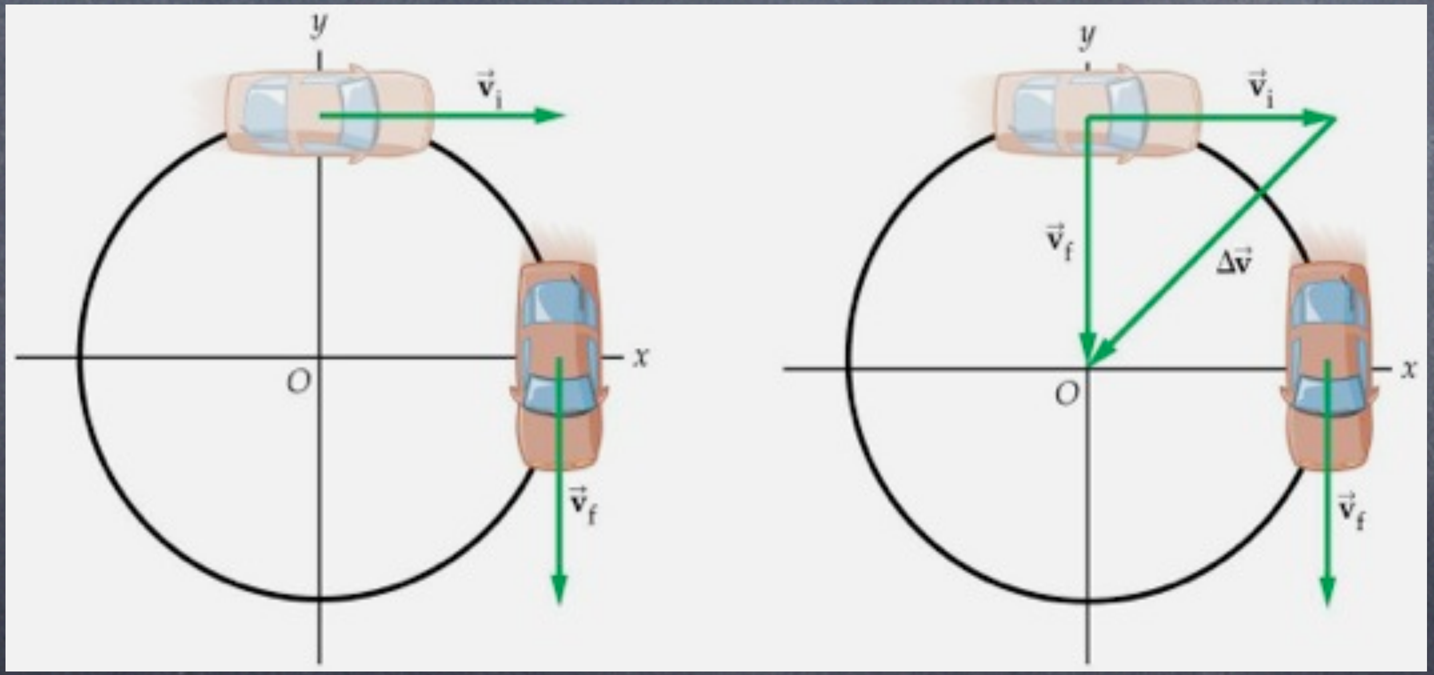
$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = 12\text{ m/s}; \qquad v_y = \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t} = (16\text{ m/s}) - 2(4.9\text{ m/s}^2)t$$

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = 0; \qquad a_y = \lim_{\Delta t \to 0} \frac{\Delta v_y}{\Delta t} = -9.8\text{ m/s}^2$$

$$\vec{v} = (12\text{ m/s})\hat{x} + [(16\text{ m/s}) - (9.8\text{ m/s}^2)t]\hat{y}; \qquad \vec{a} = (-9.8\text{ m/s}^2)\hat{y}$$

Velocity & Acceleration on a Curve

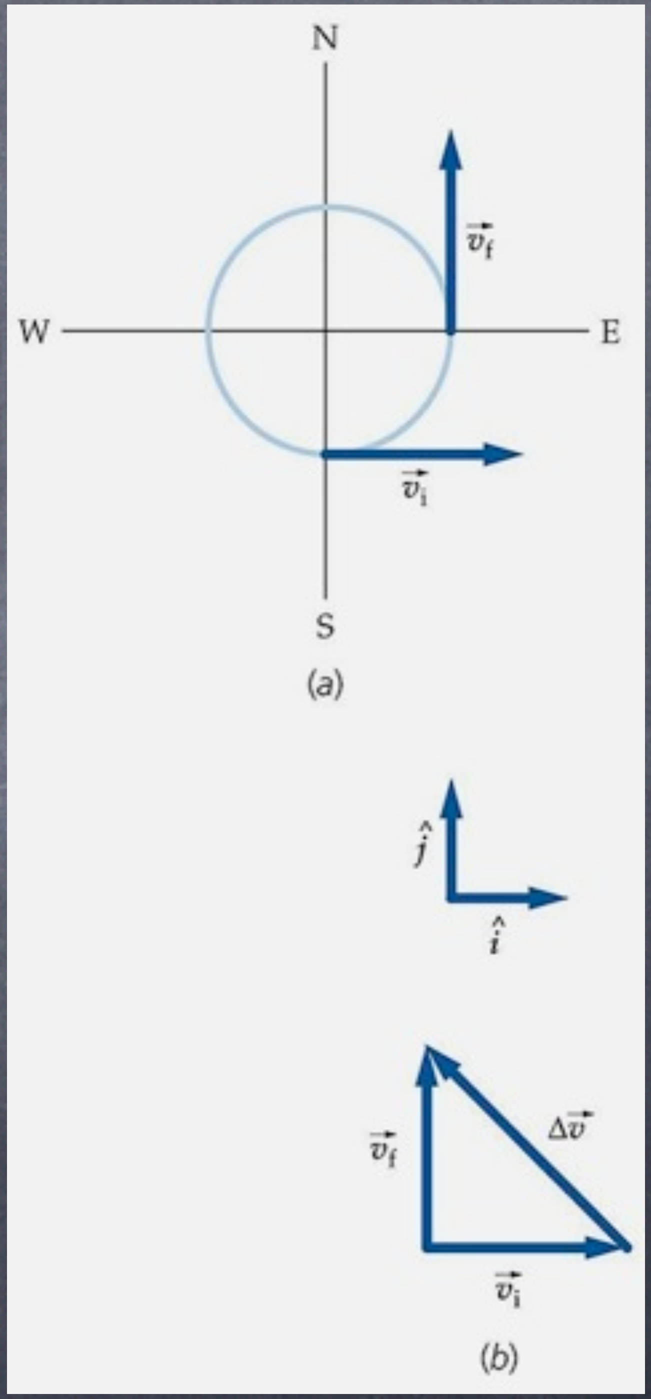
For a vehicle moving on a curve at a uniform speed, the acceleration is perpendicular to the velocity and the magnitude of the velocity stays the same, while the velocity's direction changes.



Example: Rounding a Curve

A car is traveling east at 60 km/h. It rounds a curve, and 5.0 s later it is traveling north at 60 km/h.

(a) Find the average acceleration of the car.



$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}; \qquad \vec{v}_f = \vec{v}_i + \Delta\vec{v}$$

$$\vec{a}_{av} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \frac{(60\text{ km/h})\hat{j} - (60\text{ km/h})\hat{i}}{5.0\text{ s}}$$

$$60\ \cancel{\text{km}}/\cancel{\text{h}} \times \frac{1\ \cancel{\text{h}}}{3600\text{ s}} \times \frac{1000\text{ m}}{1\ \cancel{\text{km}}} = 16.7\text{ m/s}$$

$$\vec{a}_{av} = \frac{(16.7\text{ m/s})\hat{y} - (16.7\text{ m/s})\hat{x}}{5.0\text{ s}} = (-3.4\text{ m/s}^2)\hat{x} + (3.4\text{ m/s}^2)\hat{y}$$

Example: Car Accelerating on a Curve

A car is traveling northwest at 9.0 m/s. 8 seconds later it has rounded a corner and is now headed north at 15.0 m/s.

(a) What is the magnitude and direction of the acceleration during those 8.0 s?

$$\overrightarrow{v_i} = (9.0\text{ m/s})\cos 135^\circ \hat{x} + (9.0\text{ m/s})\sin 135^\circ \hat{y} = (-6.36\text{ m/s})\hat{x} + (6.36\text{ m/s})\hat{y}$$

$$\overrightarrow{v_f} = (15.0\text{ m/s})\hat{y}$$

$$\overrightarrow{\Delta v} = \overrightarrow{v_f} - \overrightarrow{v_i} = (6.36\text{ m/s})\hat{x} + (8.64\text{ m/s})\hat{y}$$

$$\overrightarrow{a_{av}} = \frac{\overrightarrow{\Delta v}}{\Delta t} = (0.795\text{ m/s}^2)\hat{x} + (1.08\text{ m/s}^2)\hat{y}$$

$$a_{av} = \sqrt{(0.795\text{ m/s}^2)^2 + (1.08\text{ m/s}^2)^2} = 1.34\text{ m/s}^2$$

$$\theta_a = \tan^{-1}(1.08\text{ m/s}^2)/(0.795\text{ m/s}^2) = 53.6^\circ$$