Physics 170 - *Mechanics*

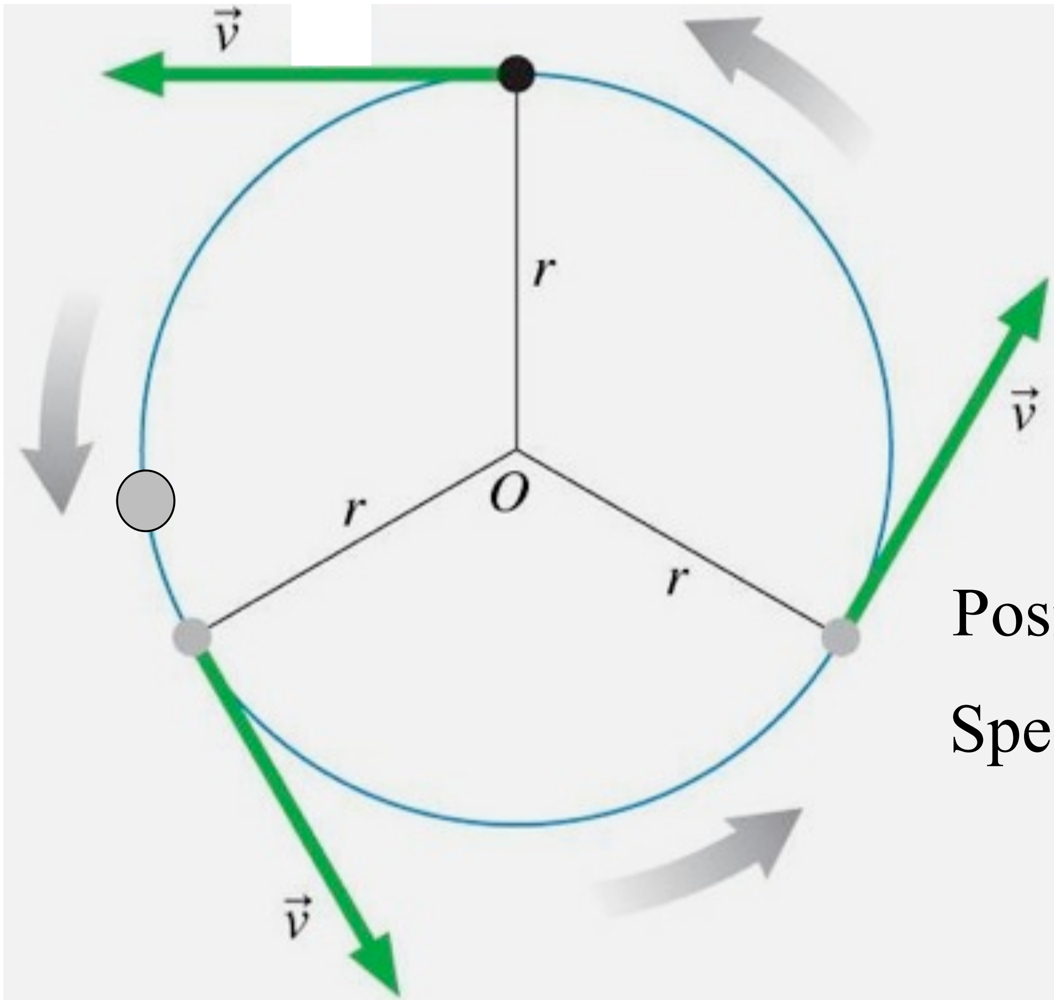
Lecture 14

Circular Motion

@ home:

- example in the book:
- 5.1, 5.2, 5.3, 5.4, 5.5, 5.6, 5.7, 5.8, 5.9,
- 5.10, 5.11,
- 5.12 (check the reference frames)
- 5.13, 5.14, 5.15, 5.16, 5.17

Circular Motion



Position: $|\vec{r}|$ = constant

Speed: $|\vec{v}|$ = constant

Angular Position θ

The angular position θ (in radiants):

$$\theta = s/r$$

s is the "arc length", i.e., the length of the arc traced by the trajectory of the particle as it moves from the x axis to its current position.

$s = r\theta$; $\theta = s/r$

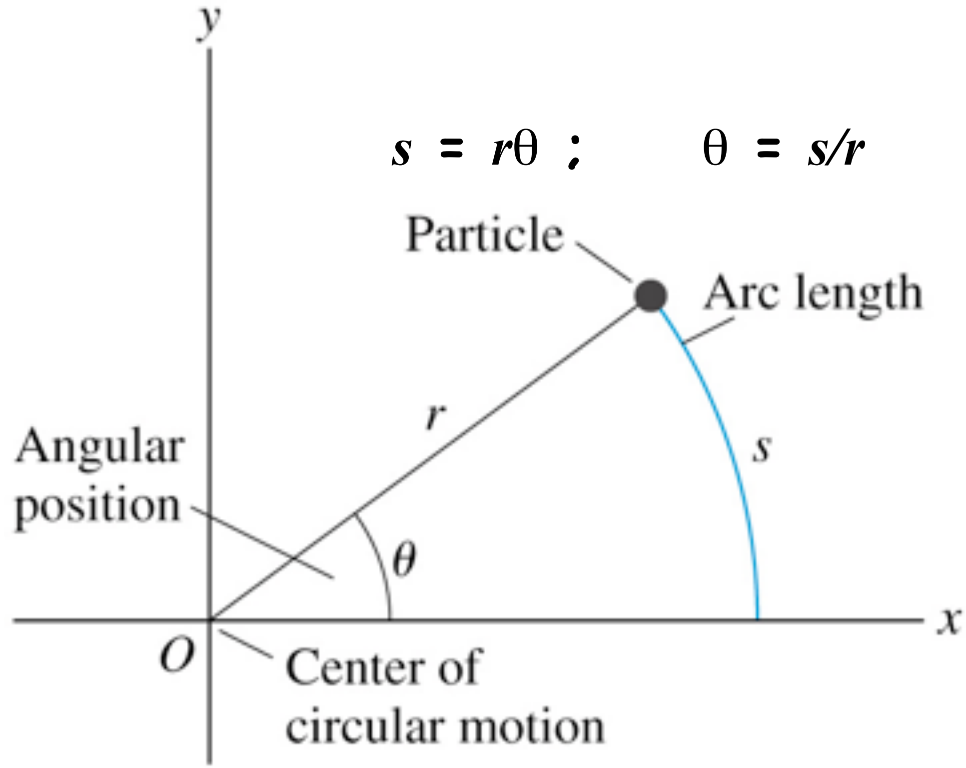


TABLE 6–2
$\frac{\sin\theta}{\theta}$ for Values of θ Approaching Zero

θ, radians	$\frac{\sin\theta}{\theta}$
1.00	0.841
0.500	0.959
0.250	0.990
0.125	0.997
0.0625	0.999

sin θ ~ θ for small angles

Angular Position θ

The angular position θ (in radiants):

$$\theta = s/r$$

$$0 \le |\theta| \le 2\pi.$$

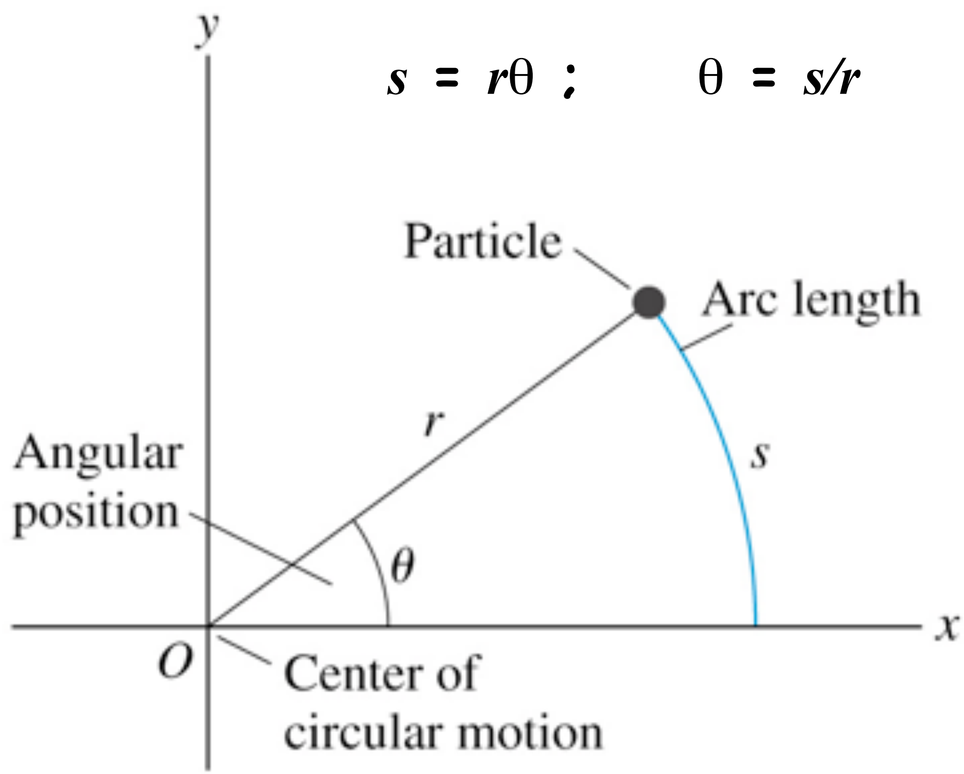
The "radians" unit is dimensionless (length/length).

$$1 \text{ rad} = 180°/\pi = 57.296°$$

The "degree" measure of angle is:

$$1° = (\pi/180°) \text{ rad} = 0.0174533 \text{ rad};$$

$$s = r\theta \; ; \qquad \theta = s/r$$



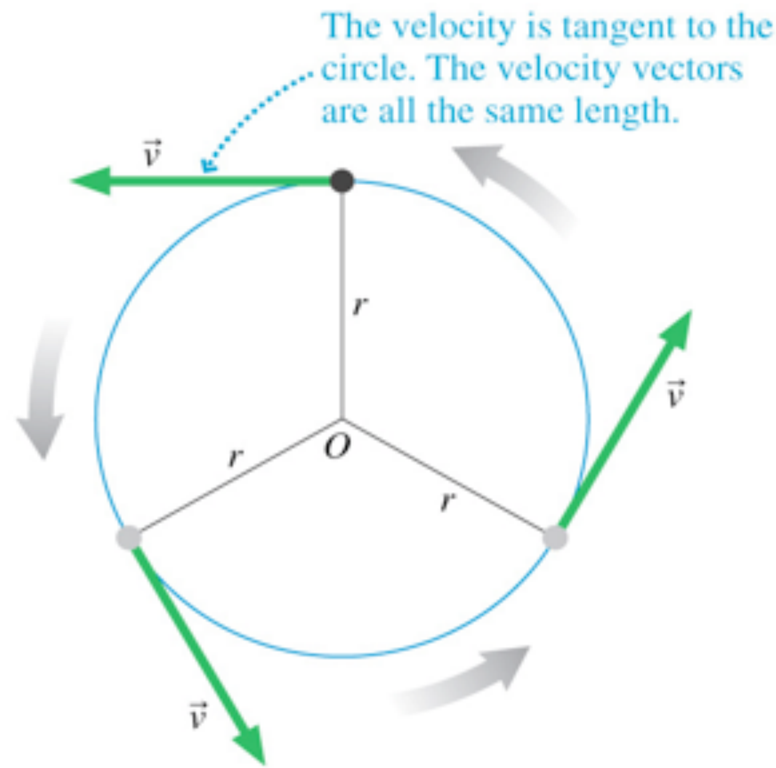
A Particle in Uniform Circular Motion

$T \equiv$ period = the time required for one complete rotation.

The velocity vector $\boldsymbol{v}$:

$$v = \frac{1 \text{ circumference}}{1 \text{ period}} = \frac{2\pi \vec{r}}{T}$$

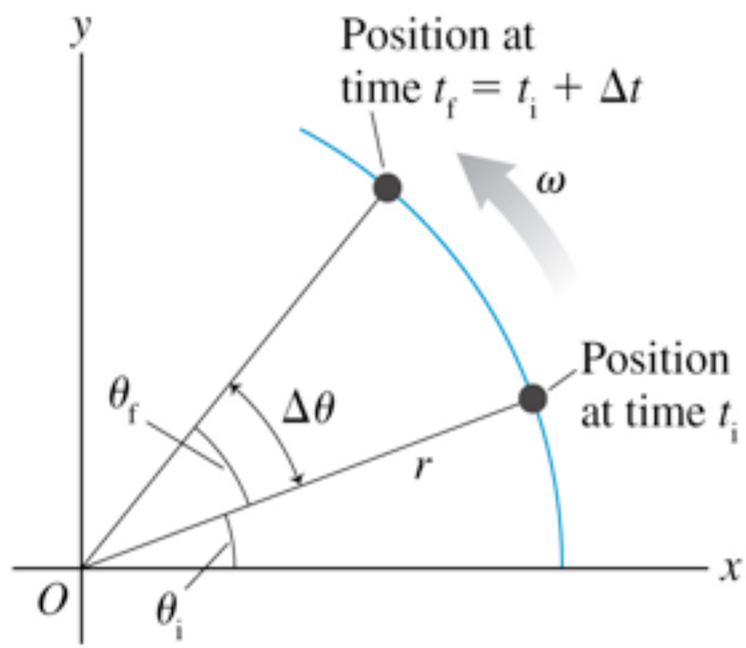
For a particle in uniform circular motion, the velocity vector $\boldsymbol{v}$ remains constant in magnitude, but it continuously changes its direction.



Angular Velocity ω

The angular velocity ω is in radians/s and it is the speed with which the angle θ changes as the particle moves in its circular path.

$$\omega_{ave} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$



Another unit used is **rpm:** revolutions per minute.

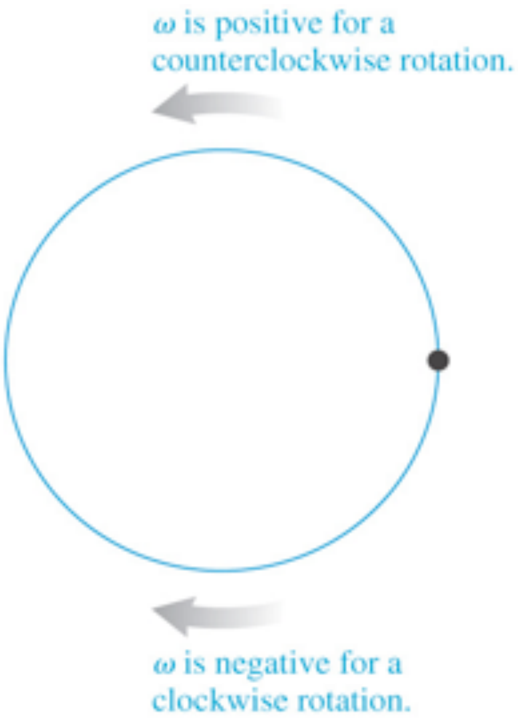
1 rpm = 2π rad/min = 2π/60 rad/s =

= 0.10472 rad/s ≈ 1/10 rad/s.

Angular Velocity ω

It is conventional to treat θ and ω as quantities that have a **sign**.

- θ increases counterclockwise
- ω describes a counterclockwise rotation.



$\theta > 0$

$\omega > 0$

Example: A Rotating Crankshaft

A 4.0 cm diameter crankshaft turns at 2400 rpm. What is the speed of a point on the surface of the crankshaft?

$$\frac{2400 \text{ rev}}{1 \text{ minute}} \times \frac{1 \text{ minute}}{60 \text{ s}} = 40 \text{ rev/s}$$

$$T = \frac{1}{40 \text{ rev/s}} = 0.025 \text{ s (per revolution)}$$

$$v = \frac{2\pi r}{T} = \frac{2\pi (0.020 \text{ m})}{0.025 \text{ s}} = 5.03 \text{ m/s}$$

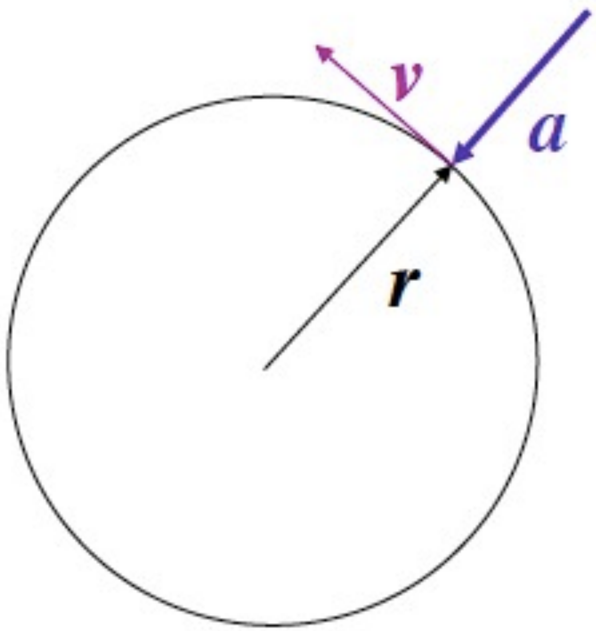
Alternative method:

$$\omega = \frac{2400 \text{ rev}}{1 \text{ minute}} \times \frac{1 \text{ minute}}{60 \text{ s}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = 80\pi \text{ rad/s}$$

$$\boxed{v = \omega r} = (80\pi \text{ rad/s})(0.020 \text{ m}) = 5.03 \text{ m/s}$$

Acceleration and Circular Motion

What keeps the body in the circle?



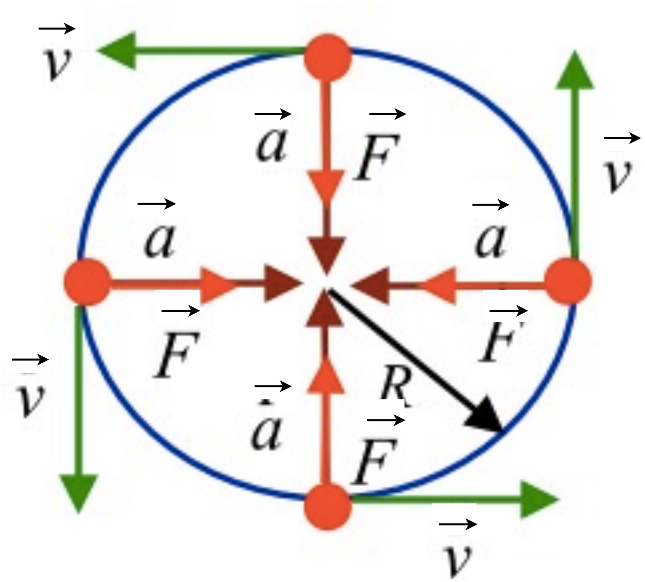
An object moving at a constant speed in a circle must have a **force** acting on it; otherwise it would move in a straight line.

It's a constant acceleration a toward the **center** of the circle:

the **centripetal acceleration**

$$a_{cp} = \frac{v^2}{r} \quad \text{(centripetal acceleration)}$$

Centripetal Acceleration



$$v_t = \frac{2\pi r}{T} \quad \text{(tangential velocity)}$$

$$a_{cp} = \frac{v^2}{r} \quad \text{(centripetal acceleration)}$$

Centripetal Force

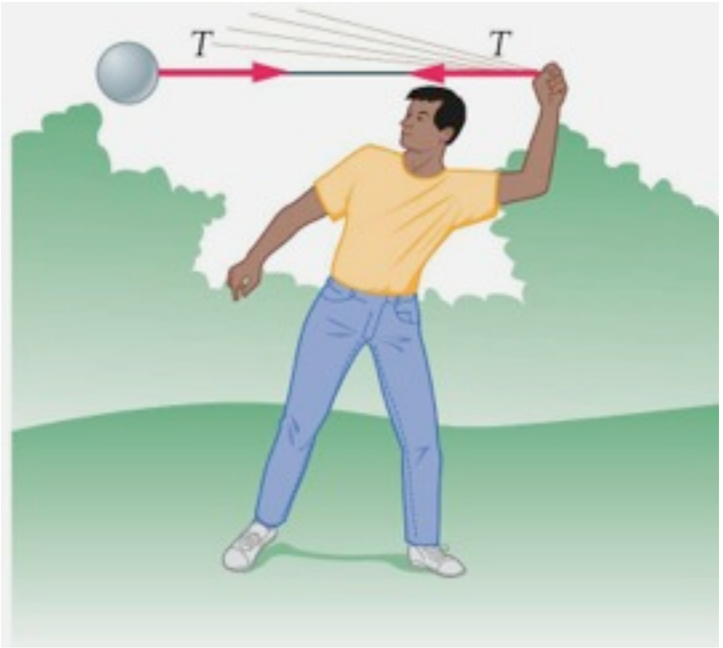
From the centripetal acceleration a_{cp}, we find the centripetal force f_{cp}, required to keep an object of mass m moving in a circle of radius r.

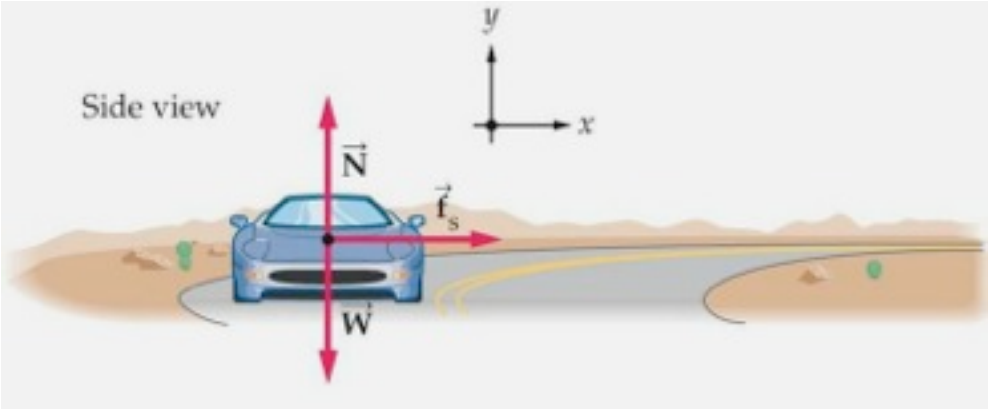
The magnitude of the force f_{cp} , called **centripetal force** because it points toward the center of rotation, is given by:

$$f_{cp} = ma_{cp} = m\frac{v^2}{r} = m\omega^2 r$$

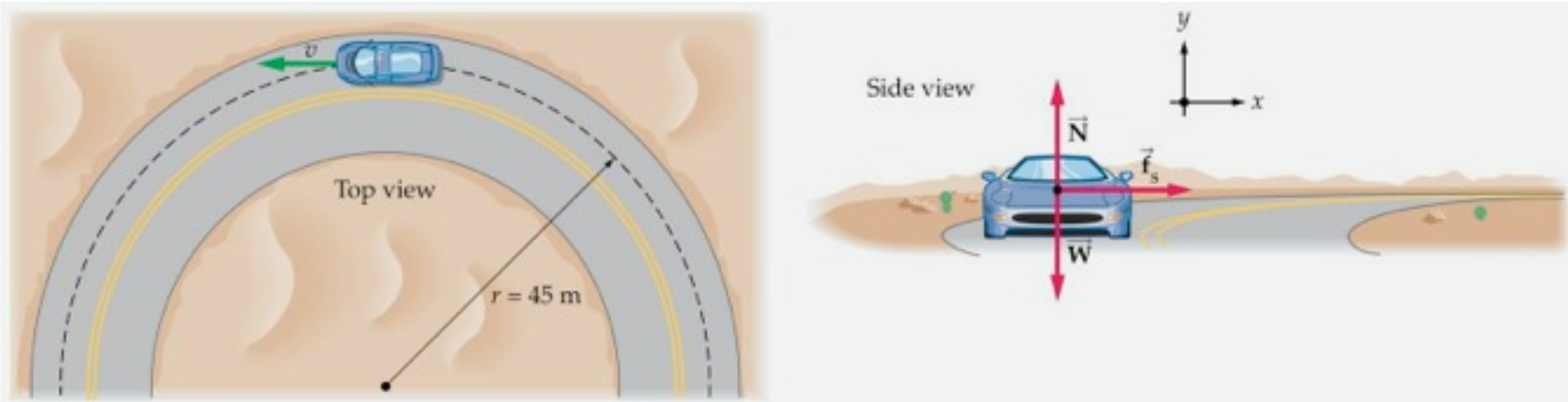
Sources of Centripetal Force

This centripetal force may be provided by the tension in a string, the normal force, or friction, among other sources.





Example: Rounding a Corner



A 1,200 kg car rounds a corner of radius r = 45.0 m. If the coefficient of friction between the tires and the road is μ_s = 0.82, what is the maximum speed the car can have on the curve without skidding?

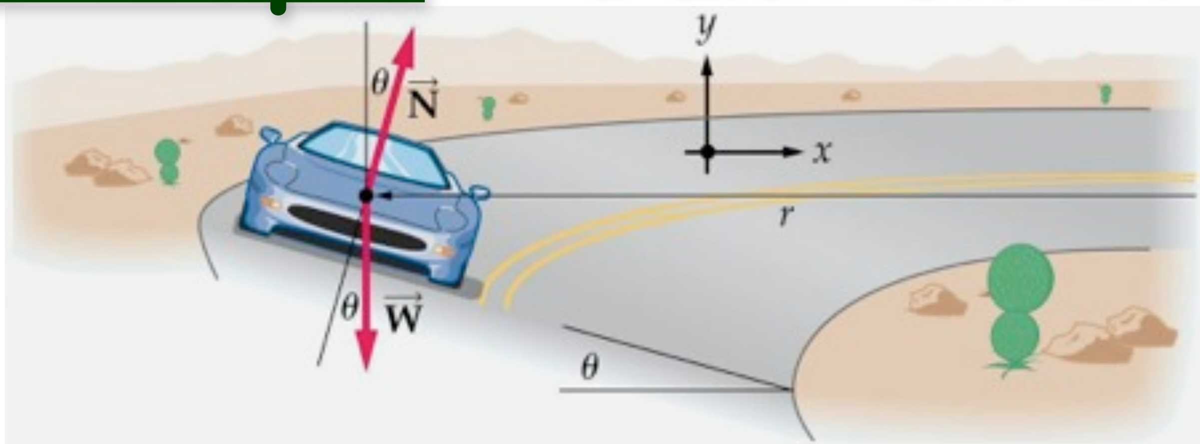
$$\sum F_x = f_s = \mu_s N = m a_x = m\frac{v^2}{r} \qquad \sum F_y = 0 = N - W = N - mg \longrightarrow N = mg$$

$$\sum F_x = \mu_s \cancel{m} g = \cancel{m}\frac{v^2}{r} \qquad v = \sqrt{\mu_s r g} = \sqrt{(0.82)(45.0\ \text{m})(9.81\ \text{m/s}^2)} = 19.0\ \text{m/s}$$

Question: How does this result depend on the weight of the car?

Banked Curves

Example: Bank on It



If the road is banked at the proper angle θ, a car can round a curve without the assistance of friction between the tires and the road and without skidding.

What bank angle θ is needed for a 900 kg car traveling at 20.5 m/s around a curve of radius 85.0 m?

$$\sum F_y = 0 = N\cos\theta - W = N\cos\theta - mg$$

$$\frac{N\sin\theta}{N\cos\theta} = \tan\theta = \frac{mv^2/r}{mg} = \frac{v^2}{gr}$$

$$\sum F_x = N\sin\theta = ma_{cp} = mv^2/r$$

$$\theta = \arctan\frac{v^2}{gr} = \arctan\frac{(20.5\text{ m/s})^2}{(9.81\text{ m/s}^2)(85.0\text{ m})} = 26.7°$$

<u>Example</u>: Spinning in a Circle

An energetic father places his 20 kg child in a 5.0 kg cart to which is attached a 2.0 m long rope. He then holds the end of the rope and spins the cart and child in a circle, keeping the rope parallel to the ground.

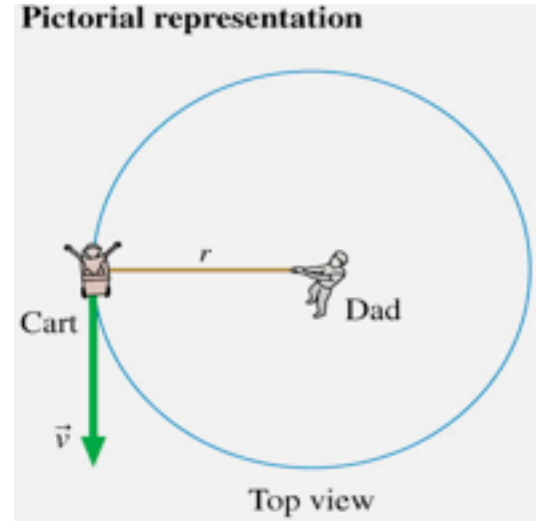
If the tension in the rope is 100 N, how many revolutions per minute does the cart make?

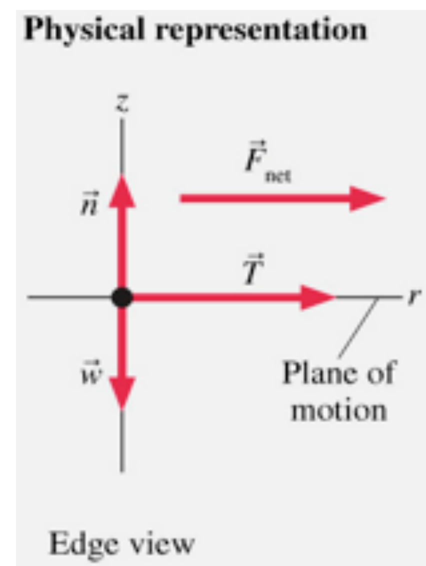
Known

$m = 25$ kg
$r = 2$ m
$T = 100$ N

Find

ω in rpm





$$(F_{net})_r = \sum F_r = T = \frac{mv^2}{r}; \quad (F_{net})_z = \sum F_r = n - w = 0;$$

$$v_t = \sqrt{\frac{rT}{m}} = \sqrt{\frac{(2.0\text{ m})(100\text{ N})}{(25\text{ kg})}} = 2.83\text{ m/s};$$

$$\omega = \frac{v_t}{r} = \frac{(2.83\text{ m/s})}{(2.0\text{ m})} = 1.41\text{ rad/s} = 13.5\text{ rpm}$$

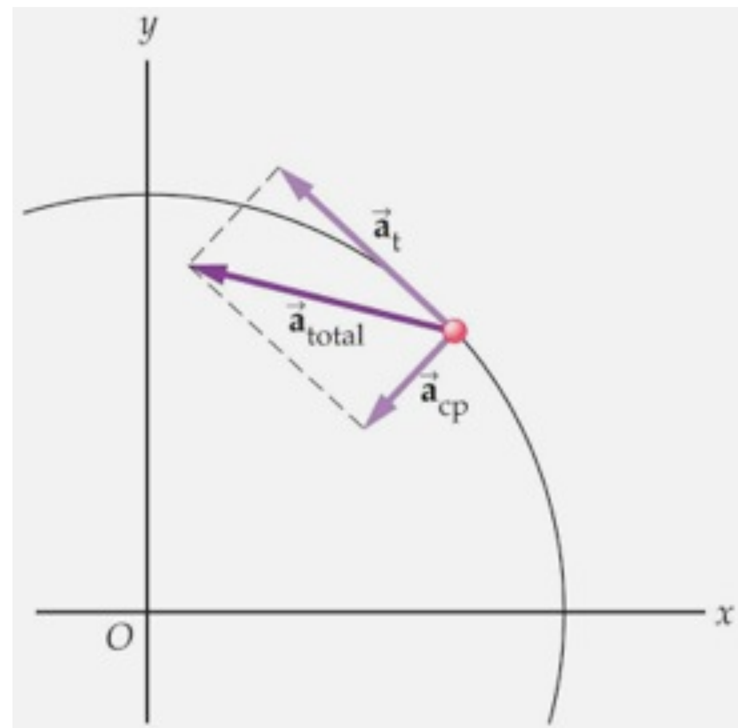
(1.41 rad/s * 1 rev/2pi * 60 sec/1min = 13.5 rpm)

Tangential & Total Acceleration

An object may be changing its speed (speeding up or slowing down) as it moves in a circular path.

In that case, there is a **tangential acceleration** as well as a centripetal acceleration.

The total acceleration a_{total} is the vector sum of the centripetal acceleration a_{cp}, which points toward the center of rotation, and the tangential acceleration a_t, which points in the direction of speed increase.

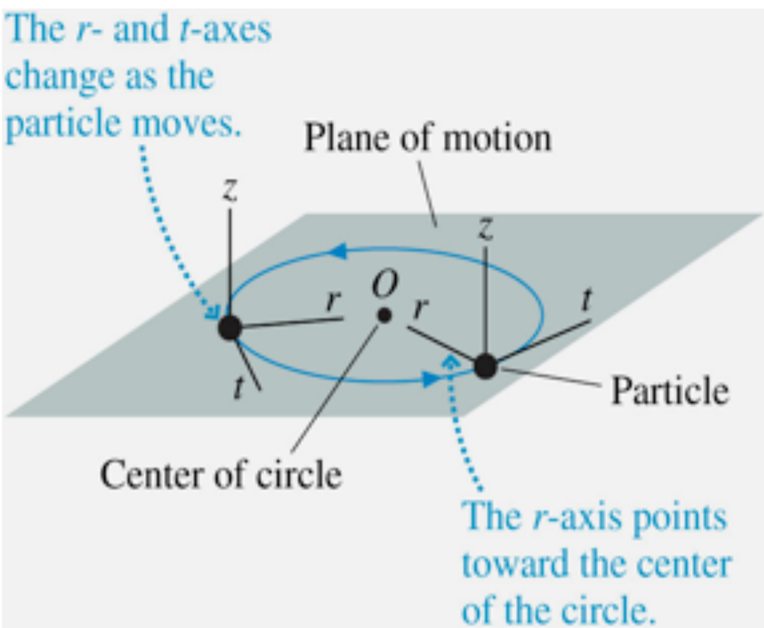


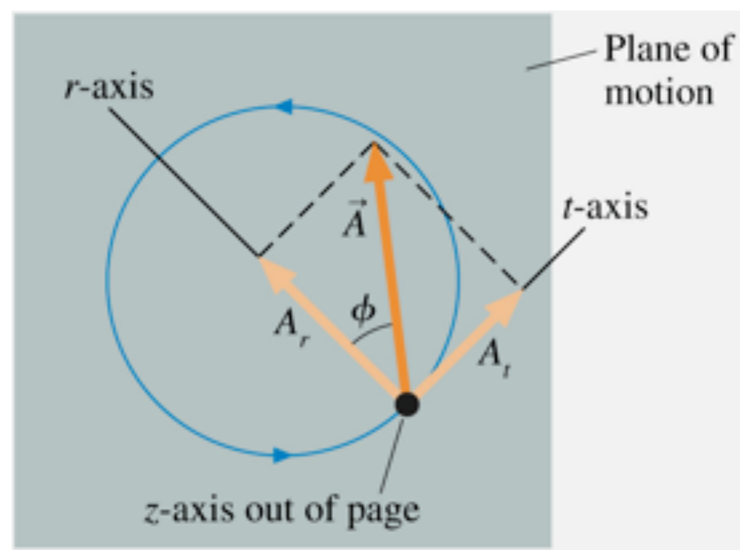
Radial-Tangential Coordinates

The r axis (radial) points from the particle to the center of rotation;
The t axis (tangential) points from the particle tangent to the circle in the counterclockwise (ccw) direction;
The z axis (axial) points up from the particle perpendicular to the plane of rotation;

The vector $\vec{A}$ can be decomposed into r and t components:

$A_r = A\cos\phi; \quad A_t = A\sin\phi;$

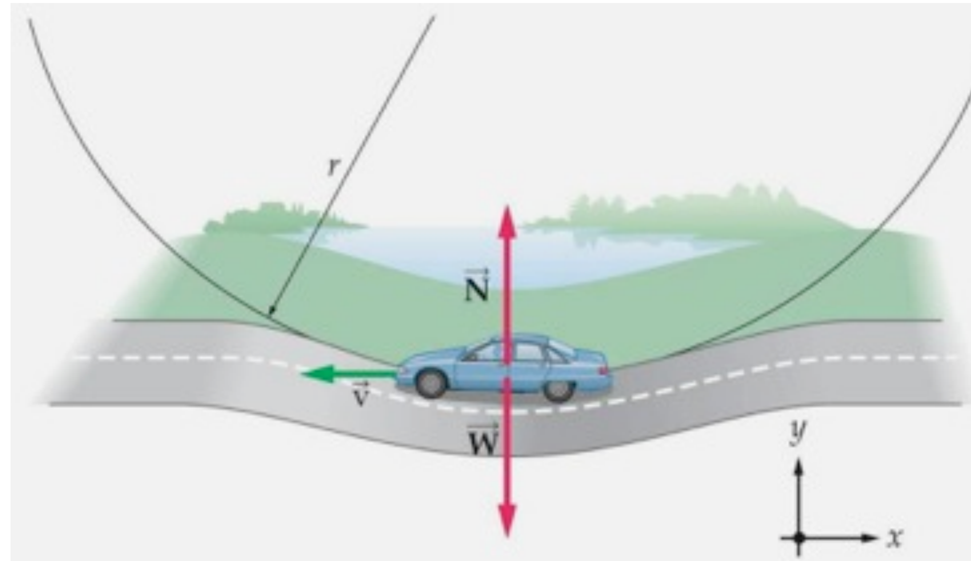




Example: Normal Force in a Dip

While driving on a country road at a constant speed of 17.0 m/s, you encounter a dip in the road. The dip can be approximated by a circular arc with a radius of 65.0 m.

What is the normal force exerted by the car seat on an 80.0 kg passenger at the bottom of the dip?



$$\sum F_y = N - mg = ma_y = mv^2/r$$

$$N = mg + mv^2/r = m(g + v^2/r)$$

$$N = (80.0\text{ kg})\left[(9.81\text{ m/s}^2) + (17.0\text{ m/s})^2/(65.0\text{ m})\right]$$

$$= (80.0\text{ kg})(14.26\text{ m/s}^2) = 1140\text{ N}$$ **(about 45% more)**

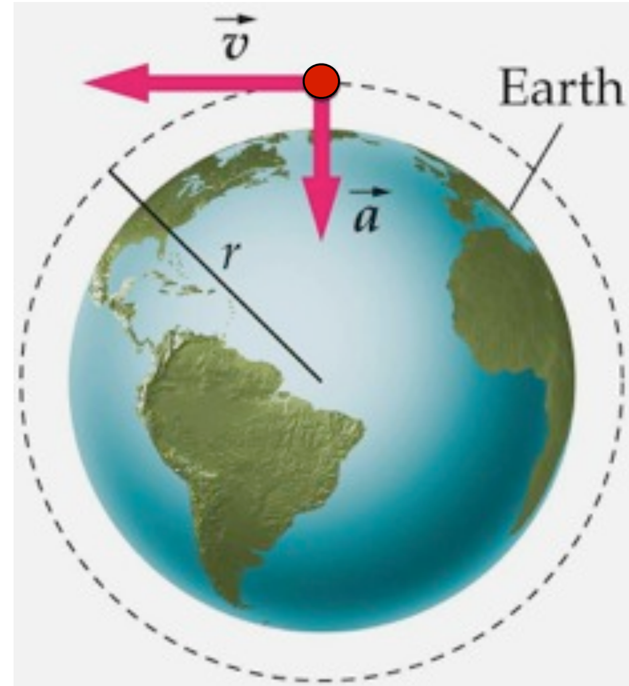
Note that **(80 kg)(9.81 m/s^2) = 785 N**

Example: A Satellite's Motion

A satellite moves at constant speed in a circular orbit about the center of the Earth and near the surface of the Earth. If the magnitude of its acceleration is g = 9.81 m/s^2 and the Earth's radius is 6,370 km, find:

(a) its speed v; and

(b) the time T required for one complete revolution.



$$a_{cp} = \frac{v^2}{r} = g$$

$$v = \sqrt{rg} = \sqrt{(6,370\times10^3\ \text{m})(9.81\ \text{m/s}^2)} = 7.91\times10^3\ \text{m/s} = 17,700\ \text{mi/h}$$

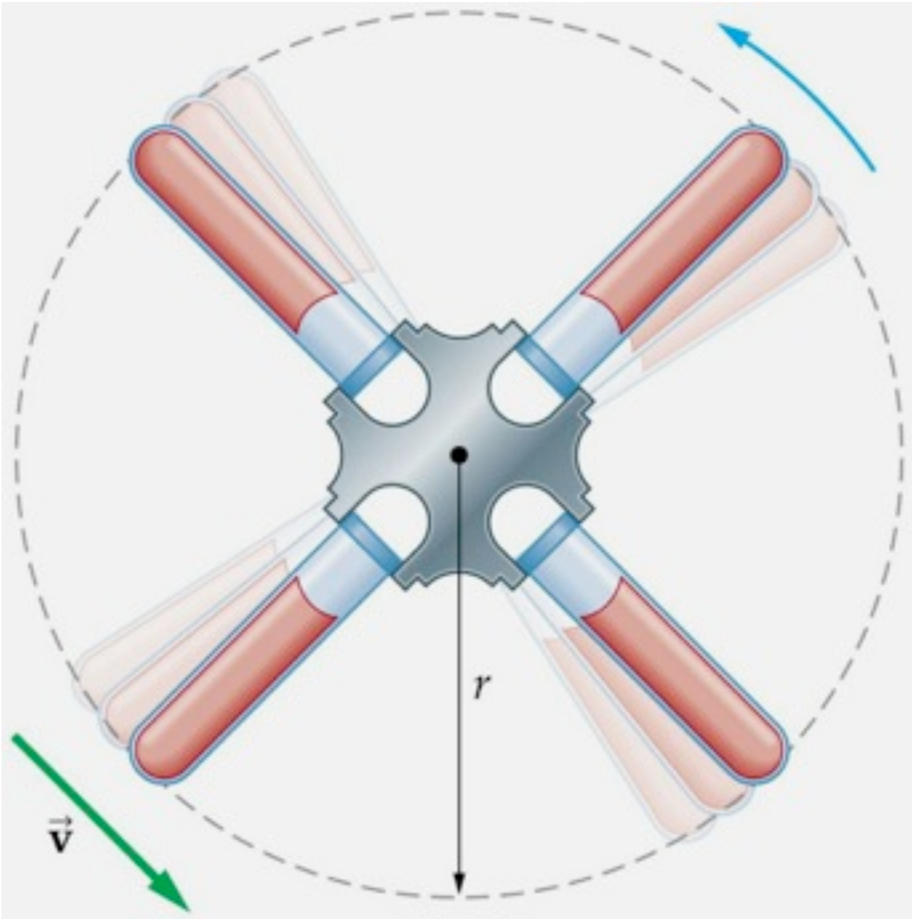
$$T = 2\pi\, r / v = 2\pi\,(6,370\times10^3\ \text{m})/(7.91\times10^3\ \text{m/s}) = 5,060\ \text{s} = 84.3\ \text{min}$$

The Centrifuge

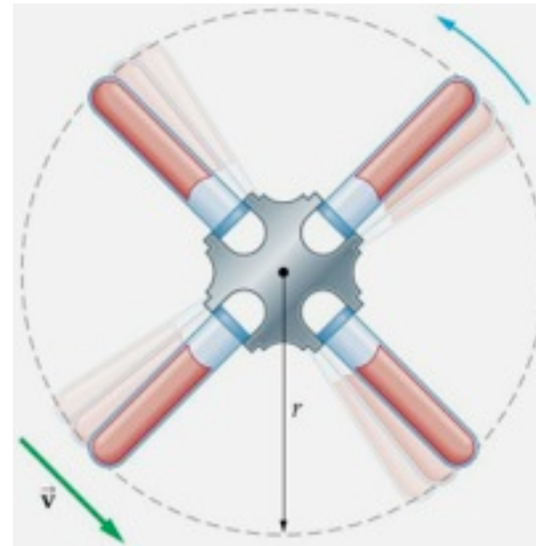
The apparent weight of an object can be greatly increased using circular motion.

A **centrifuge** is a laboratory device used in chemistry, biology, and medicine for increasing the sedimentation rate and separation of a sample by subjecting it to a very high centripetal acceleration.

Accelerations on the order of 10,000 g can be achieved.



<u>Example</u>: Big Gees



A centrifuge rotates at a rate such that the bottom of a test tube travels at a speed of 89.3 m/s. The bottom of the test tube is 8.50 cm from the axis of rotation.

What is the centripetal acceleration a_{cp} at the bottom of the test tube in m/s and in g (where 1 g = 9.81 m/s^2)?

$$a_{cp} = \frac{v^2}{r} = \frac{(89.3\ \text{m/s})^2}{(0.0850\ \text{m})} = 93{,}800\ \text{m/s}^2 = 9{,}560\ \text{g}$$

Question:

Which motion has the largest centripetal acceleration?