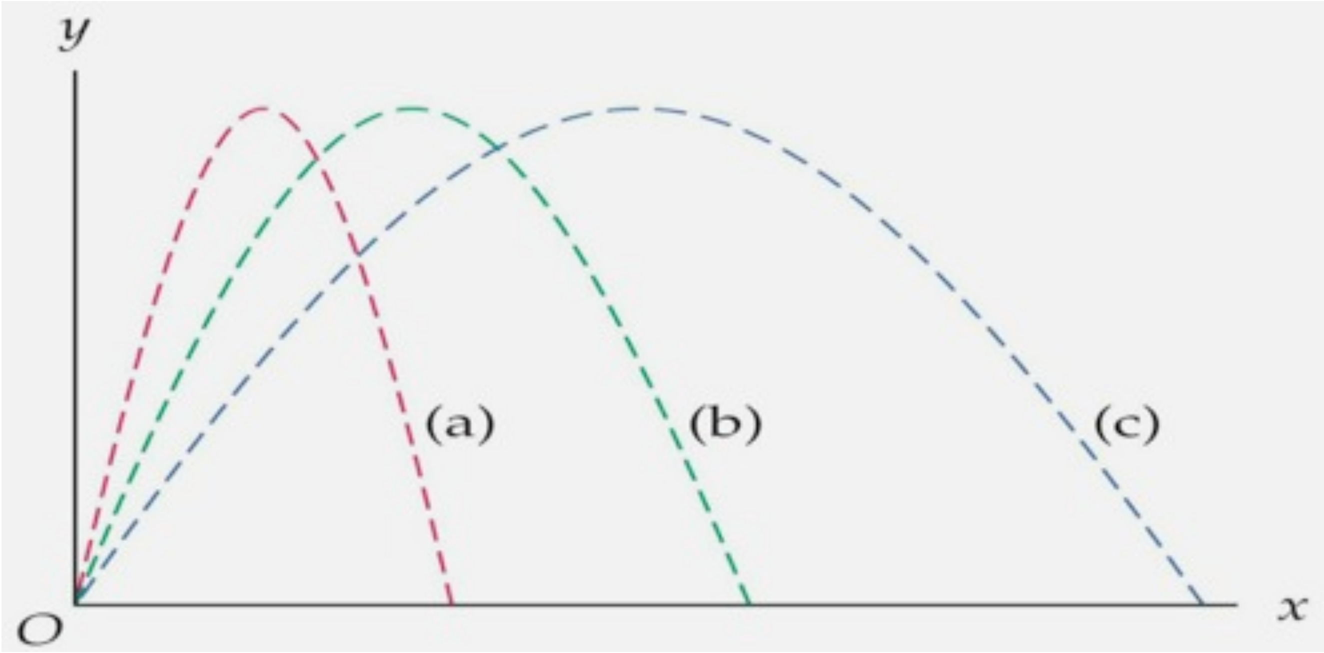
Physics 170 - *Mechanics*

Lecture 8

2D Motion Basics

Two-Dimensional Kinematics



Motion in Two Dimensions

$$\boxed{\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2}$$

$$\boxed{\vec{v} = \vec{v}_0 + \vec{a}t}$$

Motion in the x- and y-directions should be solved **separately**:

TABLE 4–1 Constant-Acceleration Equations of Motion

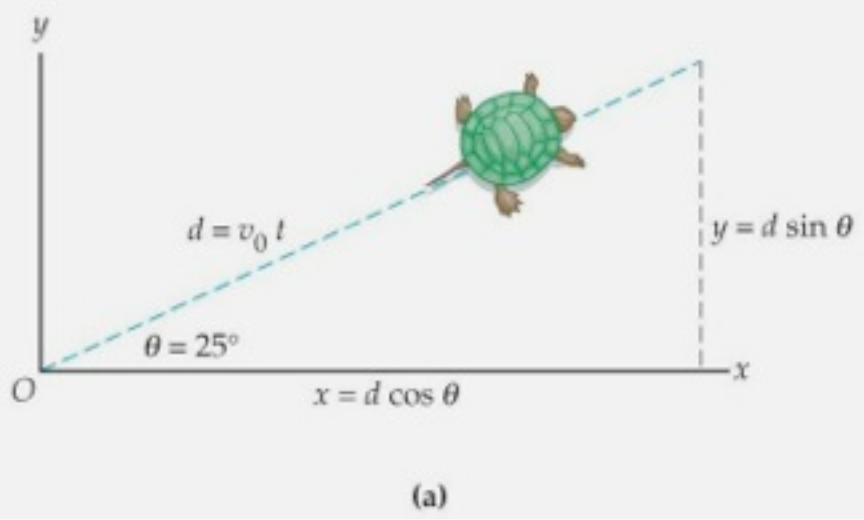
Position as a function of time	Velocity as a function of time	Velocity as a function of position
$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$	$v_x = v_{0x} + a_x t$	$v_x^2 = v_{0x}^2 + 2a_x \Delta x$
$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$	$v_y = v_{0y} + a_y t$	$v_y^2 = v_{0y}^2 + 2a_y \Delta y$

Constant Velocity

If velocity is constant, motion is along a straight line:

$$x(t) = x_0 + v_{0x}t;$$

$$y(t) = y_0 + v_{0y}t$$

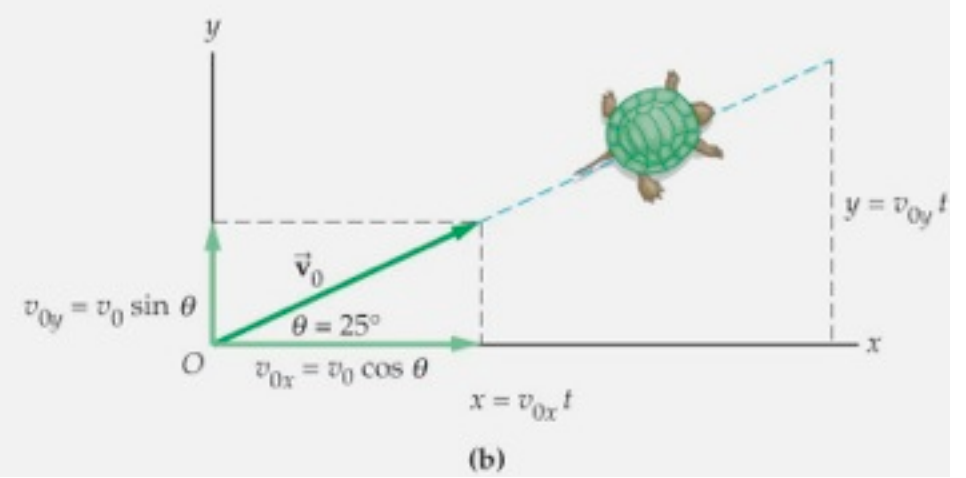


A turtle walks from the origin at a speed of v_0=0.26 m/s and an angle of 25^0.

In a time t, the turtle moves through a straight line distance of $d = v_0 t$,

with horizontal displacement $x = d\ cos\ \theta$ and a vertical displacement $y = d\ sin\ \theta$.

Equivalently, the turtle's horizontal and vertical velocity components are $v_x = v_0 cos\ \theta$ and $v_y = v_0 sin\ \theta$.

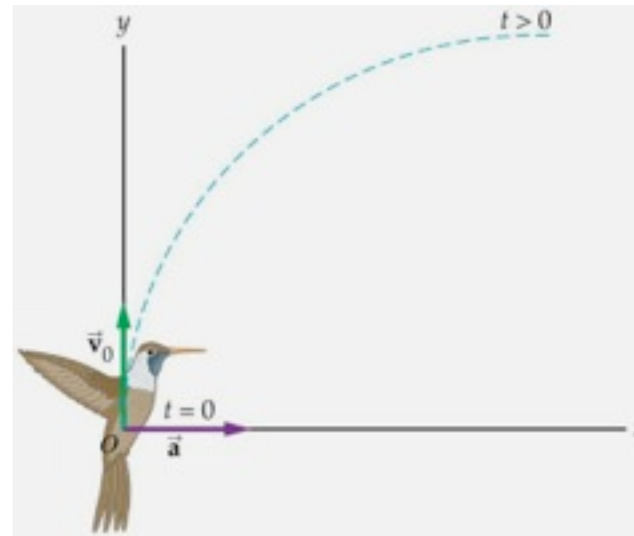


Example:

A Hummer Accelerates

A hummingbird is flying in such a way that it is initially moving vertically with a speed of 4.6 m/s and accelerating horizontally at a constant rate of 11 m/s^2.

Find the horizontal and vertical distance through which it moves in 0.55 s.

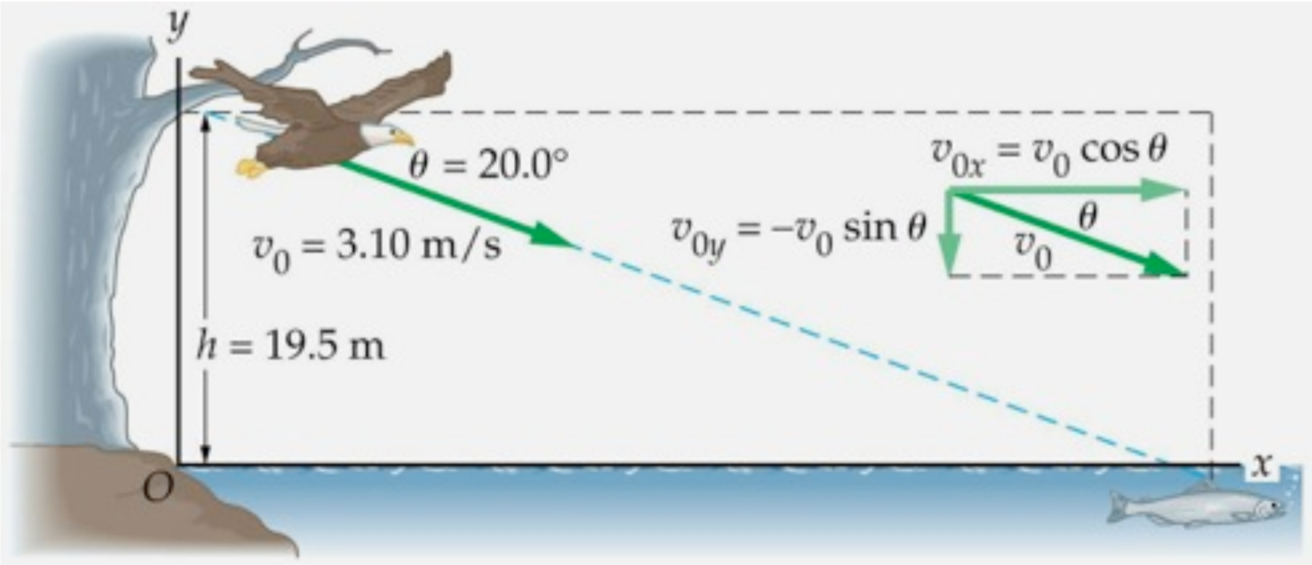


$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2 = \tfrac{1}{2}a_x t^2 = \tfrac{1}{2}(11\ \mathrm{m/s^2})(0.55\ \mathrm{s})^2 = 1.7\ \mathrm{m}$$

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 = v_{0y}t = (4.6\ \mathrm{m/s})(0.55\ \mathrm{s}) = 2.5\ \mathrm{m}$$

Example: The Eagle Descends

An eagle perched on a tree limb 19.5 m above the water spots a fish swimming near the surface. He pushes off the limb and descends toward the water, maintaining a constant speed of 3.20 m/s at 20° below horizontal.



(a) How long does it take for the eagle to reach the water?

(b) How far has the eagle traveled horizontally when it reaches the water?

$$v_{0x} = v_0 \cos\theta = (3.10 \text{ m/s})\cos(-20°) = 2.91 \text{ m/s}$$

$$v_{0y} = v_0 \sin\theta = (3.10 \text{ m/s})\sin(-20°) = -1.06 \text{ m/s}$$

$$t = h / v_{0y} = (-19.5 \text{ m}) / (-1.06 \text{ m/s}) = 18.4 \text{ s}$$

$$x = x_0 + v_{0x}t = (2.91 \text{ m/s})(18.4 \text{ s}) = 53.5 \text{ m}$$

Projectile Motion: Basic Equations

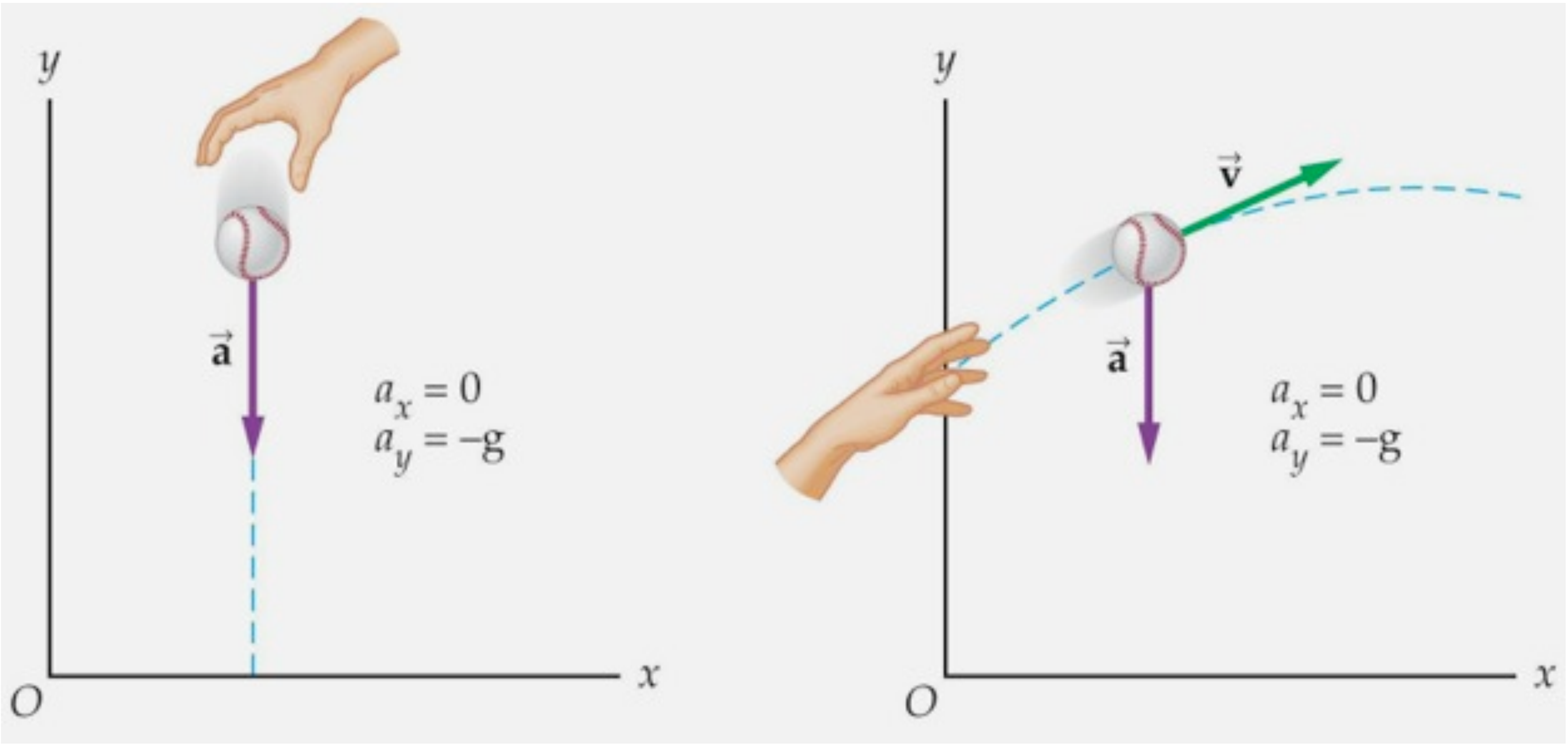
Assumptions:

- Ignore air resistance
- Use g = 9.81 m/s^2, downward
- Ignore the Earth's rotation

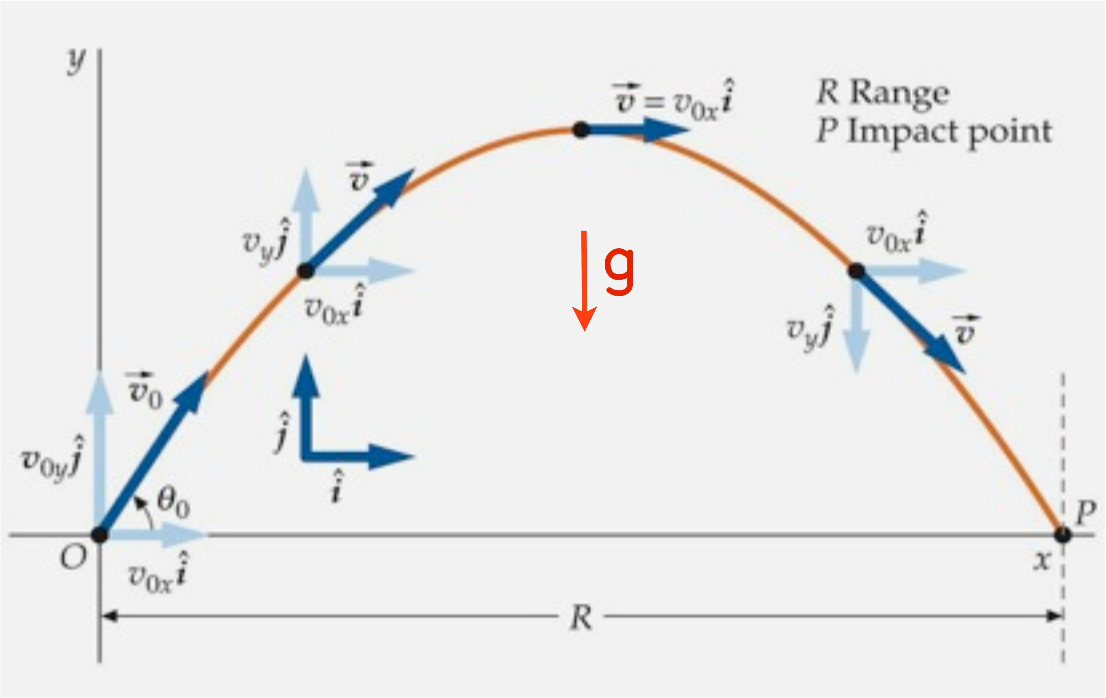
If the x-axis is horizontal and the y-axis points upward, the acceleration in the x-direction is zero and the acceleration in the y-direction is –9.81 m/s^2

Projectile Motion: Basic Equations

The acceleration is independent of the direction of the velocity:

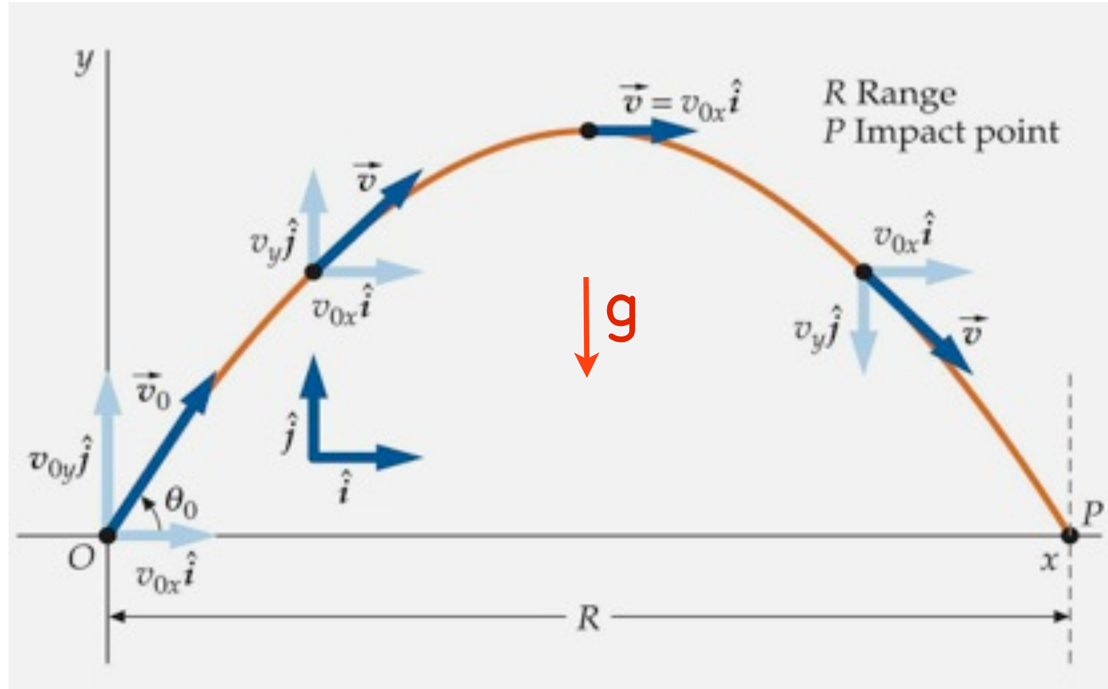


Projectile motion



- The x and y components can be treated independently
- Velocities can be broken down into its x and y components
- The x-direction is uniform motion: $a_x = 0$
- The y-direction is free fall: $|a_y| = g$

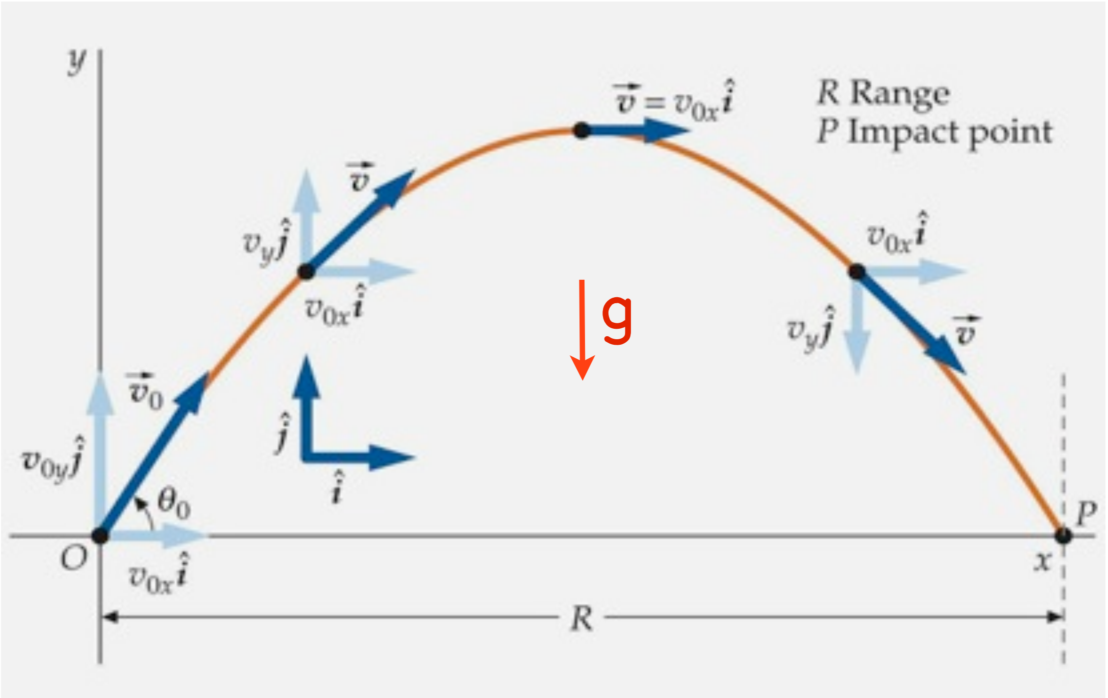
Projectile motion: X



x-direction

- $a_x = 0$
- $v_x = v_0 \cos \alpha_0$ = constant
- $x = v_x t$
- This is the only operative equation in the x-direction since there is uniform velocity in that direction

Projectile motion: X

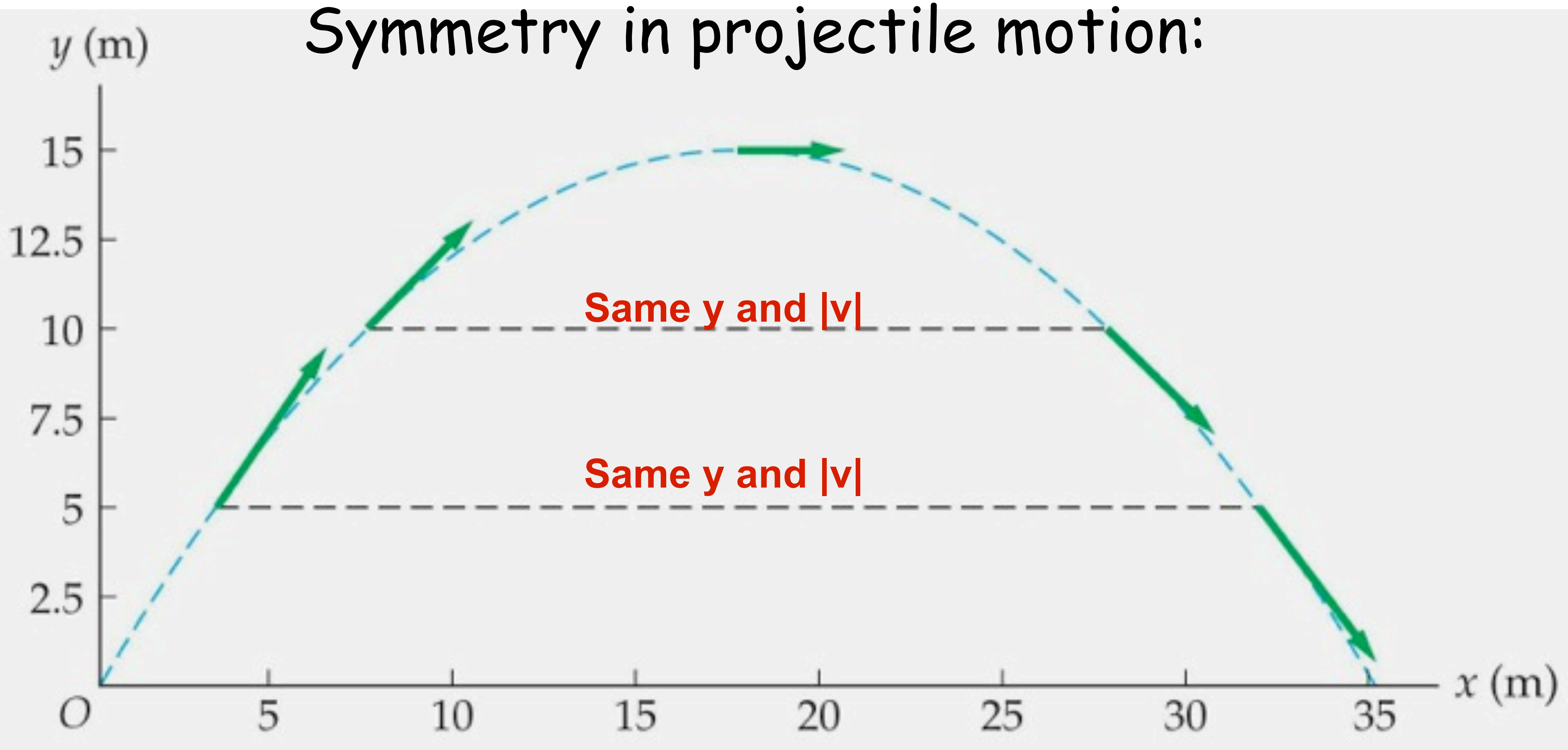


y-direction

- $v_{oy} = v_0 \sin \alpha_0$
- take the positive direction as upward
- then: free fall problem
- $a_y = -g$ (in general, $|a_y| = g$)
- uniformly accelerated motion: $y = y_0 + V_{oy}t - 1/2\ gt^2$

Projectile Motion: Symmetry

Symmetry in projectile motion:



Horizontal Range of a Projectile

$$0 = v_{0y}T - \tfrac{1}{2}gT^2; \quad T > 0$$

$$v_{0y} - \tfrac{1}{2}gT = 0$$

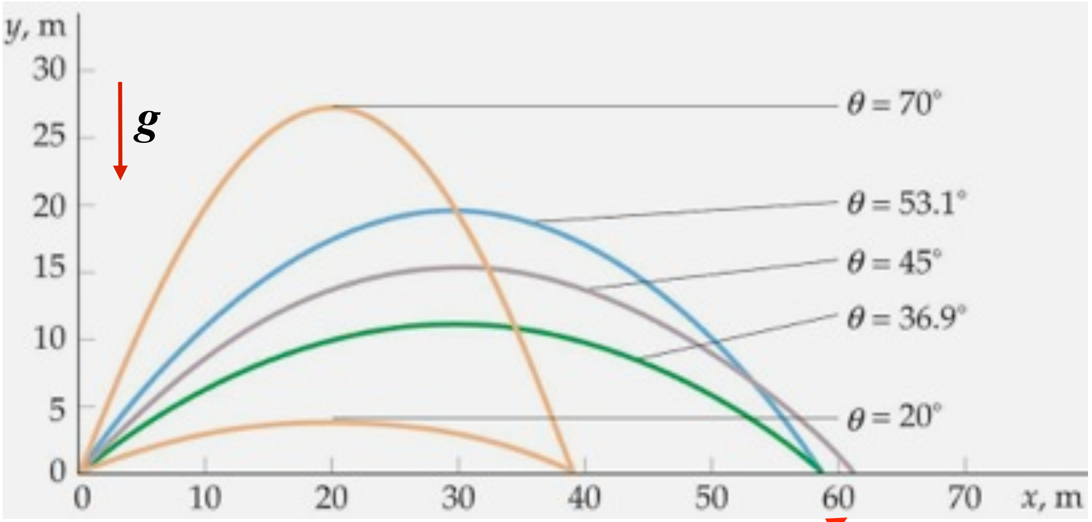
$$T = \frac{2v_{0y}}{g} = \frac{2v_0}{g}\sin\theta_0$$

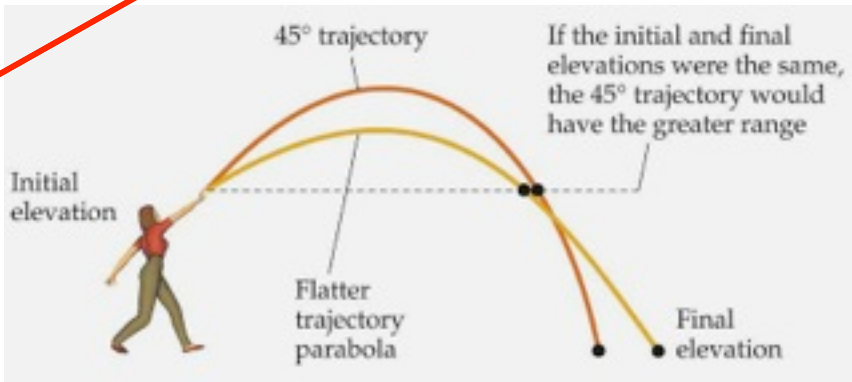
$$R = v_{0x}T = (v_0\cos\theta_0)\left(\frac{2v_0}{g}\sin\theta_0\right) = \frac{v_0^{\ 2}}{g}2\sin\theta_0\cos\theta_0$$

$$2\sin\theta_0\cos\theta_0 = \sin 2\theta_0$$

$$\boxed{R = \frac{v_0^{\ 2}}{g}\sin 2\theta_0}$$

R is maximum when θ_0=45°, so that sin $2\theta_0$ = 1.

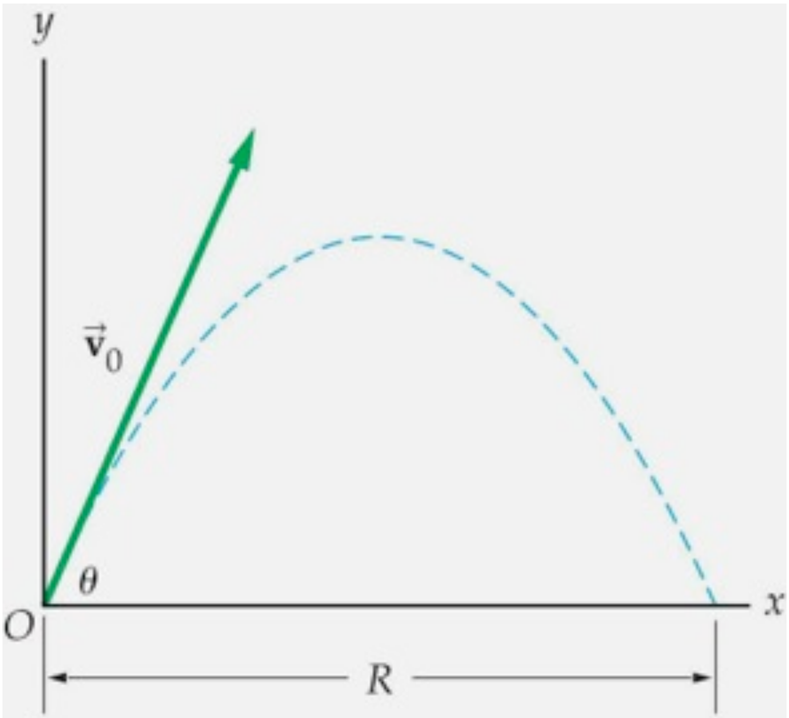




Projectile Motion: Range

Range: the horizontal distance a projectile travels

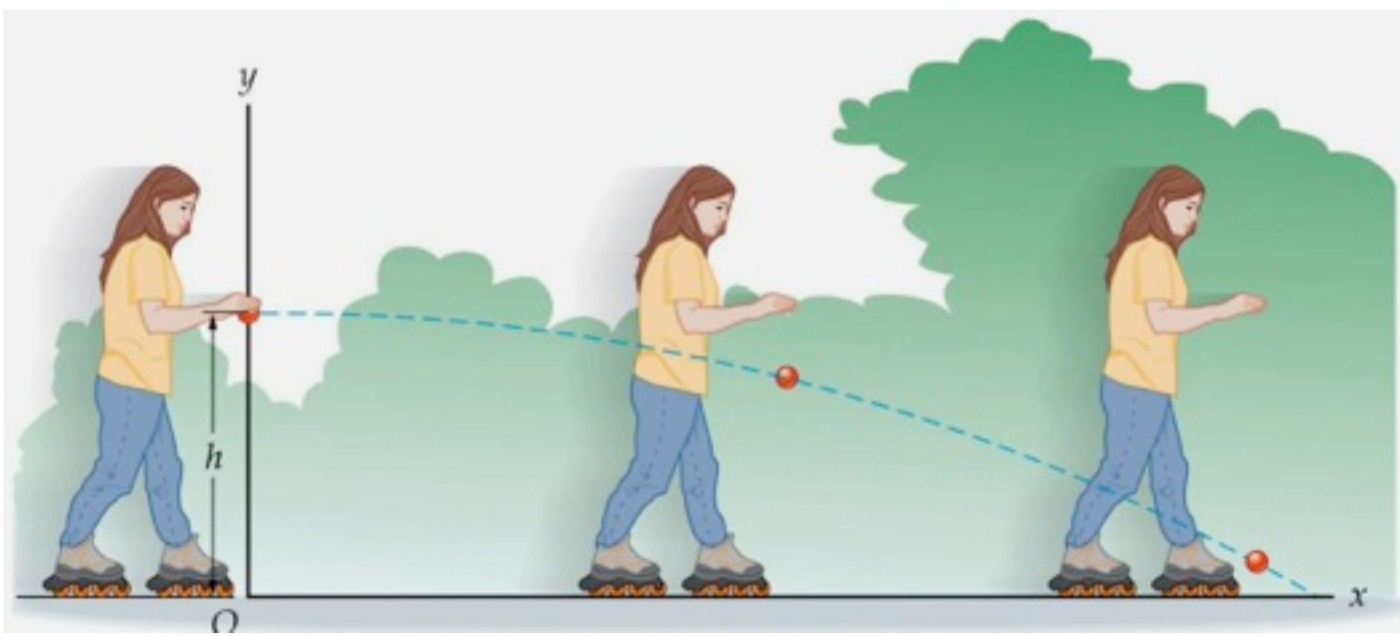
If the initial and final elevation are the same:



$$\boxed{R = \left(\frac{{v_0}^2}{g}\right) \sin 2\theta}$$

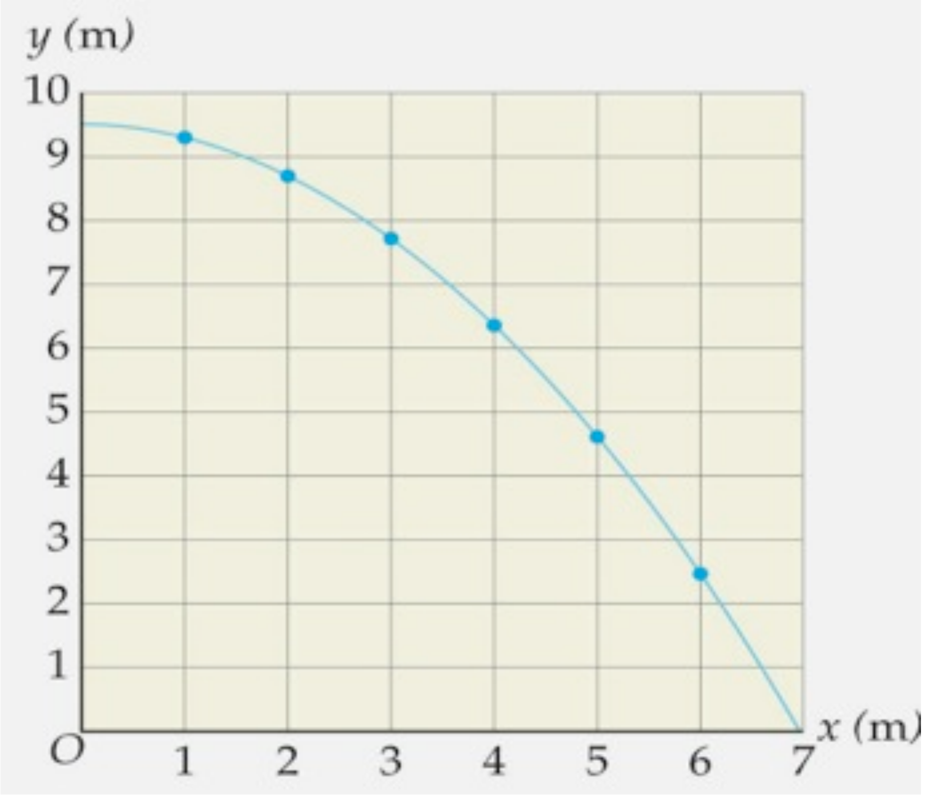
Independence of Vertical and Horizontal Motion

When you drop a ball while walking, running, or skating with constant velocity, it appears to you to drop straight down from the point where you released it. To a person at rest, the ball follows a curved path that combines horizontal and vertical motions.



Zero-Launch Trajectory

This is the **trajectory** of a projectile launched horizontally. It is a parabola.



Example: Jumping a Crevasse

A mountain climber encounters a crevasse in an ice field. The opposite side of the crevasse is 2.75 m lower, and the horizontal gap is 4.10 m. To cross the crevasse, the climber gets a running start and jumps horizontally.

(a) What is the minimum speed v_0 needed for the climber to cross the crevasse?

(b) Suppose the climber jumps at 6.0 m/s, where does he land?

(c) What is his speed on landing?

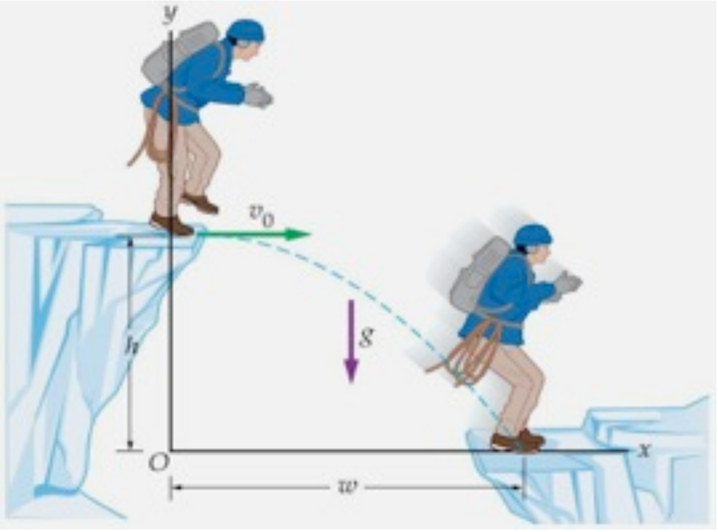
a) $y = h - \tfrac{1}{2}gt^2 = 0 \quad \Rightarrow \quad t = \sqrt{\dfrac{2h}{g}}$

$$x = v_0 t \quad \Rightarrow v_0 = \frac{x}{t} = x\sqrt{\frac{g}{2h}} = (4.10\text{ m})\sqrt{\frac{(9.81\text{ m/s}^2)}{2(2.75\text{ m})}} = 5.48\text{ m/s}$$

b) $x = v_0 t = v_0\sqrt{\dfrac{2h}{g}} = (6.00\text{ m/s})\sqrt{\dfrac{2(2.75\text{ m})}{(9.81\text{ m/s}^2)}} = 4.49\text{ m}$

c) $v_y = gt = \sqrt{2gh} = \sqrt{2(9.81\text{ m/s}^2)(2.75\text{ m})} = 7.35\text{ m/s}$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(6.00\text{ m/s})^2 + (7.35\text{ m/s})^2} = 9.49\text{ m/s}$$

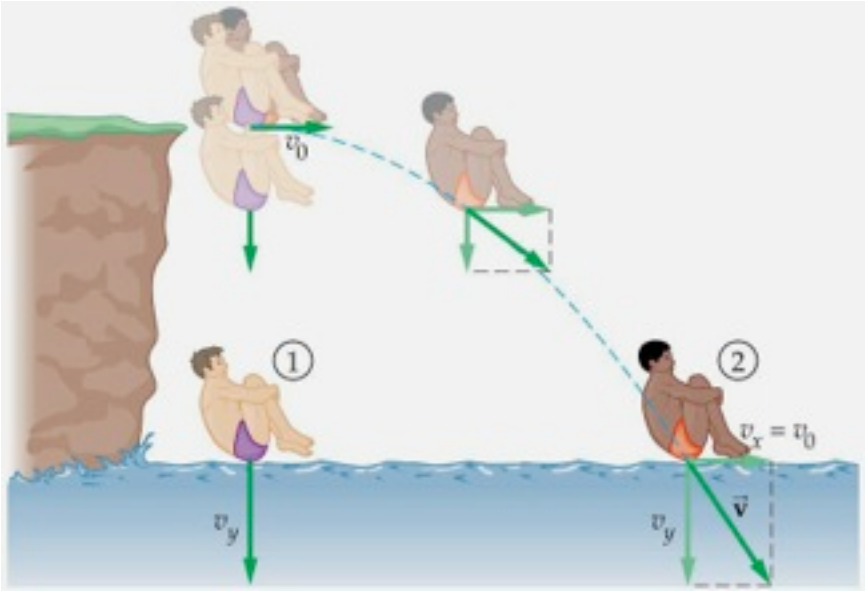


Example: Cliff Diving

Two boys, George and Sam, dive from a high overhanging cliff into a lake below.

George (1) drops straight down.

Sam (2) runs horizontally and dives outward.



(a) If they leave the cliff at the same time, which boy reaches the water first?

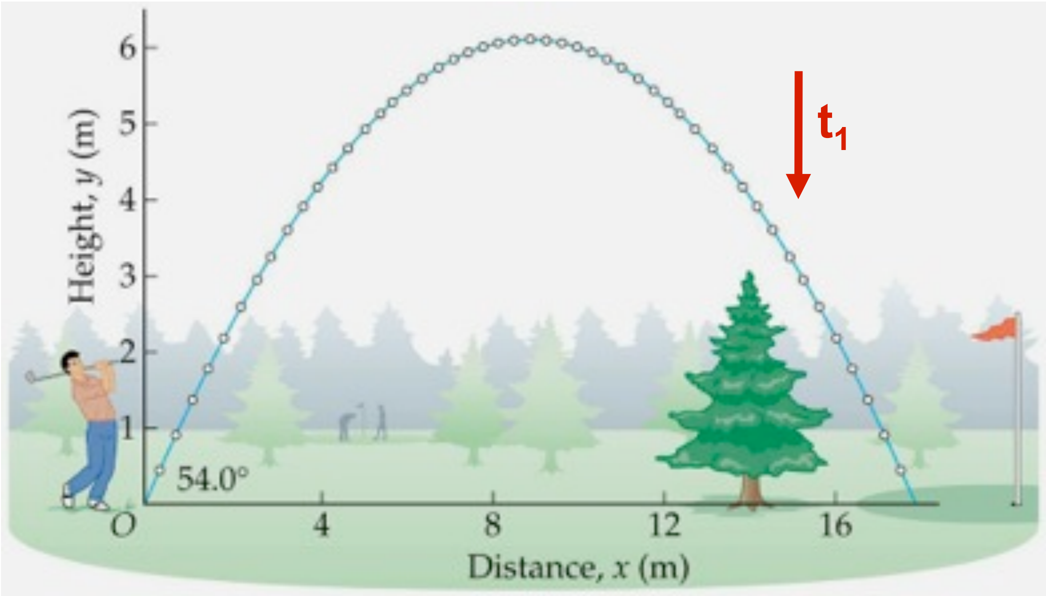
Since the vertical motion determines the time required to reach the water, both boys reach the water at the same.

(b) Which boy hits the water with the greater speed?

George reaches the water with only a vertical velocity, but Sam reaches the water with both horizontal and vertical velocity components. The vertical velocities are the same for both, so Sam's speed on entering the water is greater than that of George.

Example: A Rough Shot

Chipping from the rough, a golfer sends the ball over a 3.0 m high tree that is 14.0 m away. The ball lands on the green at the same level from which it was struck after traveling a horizontal distance of 17.8 m.



(a) If the ball left the club at 54.0° above the horizontal and landed 2.24 s later, what was its initial speed v_0?

(b) How high was the ball when it passed over the tree?

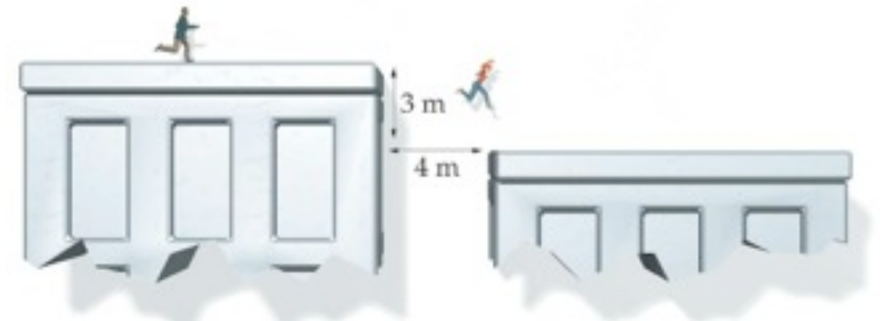
$$v_x = d/t = (17.8\text{ m})/(2.24\text{ s}) = 7.95\text{ m/s} \qquad v_0 = v_x/\cos\theta = (7.95\text{ m/s})/\cos 54.0° = \boxed{13.5\text{ m/s}}$$

$$t_1 = x/v_x = (14.0\text{ m})/(7.95\text{ m/s}) = 1.76\text{ s}$$

$$y = y_0 + v_y t_1 - \tfrac{1}{2}gt_1^2 = v_0 \sin\theta\, t_1 - \tfrac{1}{2}gt_1^2$$
$$= (13.5\text{ m/s})\sin 54.0°(1.76\text{ s}) - \tfrac{1}{2}(9.81\text{ m/s}^2)(1.76\text{ s})^2 = \boxed{4.03\text{ m}}$$

Example: To Catch a Thief

A police officer chases a master jewel thief across city rooftops. They are both running when they come to a gap between buildings that is 4.0 m wide and has a drop of 3.0 m. The thief having studied a little physics, leaps at 5.0 m/s at an angle of 45° above the horizontal and clears the gap easily. The police officer did not study physics and thinks he should maximize his horizontal velocity, so he leaps horizontally at 5.0 m/s.



(a) Does he clear the gap? **No!**

(b) By how much does the thief clear the gap?

Police officer

$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$

$$-3.0\text{ m} = 0 + 0 - \tfrac{1}{2}(9.81\text{ m/s}^2)t^2$$

$$t = \sqrt{6.0\text{ m}/9.81\text{ m/s}^2} = 0.782\text{ s}$$

$$x = x_0 + v_{0x}t = 0 + (5.0\text{ m/s})\ 0.78\text{ s} = 3.91\text{ m}$$

Thief

$$-3.0\text{ m} = 0 + (5.0\text{ m/s})\sin 45° t - \tfrac{1}{2}(9.81\text{ m/s}^2)t^2$$

$$t = -0.50\text{ s} \quad \text{or} \quad t = 1.22\text{ s}$$

$$x = x_0 + v_{0x}t = 0 + (5.0\text{ m/s})(1.22\text{ s})\cos 45° = 4.31\text{ m}$$