Physics 170 - Mechanics
Lecture 33

Waves
A wave pulse is a disturbance that propagates through a medium. It transfers energy without transferring matter; the energy is a combination of kinetic and potential energy.
Wave notion

A harmonic disturbance can set up a sinusoidal wave. The distance from crest to crest, or trough to trough, is called the wavelength, $\lambda$. 

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Wave Motion

Such a wave will have a sinusoidal form in both time and space.

Amplitude: maximum displacement

Wavelength: distance between points having the same phase

Frequency: number of waves passing per second

Period: time for one complete wave to pass
Wave Motion

Relationship between wave speed, wavelength, period, and frequency:

\[ v = \frac{\lambda}{T} = \lambda f \]
Wave Motion

MotionWaves may be either transverse (displacement perpendicular to direction of propagation) or longitudinal (displacement parallel to direction of propagation).
Wave Motion

Water waves may appear to be transverse, but they are actually a combination of transverse and longitudinal motion.
Wave Properties

When two or more waves travel through the same medium at the same time, they interfere in a process called superposition.

At any time, the combined waveform of two or more interfering waves is given by the sum of the displacements of the individual waves at each point in the medium.
This is an illustration of interference, using the principle of superposition. The displacement of any point on the rope is the sum of the individual displacements:

\[ y = y_1 + y_2 \]
Wave Properties

If the combined wave is larger than the individual ones, the interference is constructive; if smaller, it is destructive.

(a) Total constructive interference
(b) Total destructive interference
Wave Properties

Destructive interference is used in noise-canceling technology.
Wave Properties

Whether or not a wave is inverted upon reflection depends on whether the end is free to move or not.

(a) Fixed boundary, pulse inverted on reflection

(b) Free (movable) boundary, pulse not inverted on reflection
Wave Properties

When a wave enters a new medium, its speed usually changes, as the properties of the new medium are different. The direction of propagation changes also; this is called refraction.
Wave Properties

If the speed of the wave depends on its wavelength, it exhibits dispersion. The rainbow of light from a prism is an example of dispersion.
Wave Properties

Diffraction occurs when a wave passes through an opening that is comparable in size to the wavelength; the waves will “bend” around the edges of the opening.

Diffraction effects get less obvious as the gap gets larger.
Standing Waves and Resonance

On a rope with one fixed end, it is possible to set up waves that do not travel; they simply vibrate in place. These are called standing waves.
Standing Waves and Resonance

Some points on the wave remain stationary all the time; these are called nodes. Others have the maximum displacement; these are called antinodes. Adjacent nodes are separated by half a wavelength, as are adjacent antinodes.
Standing Waves and Resonance

When an integral number of half-wavelengths fit on the rope, the frequency is called the resonant frequency.
Standing Waves and Resonance

Natural frequencies for a stretched string:

\[ f_n = \frac{v}{\lambda_n} = n \left( \frac{v}{2L} \right) = nf_1 \quad \text{for } n = 1, 2, 3, \ldots \]

The wave speed is given by

\[ v = \sqrt{\frac{F_T}{\mu}} \]

where \( F_T \) is the tension and \( \mu \) is the mass per unit length.
Standing Waves and Resonance

Natural wavelengths can be varied by varying the length of a string, such as in a piano or harp; varying the mass per unit length of a string, as in a guitar; or varying the tension, which is done for fine tuning.

Driving a system at its natural frequency produces resonance; the amplitude at resonance is limited only by damping and by the strength of the materials.
Summary

• Simple harmonic motion requires a restoring force proportional to the displacement.

• The frequency is the inverse of the period.

• The total energy is proportional to the square of the amplitude.

\[ E = \frac{1}{2} kA^2 = \frac{1}{2} mv^2 + \frac{1}{2} kx^2 \]

• The equations of motion are sinusoidal.
Summary

• Velocity and period of a mass on a spring:

\[ v = \pm \sqrt{\frac{k}{m}} \left( A^2 - x^2 \right) \quad T = 2\pi \sqrt{\frac{m}{k}} \]

• Velocity and acceleration of a mass in simple harmonic motion:

\[ v = \omega A \cos \omega t \quad a = -\omega^2 A \sin \omega t \]
Summary

• A wave is a disturbance in space and time; wave motion transfers energy.

• The combined amplitude of two or more interfering waves is the sum of their individual amplitudes at each point.

• Standing waves may be produced on a string with fixed ends. Natural frequencies:

\[ f_n = \frac{v}{\lambda_n} = n \left( \frac{v}{2L} \right) = nf_1 \quad \text{for } n = 1, 2, 3, \ldots \]
Mathematical Description of a Wave

To describe a wave, we must know the position of the particles in the medium. This requires a function of the form $y(x,t)$.

$$y(x, t) = A \cos(\omega t \pm kx)$$

$+ \text{ is used for a wave traveling in the } -x \text{ direction, and } -$ is used for a wave traveling in the $+x$ direction.

$$k = \frac{2\pi}{\lambda}$$

is called the wave number.

$$\left(\omega t \pm kx\right)$$

is called the phase.

Note: it would also be valid to use the sine function in the above description.
Example: A wave on a string has an equation:

\[ y(x,t) = (4.00 \text{ mm}) \sin((600 \text{ rad/sec})t - (6.00 \text{ rad/m})x) \]

Compare this to \[ y(x,t) = A \sin(\omega t - kx) \]

(a) What is the amplitude of the wave?

\[ A = 4.00 \text{ mm} \]

(b) What is the wavelength?

\[ \lambda = \frac{2\pi}{k} = \frac{2\pi}{6.00 \text{ rad/m}} = 1.05 \text{ m} \]
(c) What is the period?

\[ T = \frac{2\pi}{\omega} = \frac{2\pi}{600 \text{ rad/sec}} = 1.05 \times 10^{-2} \text{ sec} \]

(d) What is the wave speed?

\[ v = \frac{\lambda f}{\lambda} = \left( \frac{\lambda}{2\pi} \right)(2\pi f) = \frac{\omega}{k} = \frac{600 \text{ rad/sec}}{6.00 \text{ rad/m}} = 100 \text{ m/s} \]

(e) What direction is the wave traveling.

Along the +x direction.