

# **Physics 170 - *Mechanics***

## **Lecture 29**

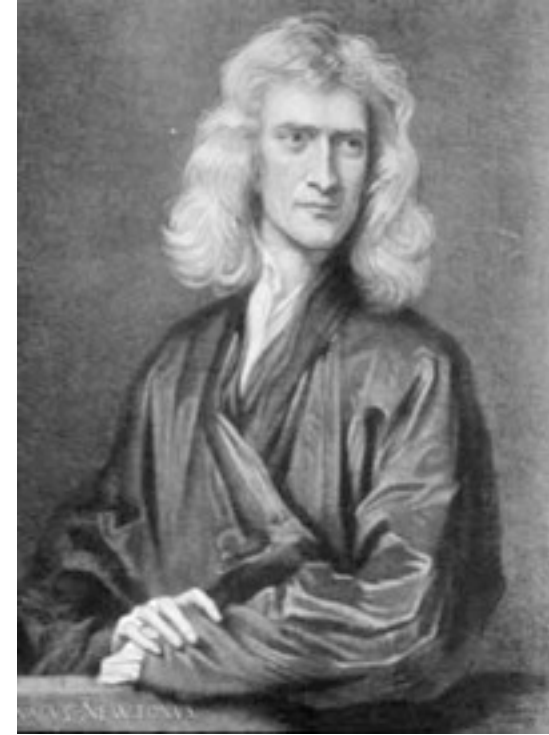
### **Gravitation**

# Isaac Newton (1642 - 1727)

Isaac Newton was born in 1642, the year of Galileo's death. He entered Trinity College of Cambridge University at the age of 19 and graduated in 1665, at the age of 23. Because the Black Death was ravaging Europe at the time, he then returned to his family's farm estate for two years to escape the pestilence.

It was during this period that he did his greatest work. He performed experiments in optics and laid the foundations of his theories of mechanics and gravitation. Because he needed it for his studies, he invented the **calculus** as a new branch of mathematics.

Newton, following an idea suggested by Robert Hooke, hypothesized that the force of gravity acting on the planets is inversely proportional to their distances from the Sun. This is now called Newton's Law of Gravity.



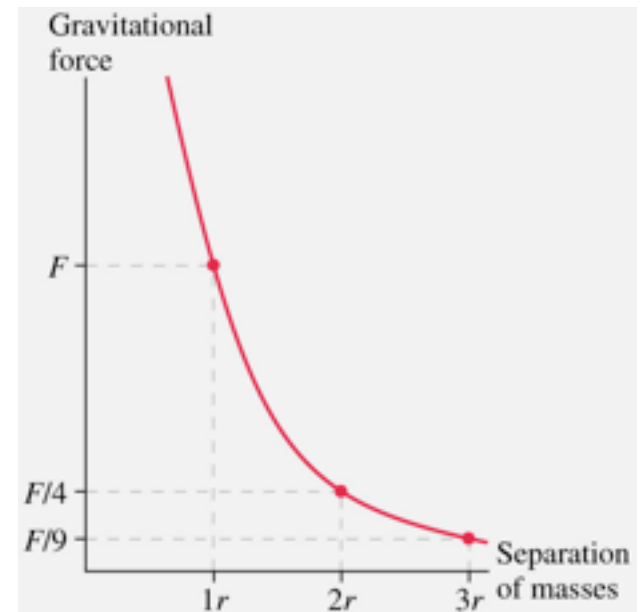
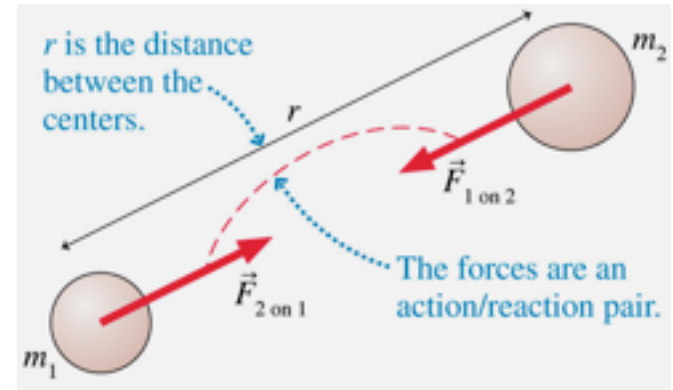
# Newton's Law of Gravity

Newton proposed that every object in the universe attracts every other object with a force that has the following properties:

1. The force is inversely proportional to the square of the distance between the objects.
2. The force is directly proportional to the product of the masses of the two objects.

$$F_{1 \text{ on } 2} = F_{2 \text{ on } 1} = G \frac{m_1 m_2}{r^2}$$

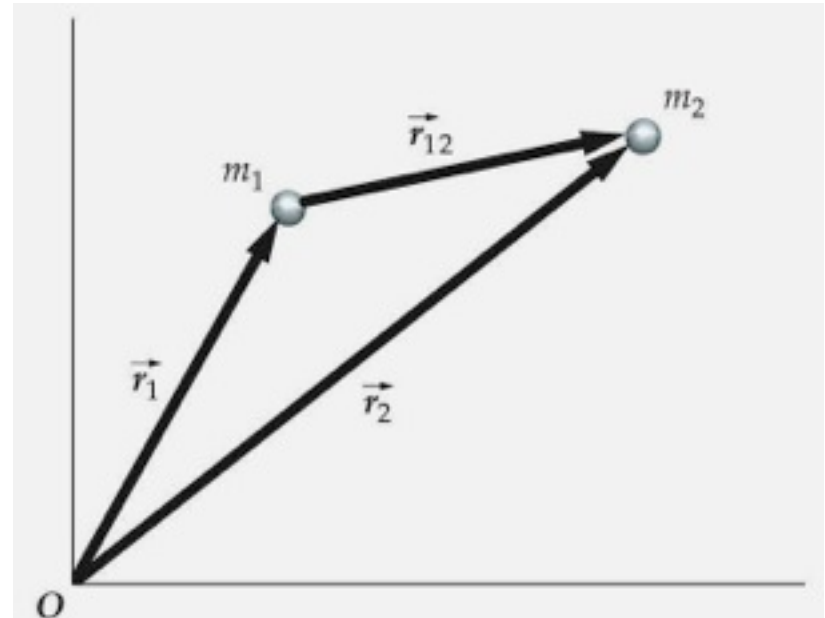
$$G = 6.673 \times 10^{-11} \text{ N m}^2 / \text{kg}^2$$



# Newton's Law of Gravity



$$\vec{F}_{12} = G \frac{m_1 m_2}{r_{12}^2} \hat{r}_{12}$$



$$G = 6.67 \times 10^{-11} \text{ N m}^2 / \text{kg}^2$$

For a particle near the Earth,  $F_g = G \frac{M_E m}{r^2}$  and  $a = \frac{F_g}{m} = G \frac{M_E}{r^2}$

At the Earth's surface ( $r = R_E$ ),  $a = g = G \frac{M_E}{R_E^2}$



# Superposition in Newton's Law of Universal Gravitation

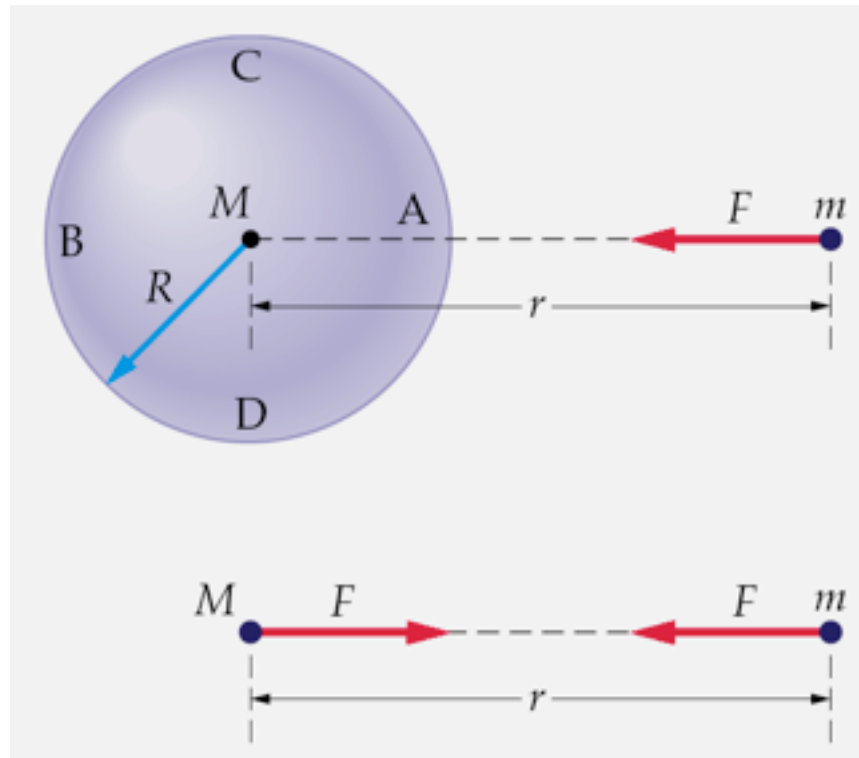
$G$  is a very small number; this means that the force of gravity is negligible, unless there is a very large mass involved (such as the Earth).

If an object is being acted upon by several different gravitational forces, the net force on it is the vector sum of the individual forces.

This is called the **principle of superposition**.

# Gravitational Attraction of Spherical Bodies

Gravitational force between a point mass and a sphere: the force is exactly the **same** as if all the mass of the sphere were concentrated **at its center**.



# The Earth's Gravitation

What about the gravitational force on objects at the surface of the Earth? The center of the Earth is one Earth radius away, so this is the distance we use:

$$F = G \frac{mM_E}{R_E^2} = m \left( \frac{GM_E}{R_E^2} \right)$$

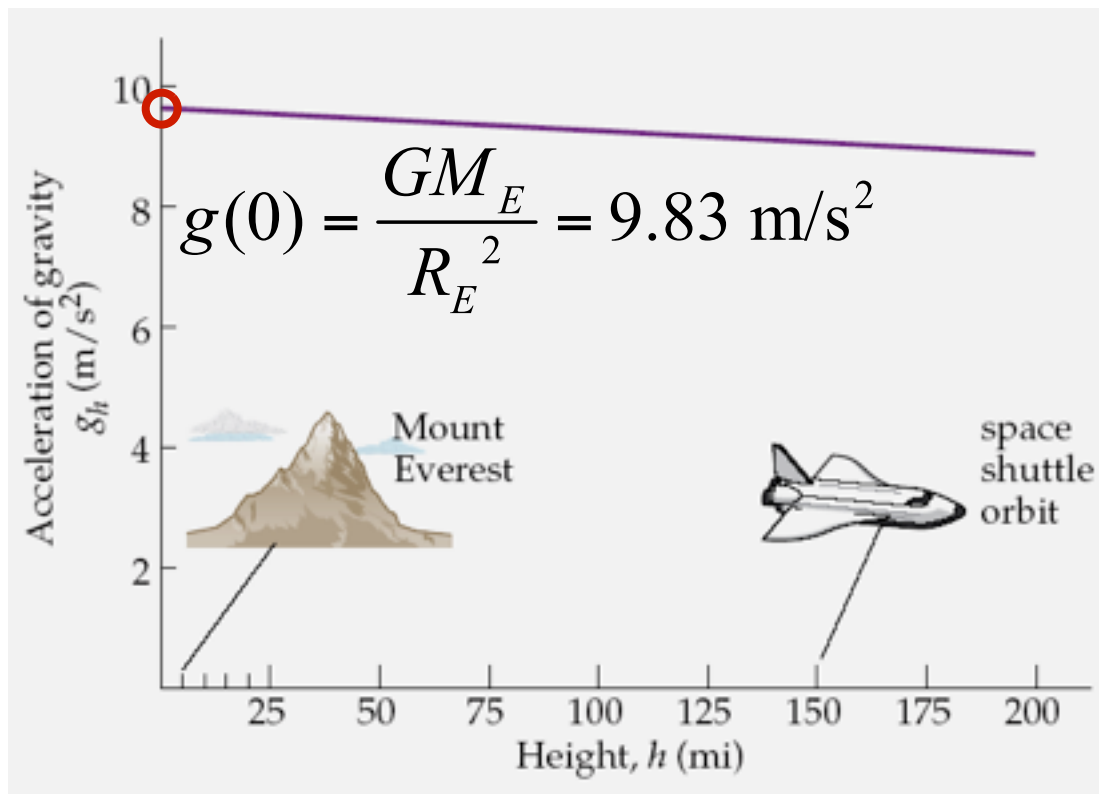
Therefore,

$$m \left( \frac{GM_E}{R_E^2} \right) = mg \quad \text{and}$$

$$g = \frac{GM_E}{R_E^2}$$

# Gravitation vs. Altitude

The acceleration of gravity decreases slowly with altitude:

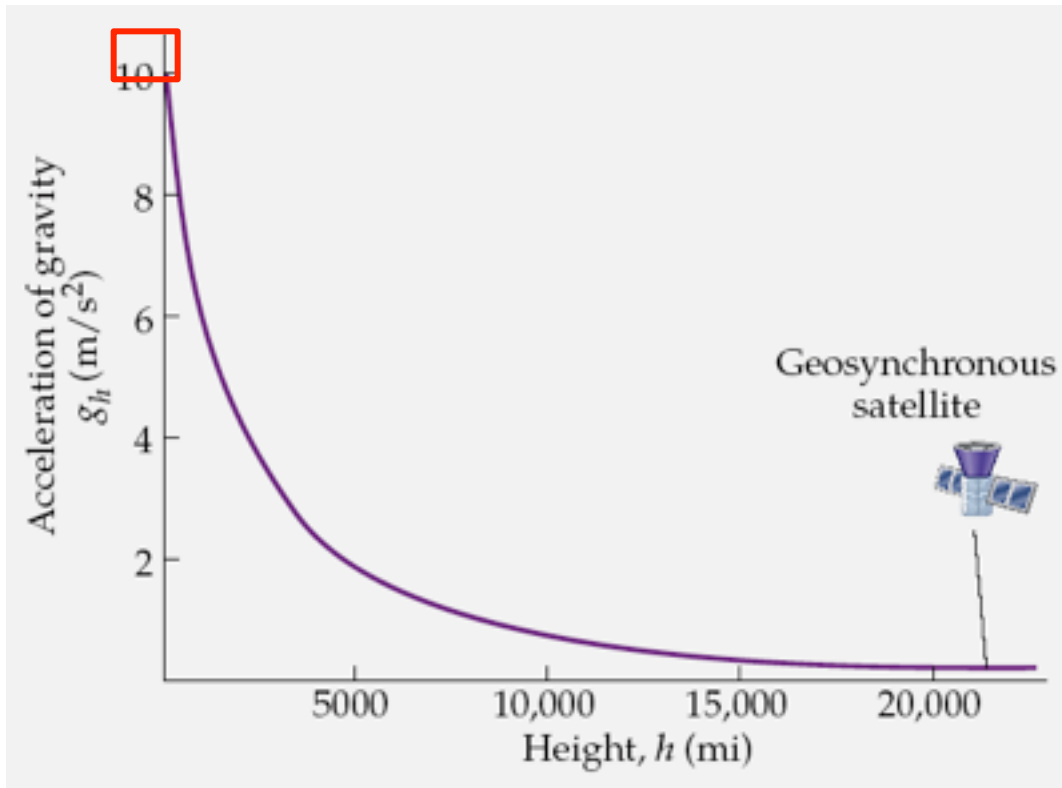


$$g(h) = \frac{GM_E}{(R_E + h)^2}$$



# Gravitation vs. Altitude

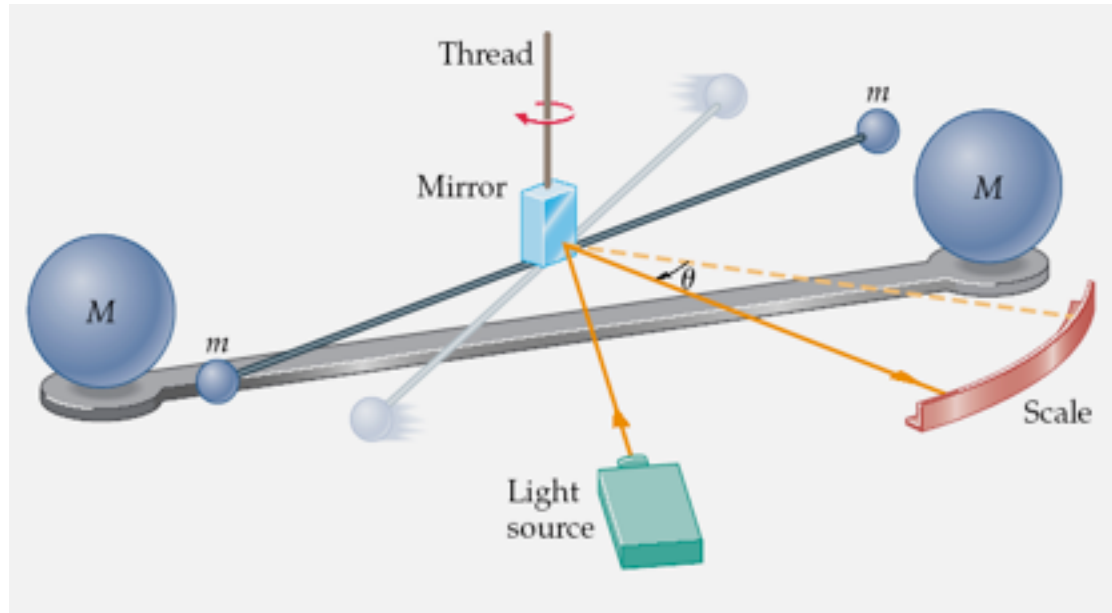
Once the altitude becomes comparable to the radius of the Earth, the decrease in the acceleration of gravity is much larger:



Earth radius: 3,959 miles (6,371 km)

$$g(r) = \frac{GM_E}{r^2}$$











# Measuring $G$



Newton's gravitational constant  $G$  must be measured in the laboratory. Henry Cavendish made the first accurate measurement of this quantity, using a "Cavendish balance". The forces between masses are measured using their action in twisting a thin fiber.  $G$  is calculated from the measured force.



# Clicker Question 1



Which of these systems has the largest force of gravitational attraction ?



- (a)  $m_1 = 2$    $r = 4$    $m_2 = 2$
- (b)  $m_1 = 1$    $r = 1$    $m_2 = 1$
- (c)  $m_1 = 1$    $r = 2$    $m_2 = 1$
- (d)  $m_1 = 1$    $r = 4$    $m_2 = 4$
- (e)  $m_1 = 4$    $r = 8$    $m_2 = 4$



# Clicker Question 1


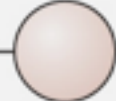
Which of these systems has the largest force of gravitational attraction ?

(a)  $m_1 = 2$    $r = 4$    $m_2 = 2$   $4/16 = 1/4$

(b)  $m_1 = 1$    $r = 1$    $m_2 = 1$   $1/1 = 1$

(c)  $m_1 = 1$    $r = 2$    $m_2 = 1$   $1/4$

(d)  $m_1 = 1$    $r = 4$    $m_2 = 4$   $4/16 = 1/4$

(e)  $m_1 = 4$    $r = 8$    $m_2 = 4$   $16/64 = 1/4$

# The Solar System



$$1 \text{ AU} = 15.0 \times 10^{10} \text{ m} = 93.0 \times 10^6 \text{ mi}$$

What's going on?

Table 11-1

Mean Orbital Radii and Orbital Periods for the Planets

Planet	Mean Radius $r$ ( $\times 10^{10}$ m)	Period $T$ (y)
Mercury	5.79	0.241
Venus	10.8	0.615
Earth	15.0	1.00
Mars	22.8	1.88
Jupiter	77.8	11.9
Saturn	143	29.5
Uranus	287	84
Neptune	450	165
Pluto	590	248

# Johannes Kepler (1571-1630)

Johannes Kepler inherited (or stole) the detailed observations of planetary positions of Danish astronomer Tycho Brahe and tried to make sense of them, using algebra, trigonometry, and geometry.

After a decade of work, he was forced to conclude that planetary orbits were better described by ellipses than by circles, and that the planets travel in these orbits with a varying speed.

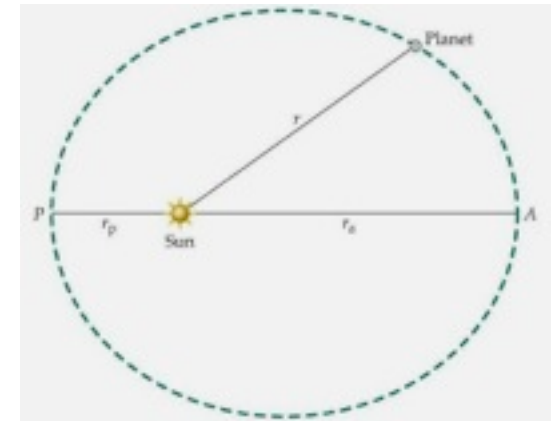




# Kepler's Laws of Orbital Motion

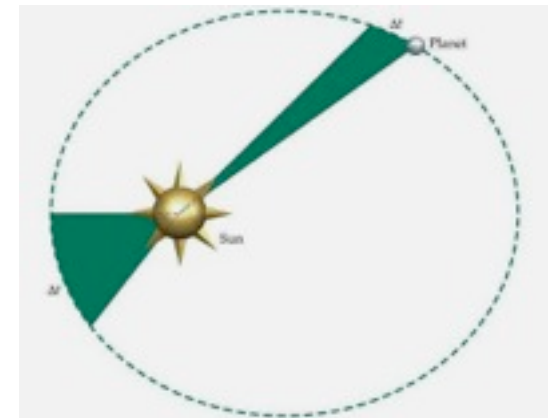
## Kepler's 1st Law:

All planets move in elliptic orbits, with the Sun at one focus of the ellipse.



## Kepler's 2nd Law:

A line joining any planet to the Sun sweeps out equal areas in equal times.



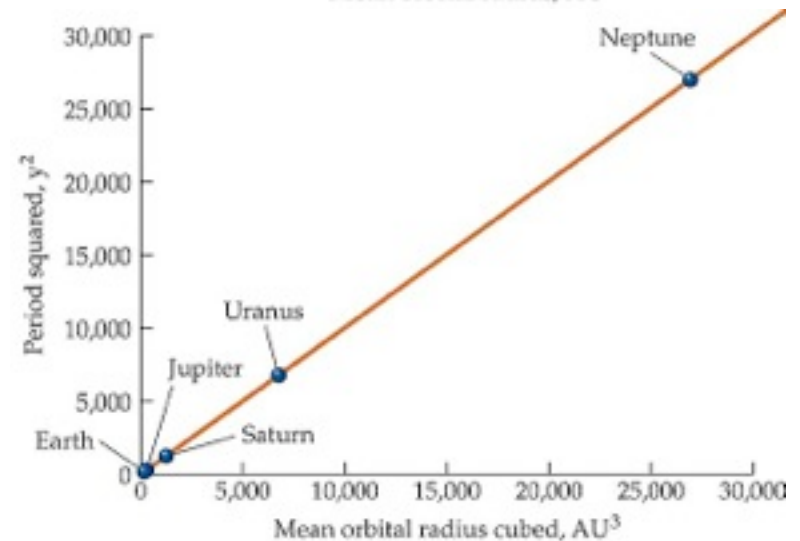
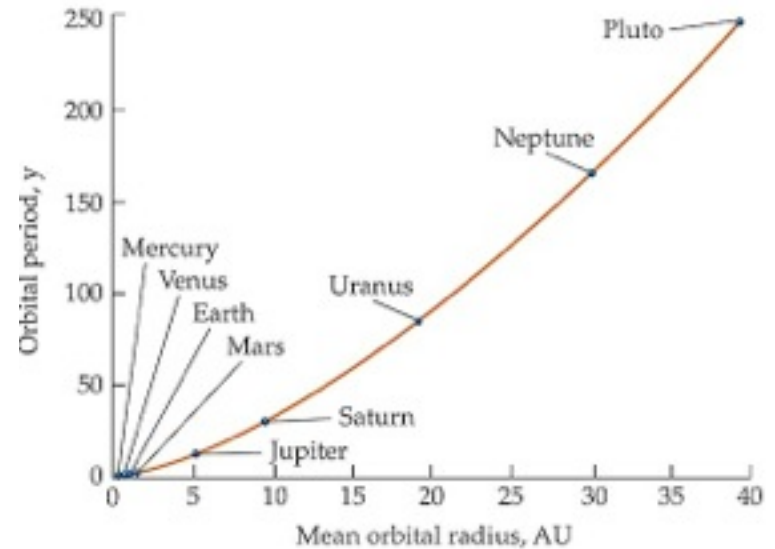
# Kepler's 3<sup>rd</sup> Law

## Kepler's 3<sup>rd</sup> Law:

The square of the period of any planet is proportional to the cube of the semi-major axis of its elliptical orbit.

$$T^2 = C r^3$$

From Newton's  
Laws, we can show that  
Kepler's  $C = 4\pi^2/GM$





# Example: Jupiter's Orbit

Jupiter's mean orbital radius is  $r_J = 5.20$  AU (Earth's mean orbital radius is 1 AU).

What is the period  $T_J$  of Jupiter's orbit around the Sun?

$$T^2 = Cr^3 \quad \text{so} \quad \left(\frac{T_J}{T_E}\right)^2 = \left(\frac{r_J}{r_E}\right)^3$$

$$T_J = T_E \left(\frac{r_J}{r_E}\right)^{3/2} = (1.0 \text{ yr}) \left[\frac{(5.20 \text{ AU})}{(1.00 \text{ AU})}\right]^{3/2} = 11.9 \text{ yr}$$

# Geosynchronous Orbits

In 1945, the British science fiction author Arthur C. Clarke pointed out that it was possible to put a satellite in an orbit above the equator with a period of exactly 24 hours = 86,400 s, so that it rotates around the Earth at just the rate that the Earth rotates under it. Such "geosynchronous" satellites hang above a particular point on the equator and are now widely used for communications.



Arthur C. Clarke  
(1917 – 2008)

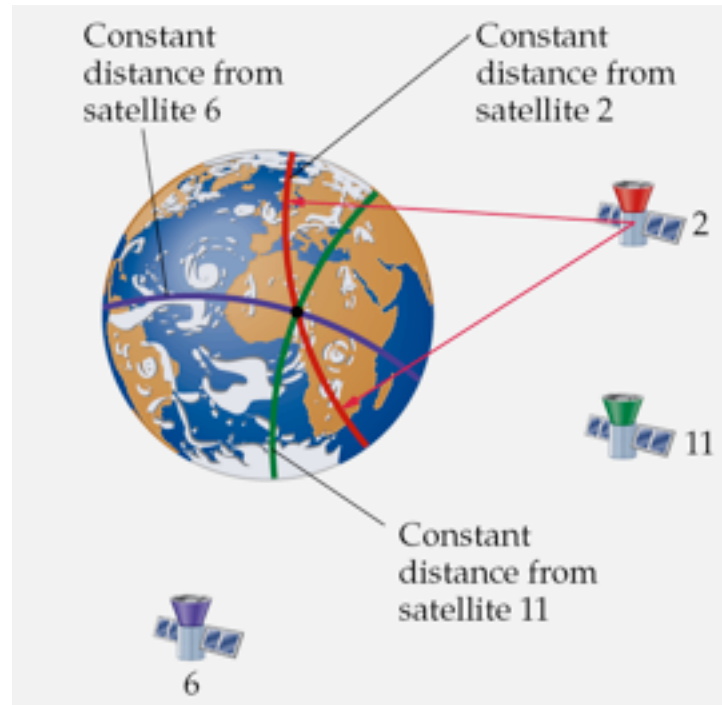
$$\begin{aligned} r_{\text{geo}} &= R_E + h_{\text{geo}} = \sqrt[3]{\frac{GM}{4\pi^2} T^2} \\ &= \sqrt[3]{\frac{(6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{4\pi^2} (86,400 \text{ s})^2} \\ &= 4.225 \times 10^7 \text{ m} \quad h_{\text{geo}} = r_{\text{geo}} - R_E = (4.225 \times 10^7 \text{ m}) - (6.37 \times 10^6 \text{ m}) \\ &= 3.59 \times 10^7 \text{ m} = 2.23 \times 10^4 \text{ mi} \end{aligned}$$

Notice that  
this is a  
**cube-root.**

Clarke also envisioned lowering a 22,300 mile long "rope" from a geosynchronous space station to the ground, which could then be used to lift objects into space without rockets, using a "space elevator". This is now being seriously considered using super-strong cables, perhaps made from carbon nanotubes.

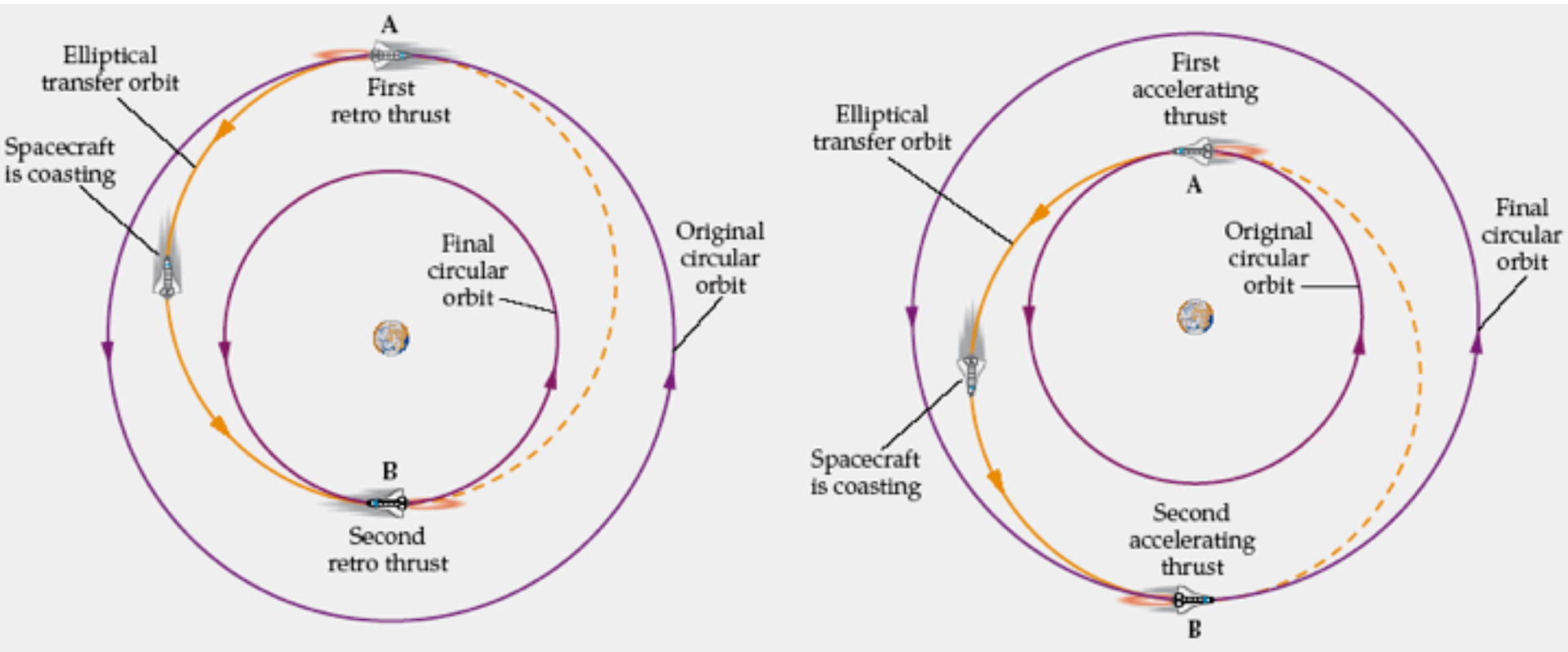
# The GPS Orbits

GPS satellites are not in geosynchronous orbits; their orbit period is 12 hours. Triangulation of time signals from several satellites simultaneously allows precise location of objects on Earth in position and altitude.



# Orbital Maneuvers

Kepler's laws also give us an insight into possible orbital maneuvers. Slowing down moves you inward, while speeding up moves you outward.

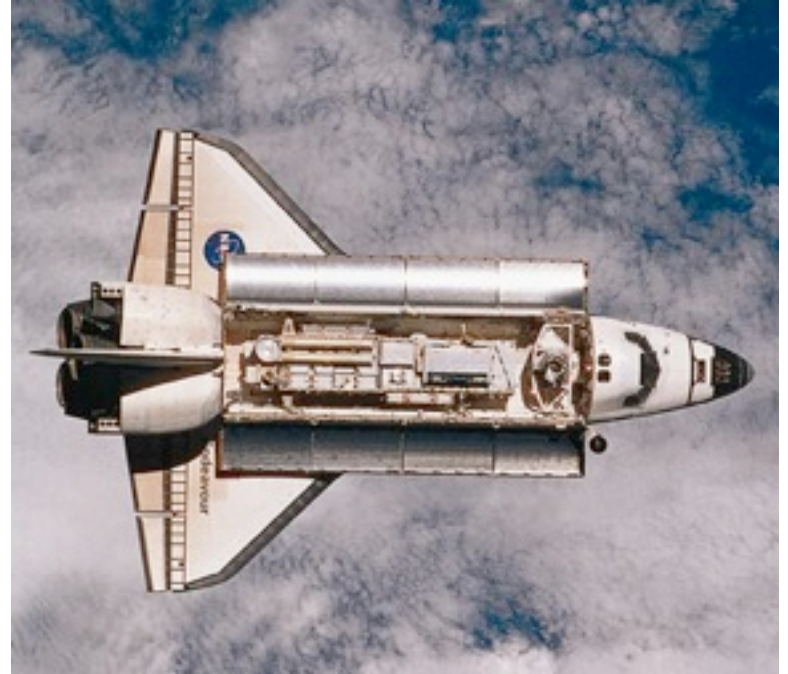


# Example: Falling to Earth

What is the acceleration due to gravity of an object at the altitude of the space shuttle's orbit, about 400 km above the Earth's surface?

$$r = R_E + h = (6370 \text{ km}) + (400 \text{ km}) = 6770 \text{ km}$$

$$\begin{aligned} a &= \frac{F_g}{m} = \frac{GmM_E}{r^2} / m = \frac{GM_E}{r^2} \\ &= \frac{(6.67 \times 10^{-11} \text{ m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{(6.77 \times 10^6 \text{ m})^2} = 8.70 \text{ m/s}^2 \end{aligned}$$



Why, then is there **zero gravity** within the Shuttle's living area?  
Because it is in free fall and freely responding to the pull of gravity.

# Example:

## The Orbiting Space Station

You are trying to view the International Space Station (ISS), which travels in a roughly circular orbit about the Earth.

If its altitude is 385 km above the Earth's surface, how long do you have to wait between sightings?

$$2\pi r = vT \Rightarrow v = \frac{2\pi r}{T}$$

$$F_g = G \frac{M_E m}{r^2} = \frac{mv^2}{r} \Rightarrow \frac{GM_E}{r} = \frac{4\pi^2 r^2}{T^2}$$

$$T = \sqrt{\frac{4\pi^2}{GM_E} r^3} = 2\pi \sqrt{\frac{r^3}{GM_E}} = 2\pi \sqrt{\frac{(R_E + h)^3}{GM_E}}$$

$$T = 2\pi \sqrt{\frac{(6375 \text{ km} + 385 \text{ km})^3}{(6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}} = 5,528 \text{ s} = 92.1 \text{ min}$$

