Physics 170 - *Mechanics*

Lecture 28

Rotational Dynamics

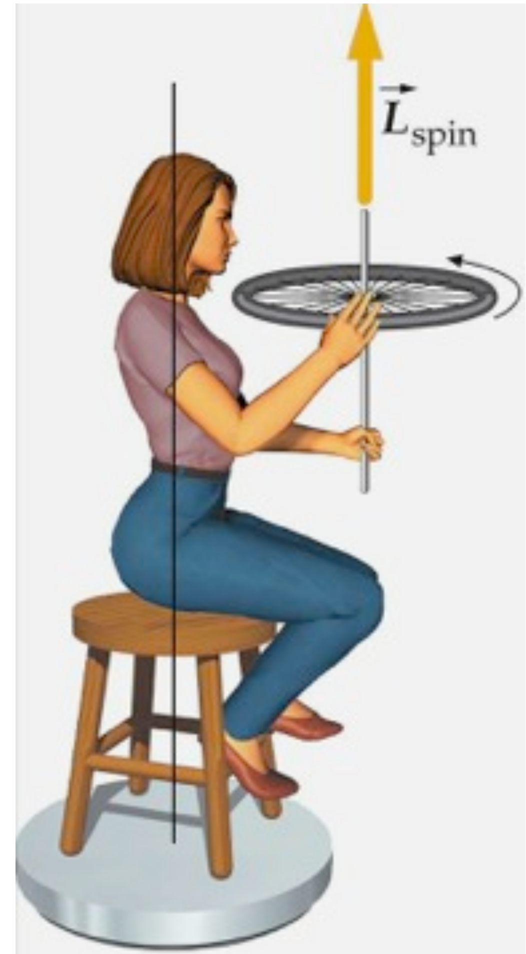
<u>Example</u>: Spinning the Wheel

You are sitting on a stool on a frictionless turntable holding a bicycle wheel. Initially, neither the wheel nor the turntable is spinning. You hold the axel vertical with one hand and spin the wheel counterclockwise with the other hand.

You observe that the stool and turntable begin to rotate clockwise. Then you stop the wheel with your free hand.

What happens to the turntable rotation?

The turntable stops!



Example: Tipping the Wheel

A student sitting on a stool on a frictionless turntable is holding a rapidly spinning bicycle wheel. Initially, the axis of the wheel is horizontal, with the angular momentum vector $\boldsymbol{L}$ pointing to the right.

What happens when the student tips the wheel so that the spin axis is vertical, with the wheel spinning counterclockwise?

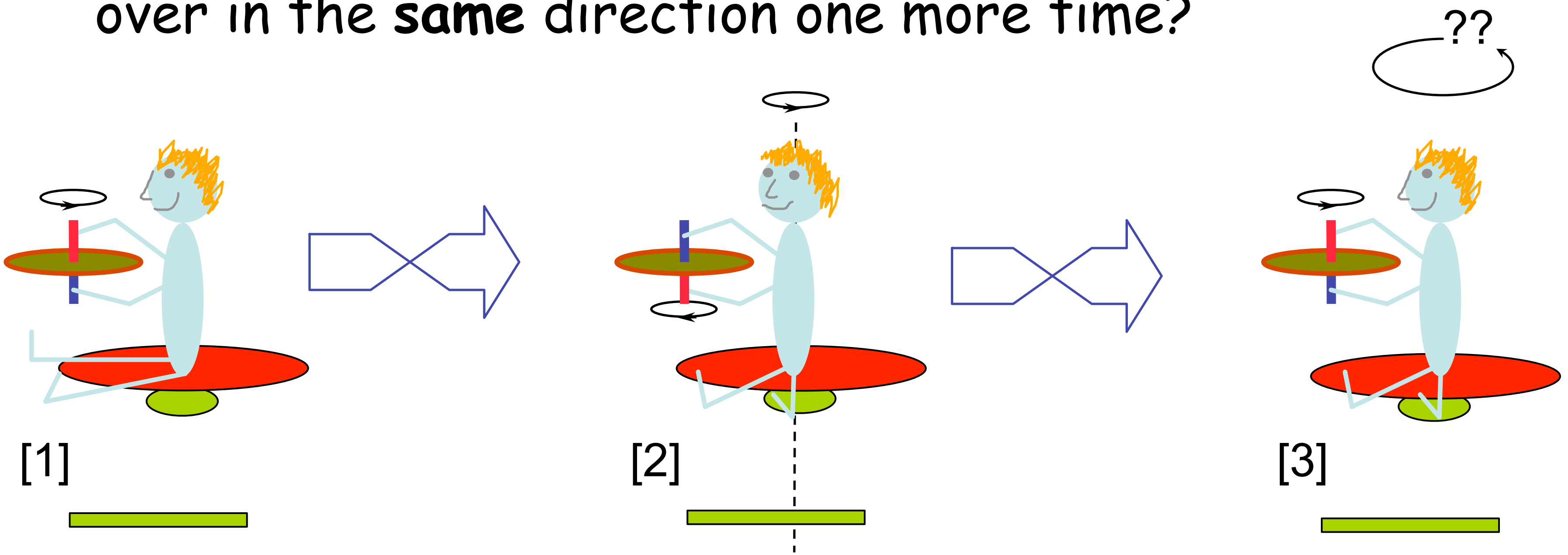
The system is free to rotate about the vertical axis (no vertical torques) and initially the angular momentum is zero along that axis. Therefore, vertical angular momentum is conserved and the final angular momentum must also be zero.

In the final state, the wheel has a large angular momentum pointing vertically upward, so the stool and student must rotate clockwise to have an equal and opposite downward angular momentum.

Question

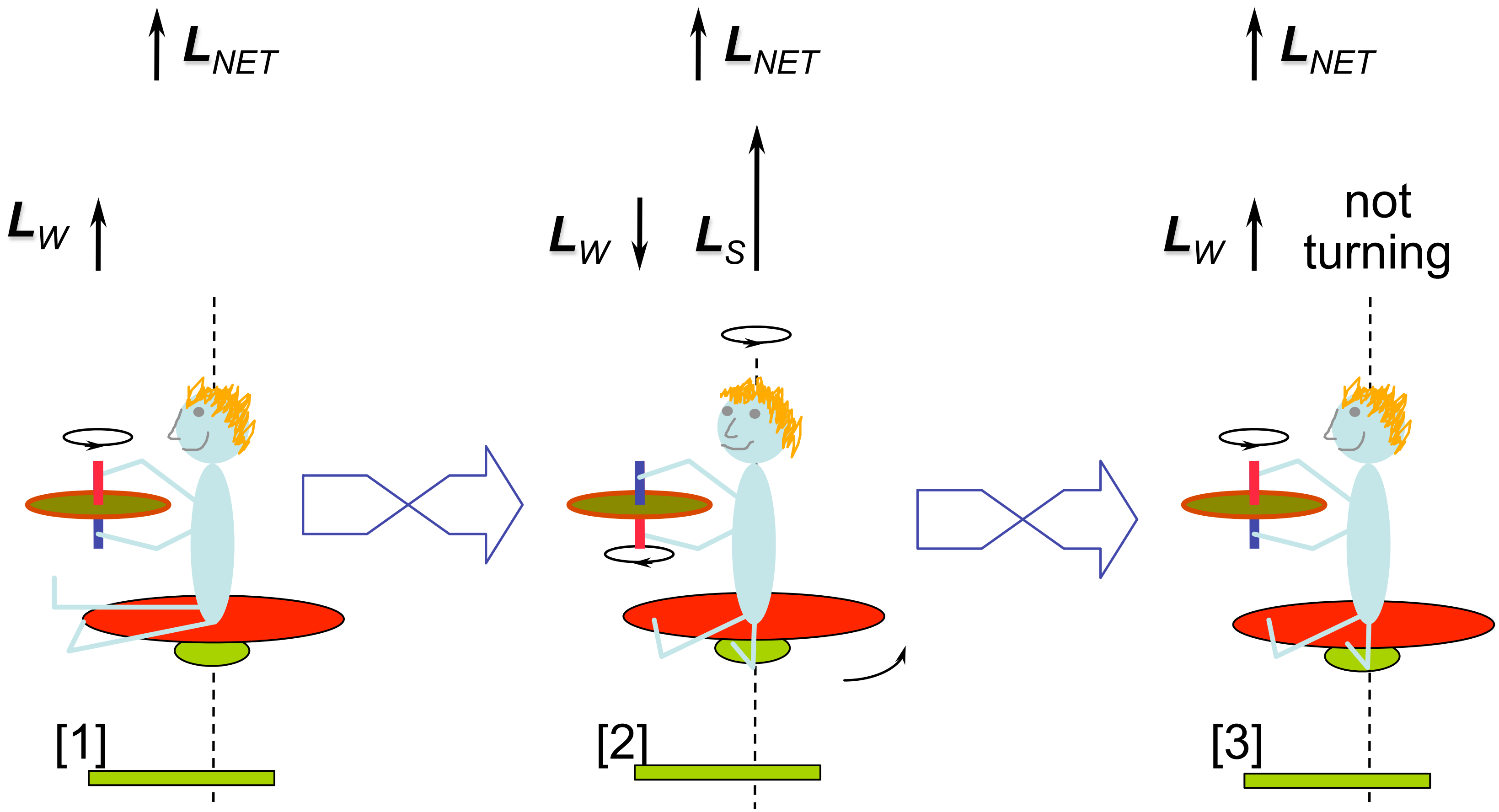
I am initially at rest on the rotatable chair, holding the wheel spinning as shown in [1]. Now [2] I turn it over and the chair starts to rotate.

Question: What happens if [3] the wheel is turned over in the **same** direction one more time?

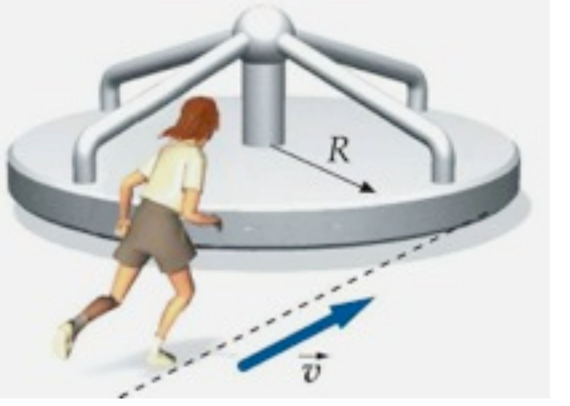


(a) rotation stops; (b) rotation doubles; (c) rotation stays the same

Turning the Bike Wheel Twice



Example:
Ride the Merry-Go-Round



A 25 kg child at a playground runs with an initial speed of 2.5 m/s along a path tangent to the rim of a merry-go-round with a radius of 2.0 m and jumps on. The merry-go-round, which is initially at rest, has a moment of inertia of 500 kg m^2.

Find the angular velocity of the child and merry-go-round.

$$L_f = L_i \quad \Rightarrow \quad \left(I_{mgr} + m_c R^2\right)\omega_f = \vec{R} \times m_c \vec{v}_i = R m_c v_i$$

$$\omega_f = \frac{R m_c v_i}{I_{mgr} + m_c R^2} = \frac{(2.0\text{ m})(25\text{ kg})(2.5\text{ m/s})}{(500\text{ kg m}^2) + (25\text{ kg})(2.0\text{ m})^2} = \boxed{0.21\text{ rad/s}}$$

Example: merry-go-round

A nearby park has a merry-go-round with a 3.0 m diameter turntable that has a 130 kg m^2 moment of inertia. Initially five friends stand near the rim while the turntable rotates at 20 rev/min. Four friends move to within 0.3 m of the center, leaving Gene at the rim. Gene is quick and strong, so it would require an acceleration of 4.0 g to throw him off. Assume everybody has a mass of 60 kg will Gene stay on the merry go round?

$$L_f = L_i \quad \Rightarrow \quad I_f \omega_f = I_i \omega_i \qquad \omega_i = (20 \text{ rev/min})(2\pi \text{ rad/rev}) / (60 \text{ s/min}) = 2.09 \text{ rad/s}$$

$$I_i = 5mR^2 + I_{\text{mgr}} = 5(60 \text{ kg})(1.5 \text{ m})^2 + (130 \text{ kg m}^2) = 805 \text{ kg m}^2$$

$$I_f = mR^2 + 4mr^2 + I_{\text{mgr}} = (60 \text{ kg})(1.5 \text{ m})^2 + 4(60 \text{ kg})(0.3 \text{ m})^2 + (130 \text{ kg m}^2) = 287 \text{ kg m}^2$$

$$\omega_f = \omega_i I_i / I_f = (2.09 \text{ rad/s})(805 \text{ kg m}^2)/(287 \text{ kg m}^2) = 5.88 \text{ rad/s}$$

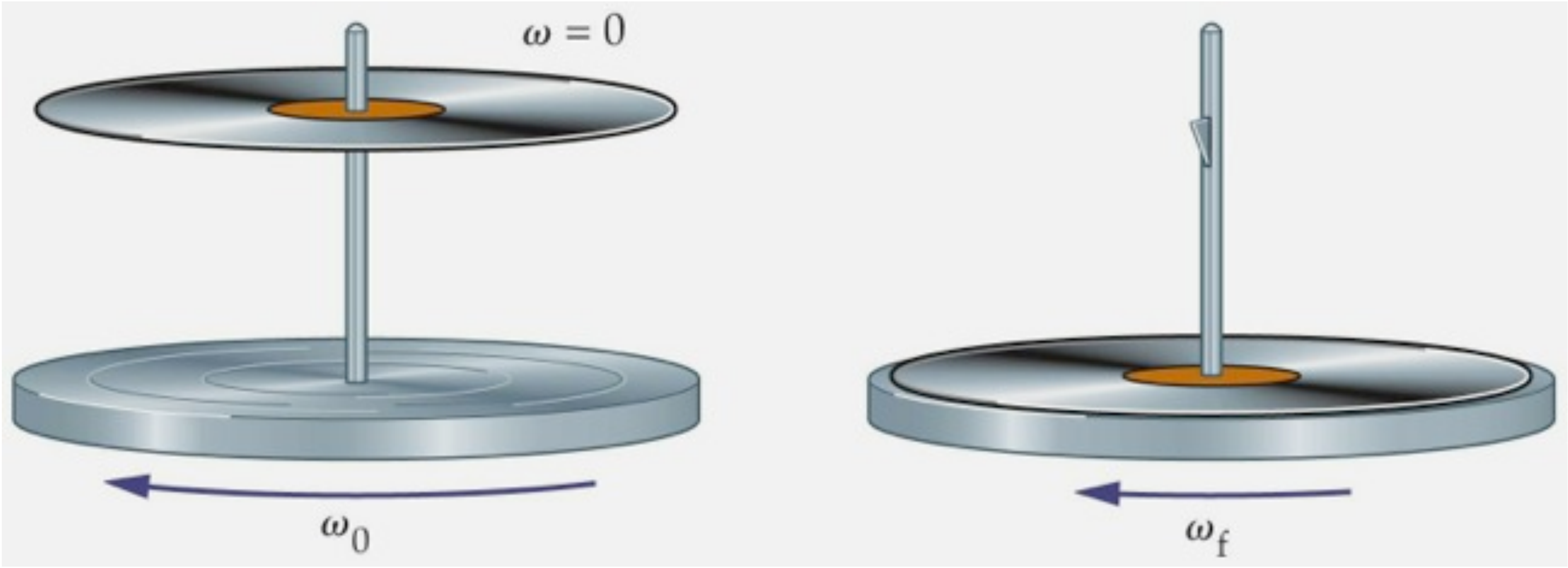
$$a_c = \omega^2 R = (5.88 \text{ rad/s})^2 (1.5 \text{ m}) = 51.9 \text{ m/s}^2 = 5.29 \text{ g}$$

No!

Rotational Collisions

If the moment of inertia increases, the angular speed decreases, so the angular momentum does not change.

Angular momentum is conserved in rotational collisions:



Example: Two Interacting Disks

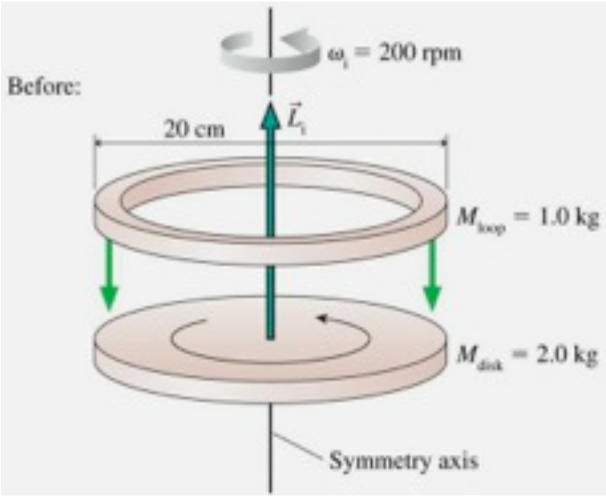
A 20 cm diameter 2.0 kg solid disk is rotating at 200 rpm. A 20 cm diameter 1.0 kg circular loop is dropped straight down on the rotating disk. Friction causes the loop to accelerate until it is "riding" on the disk.

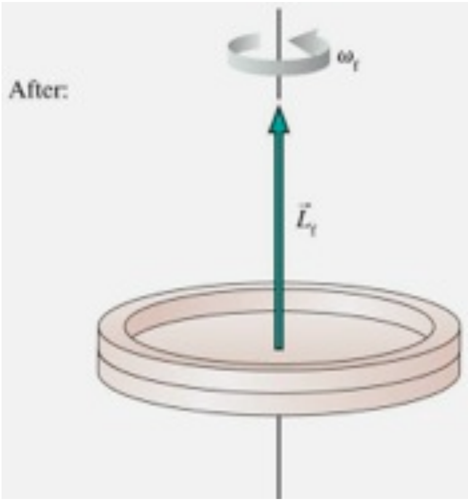
What is the final angular velocity of the combined system?

$$L_f = I_{\text{disk}}\omega_f + I_{\text{loop}}\omega_f = L_i = I_{\text{disk}}\omega_i$$

$$\omega_f = \omega_i \frac{I_{\text{disk}}}{I_{\text{disk}} + I_{\text{loop}}}$$

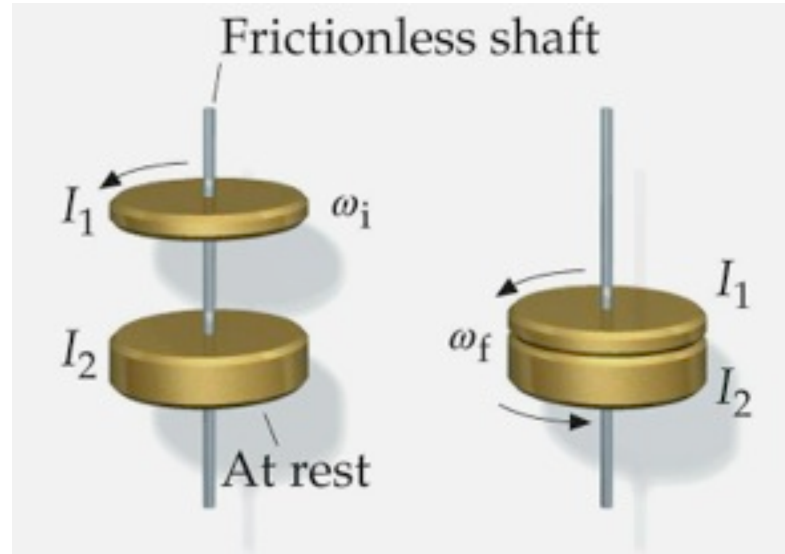
$$= \omega_i \frac{\frac{1}{2} M_{\text{disk}} R^2}{\frac{1}{2} M_{\text{disk}} R^2 + M_{\text{loop}} R^2}$$





Example: A Rotating Disk

Disk 1 is rotating freely and has angular velocity ω_i and moment of inertia I_1 about its symmetry axis, as shown. It drops onto disk 2 of moment of inertia I_2, initially at rest. Because of kinetic friction, the two disks eventually attain a common angular velocity ω_f.



(a) What is ω_f?

(b) What is the ratio of final to initial kinetic energy?

$$L_f = L_i \qquad L = I\omega$$

$$(I_1 + I_2)\omega_f = I_1\omega_i$$

$$\omega_f = \frac{I_1}{I_1 + I_2}\omega_i = \boxed{\frac{1}{1 + (I_2/I_1)}\omega_i}$$

$$K = \tfrac{1}{2} I\omega^2 = \frac{(I\omega)^2}{2I} = \frac{L^2}{2I}$$

$$\frac{K_f}{K_i} = \left(\frac{L^2}{2I_f}\right) / \left(\frac{L^2}{2I_i}\right) = \frac{I_i}{I_f} = \boxed{\frac{I_1}{I_1 + I_2}}$$

Rotational Work & Power

A force F acting through a linear displacement Δx does work: $W = F\Delta x$.

Similarly, a torque τ acting through an angular displacement $\Delta\theta$ does work:

Work Done by Torque

$$W = \tau\Delta\theta$$

The work-energy theorem applies as usual.

Rotational Work & Power

Power is the rate at which work is done, for rotational motion as well as for translational motion.

Power Produced by a Torque

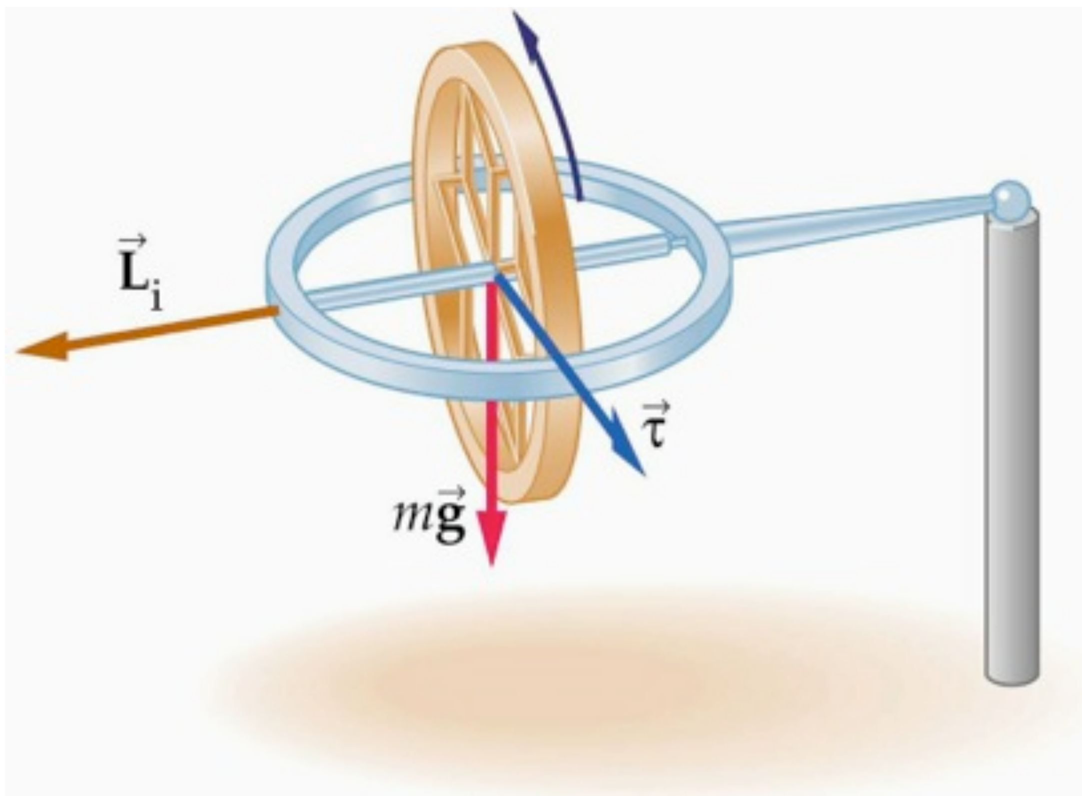
$$P = \frac{W}{\Delta t} = \tau \frac{\Delta \theta}{\Delta t} = \tau \omega$$

Again, note the analogy to the linear form:

$$P = Fv$$

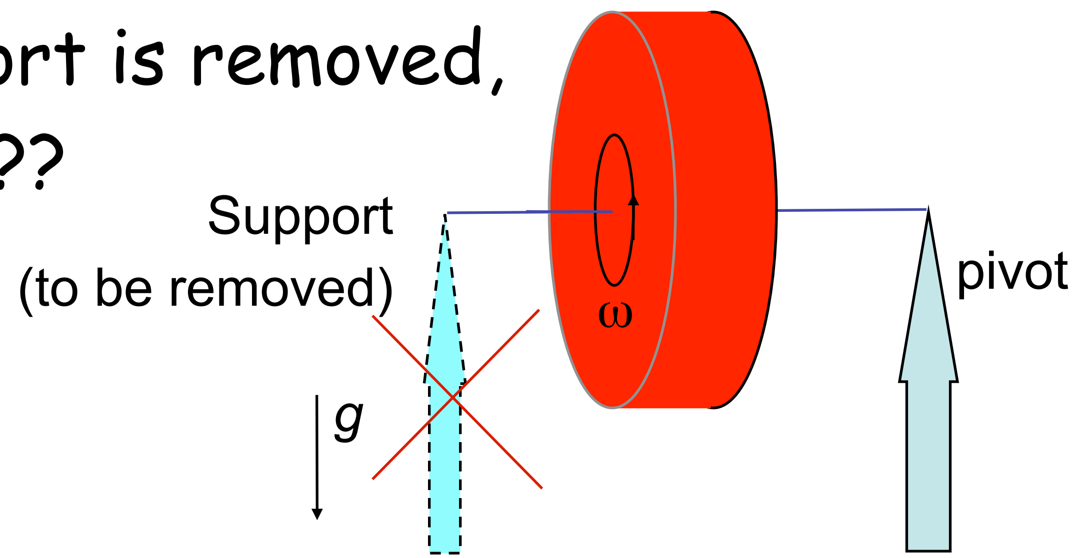
The Vector Nature of Rotational Motion

Conservation of angular momentum means that the total angular momentum around any axis must be constant. This is why gyroscopes are so stable.



Gyroscopic Motion

- Suppose you have a spinning gyroscope in the configuration shown below:
- If the left support is removed, what will happen??

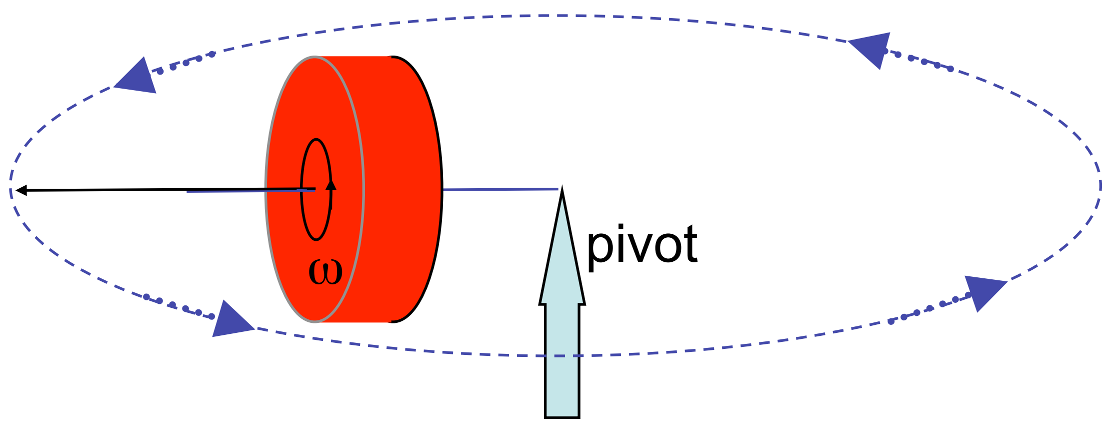


- The gyroscope does not fall down!

Gyroscopic Motion

- The gyroscope precesses around its pivot axis !
- This rather odd phenomenon can be understood by using the vector character of :

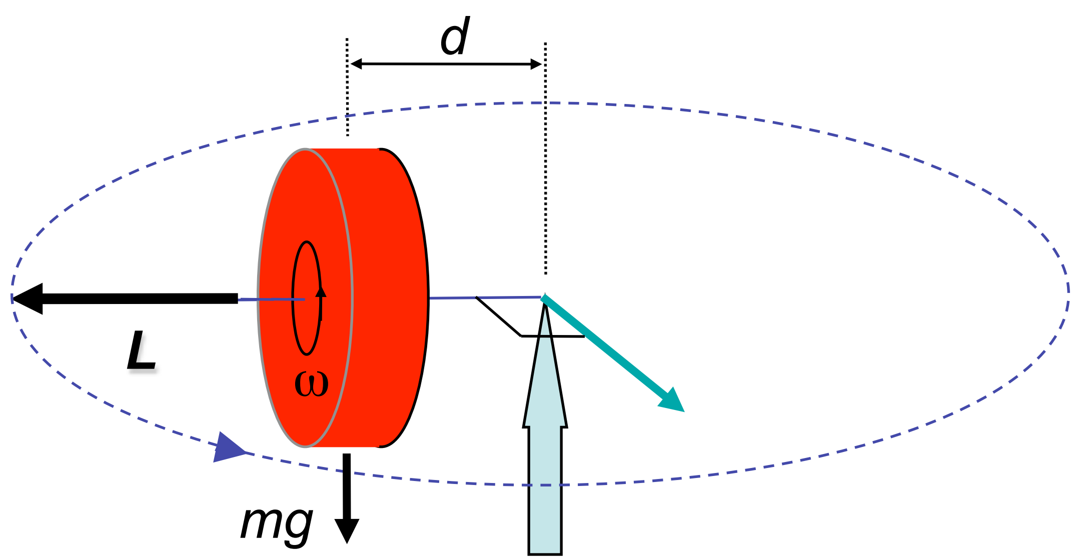
$$\tau = \frac{\Delta \boldsymbol{L}}{\Delta t}$$



Gyroscopic Motion

- The magnitude of the torque about the pivot is $\tau = mgd$.
- The direction of this torque at the instant shown is out of the page (using the right hand rule).
  - The change in angular momentum at the instant shown must also be out of the page!

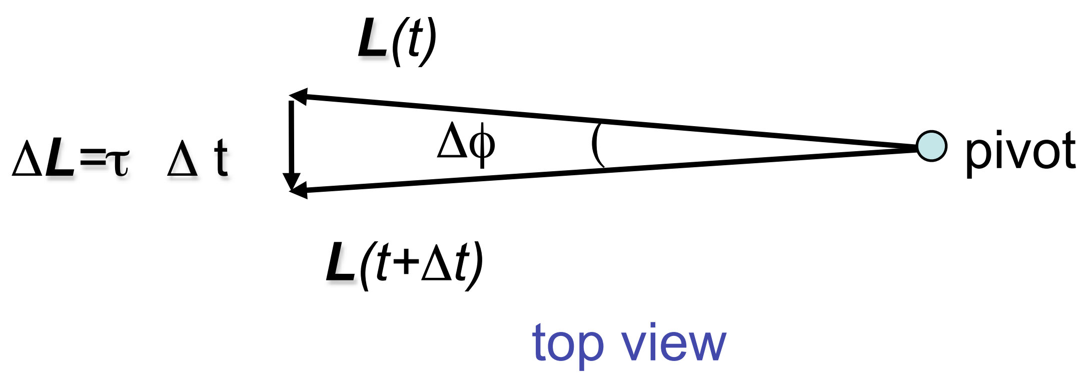
$$\tau = \frac{\Delta L}{\Delta t}$$



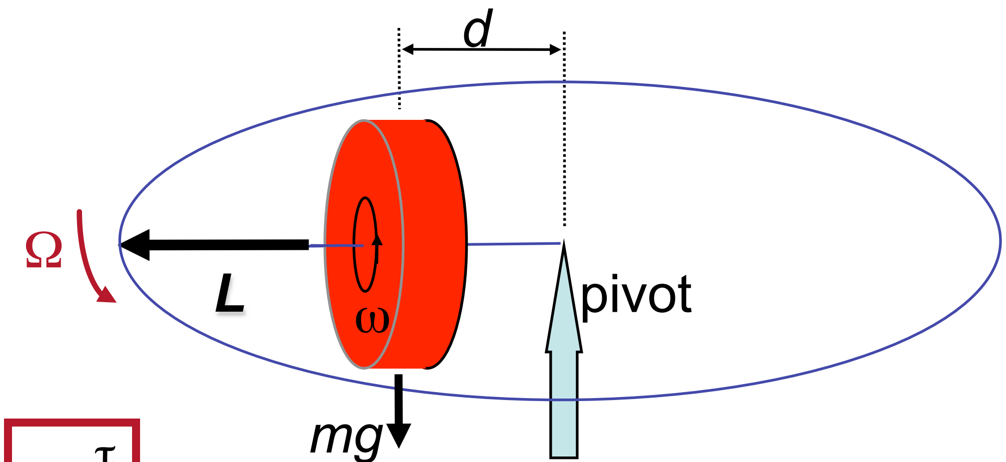
Gyroscopic Motion

- Consider a view looking down on the gyroscope.
  - The magnitude of the change in angular momentum in a time Δt is $\Delta L = L\Delta\phi$.
  - So $$\frac{\Delta L}{\Delta t} = L\frac{\Delta\phi}{\Delta t} \equiv L\Omega$$

    where $\Omega=\Delta\phi/\Delta t$ is the "precession frequency"



Gyroscopic Motion



- So $\tau = \frac{\Delta L}{\Delta t} = L\Omega$ ⇒ $\Omega = \frac{\tau}{L}$
- In this example, $\tau = mgd$ and $L = I\omega$: ⇒ $\Omega = \frac{mgd}{I\omega}$
- The direction of precession is given by applying the right hand rule to find the direction of τ and hence of $\Delta \mathbf{L}/\Delta t$.