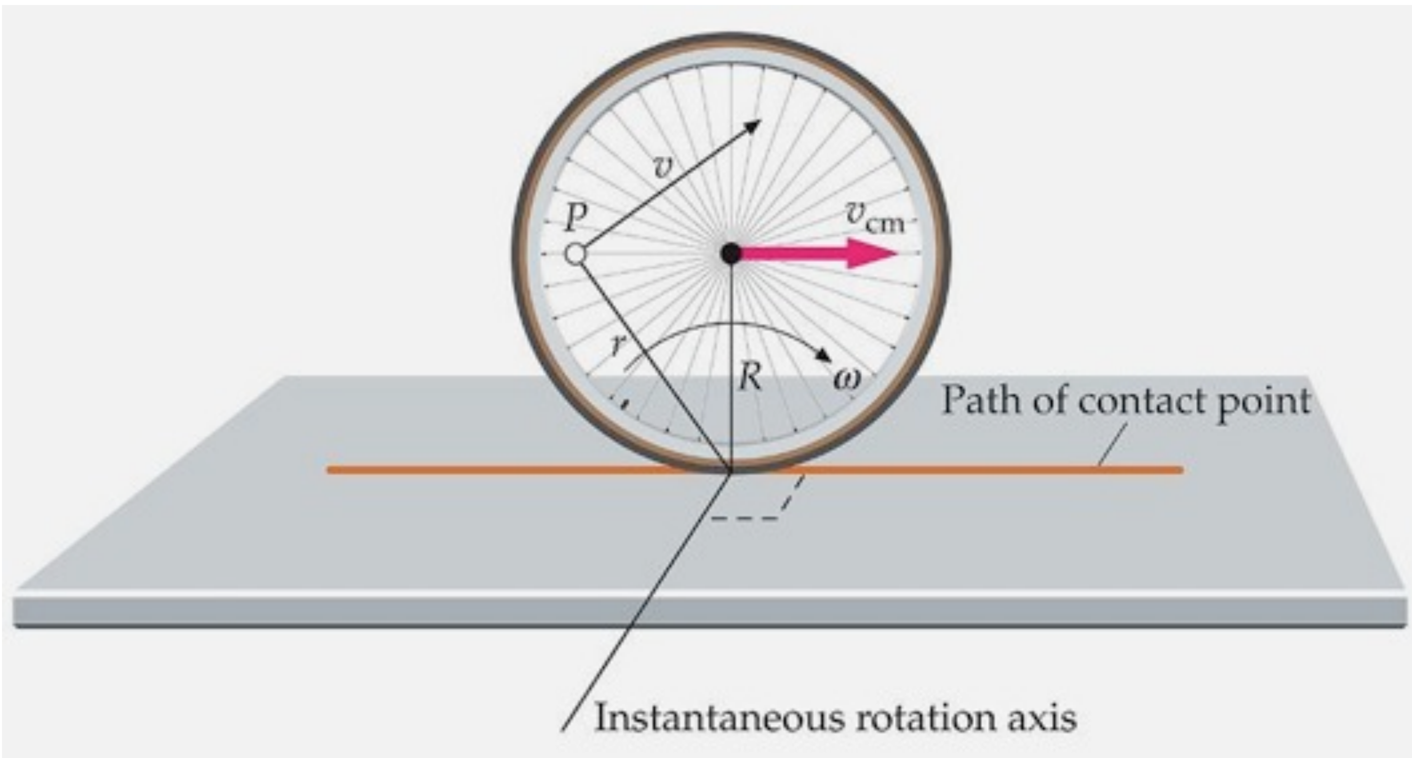
Physics 170 - *Mechanics*

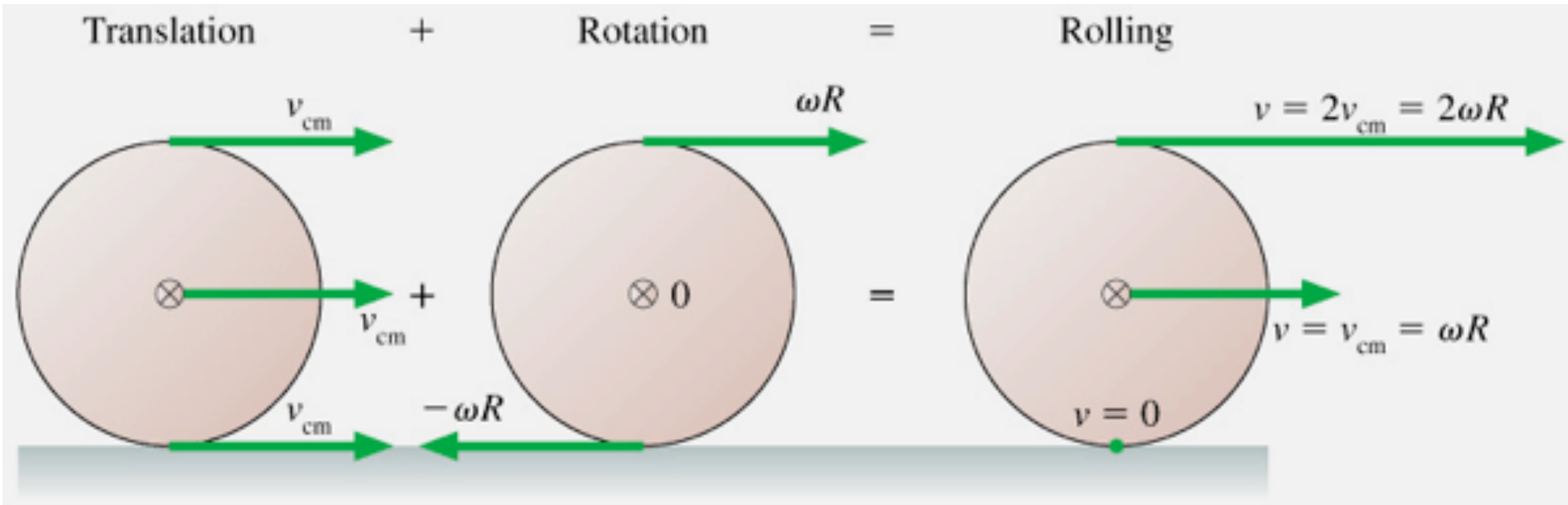
Lecture 24

Rotational Energy Conservation

Rotation Plus Translation

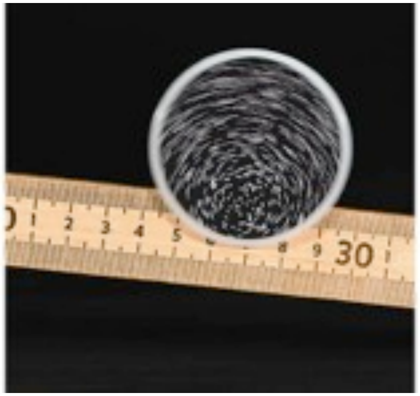


Rotation Plus Translation

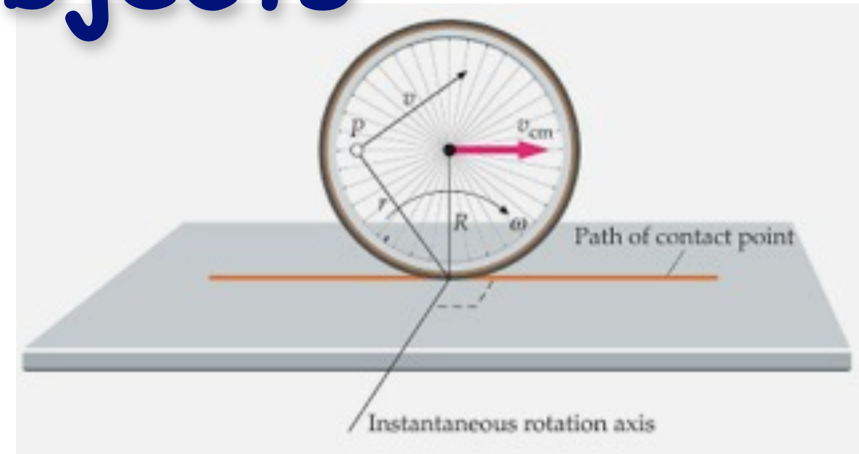


$$\boxed{\vec{v}_i = \vec{v}_{\text{cm}} + \vec{v}_{i,\text{rel}}}$$

$v_{\text{bottom}} = 0$ $\quad v_{\text{axel}} = v_{\text{cm}} = \omega R$ $\quad v_{\text{top}} = 2v_{\text{cm}} = 2\omega R$



Rolling Objects



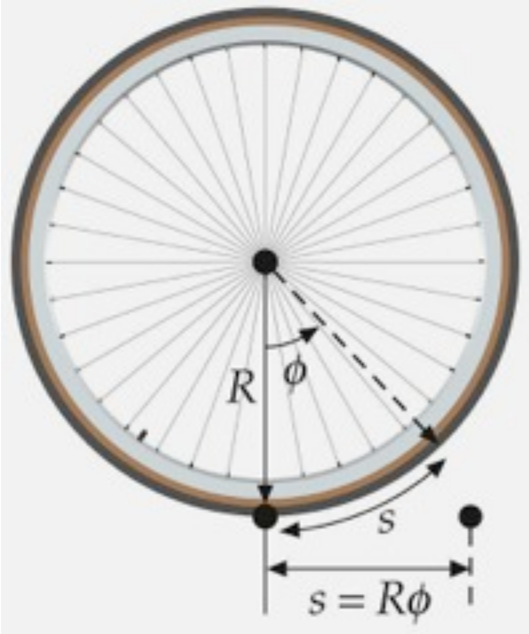
$$v = r\omega$$

$$v_{\mathrm{cm}} = R\omega$$

$$a_{\mathrm{cm}} = R\alpha$$

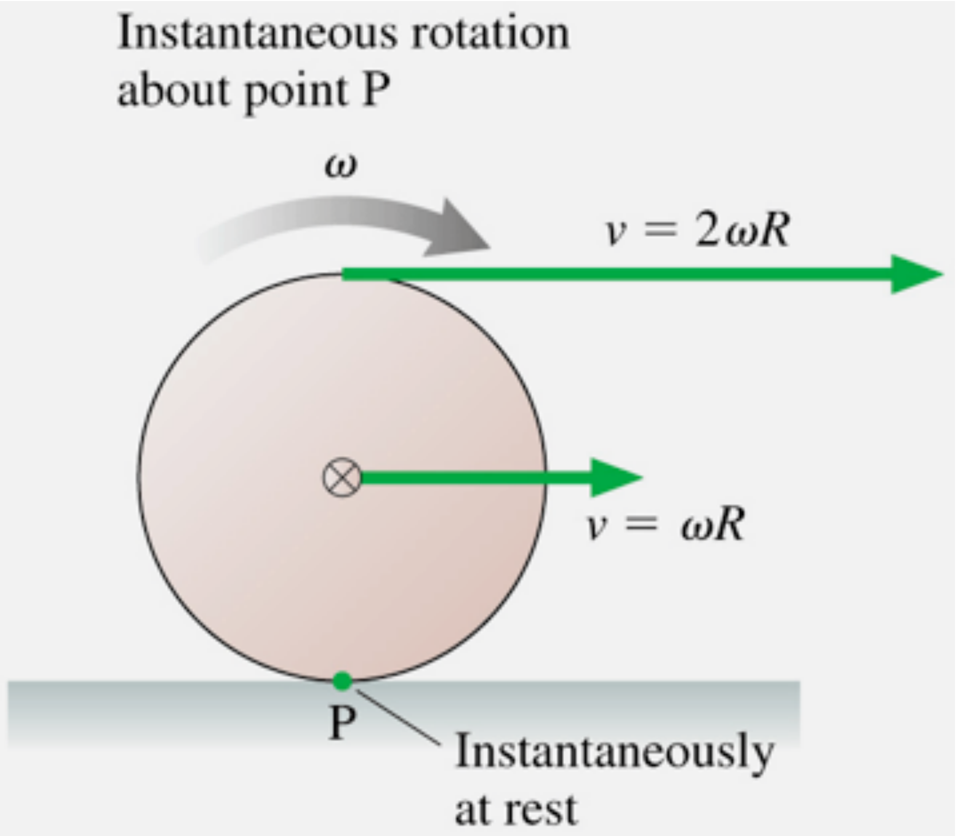
$$s = R\phi$$

$$K = \frac{1}{2}mv_{\mathrm{cm}}^2 + \frac{1}{2}I_{\mathrm{cm}}\omega^2$$



Kinetic Energy of Rolling

$$E_{\text{mech}} = K_{\text{cm}} + K_{\text{rot}} + U_g$$
$$= \tfrac{1}{2} M v_{\text{cm}}{}^2 + \tfrac{1}{2} I \omega^2 + M g y_{\text{cm}}$$



Trick: Instead of treating the rotation and translation separately, combine them by considering that instantaneously the system is rotating **abut the point of contact**.

$$E_{\text{mech}} = K_{\text{rot,P}} = \tfrac{1}{2} I_{\text{P}} \omega^2$$

$$I_{\text{P}} = \tfrac{1}{2} M R^2 + M R^2 = \tfrac{3}{2} M R^2$$

$$K_{\text{rot,P}} = \tfrac{1}{2} (\tfrac{3}{2} M R^2) \omega^2 = \tfrac{3}{4} M R^2 \omega^2 = \tfrac{3}{4} M v^2$$

Conservation of Energy

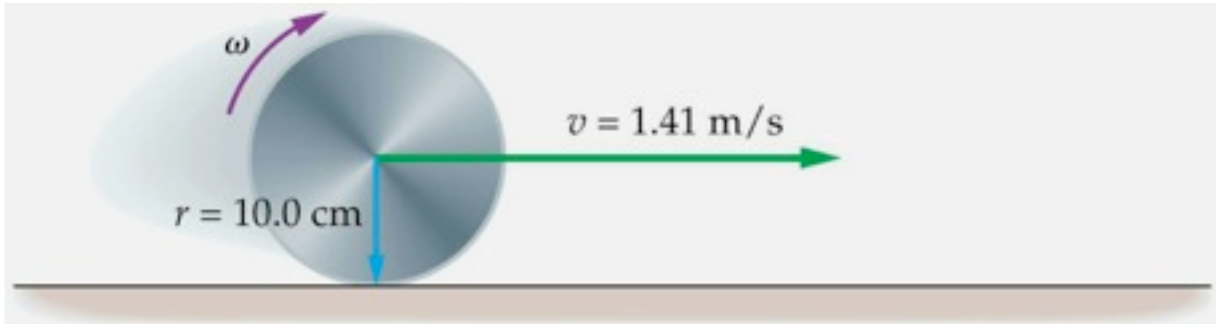
The total kinetic energy of a rolling object is the sum of its linear and rotational kinetic energies:

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{r}\right)^2 = \frac{1}{2}mv^2\left(1 + \frac{I}{mr^2}\right)$$

The second equation makes it clear that the kinetic energy of a rolling object is a **multiple** of the kinetic energy of translation.

Example: Like a Rolling Disk



A 1.20 kg disk with a radius 0f 10.0 cm rolls without slipping. The linear speed of the disk is v = 1.41 m/s.

(a) Find the translational kinetic energy.

(b) Find the rotational kinetic energy.

(c) Find the total kinetic energy.

$$K_t = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(1.20\text{ kg})(1.41\text{ m/s})^2 = 1.19\text{ J}$$

$$K_r = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(\tfrac{1}{2}mr^2)(v/r)^2 = \tfrac{1}{4}(1.20\text{ kg})(1.41\text{ m/s})^2 = 0.595\text{ J}$$

$$K_\Sigma = K_t + K_r = (1.19\text{ J}) + (0.595\text{ J}) = 1.79\text{ J}$$

Question 1

A solid sphere and a hollow sphere of the same mass and radius roll forward without slipping at the **same** speed.

How do their kinetic energies compare?

(a) $K_{solid} > K_{hollow}$

(b) $K_{solid} = K_{hollow}$

(c) $K_{solid} < K_{hollow}$

(d) Not enough information to tell

Rolling Down an Incline

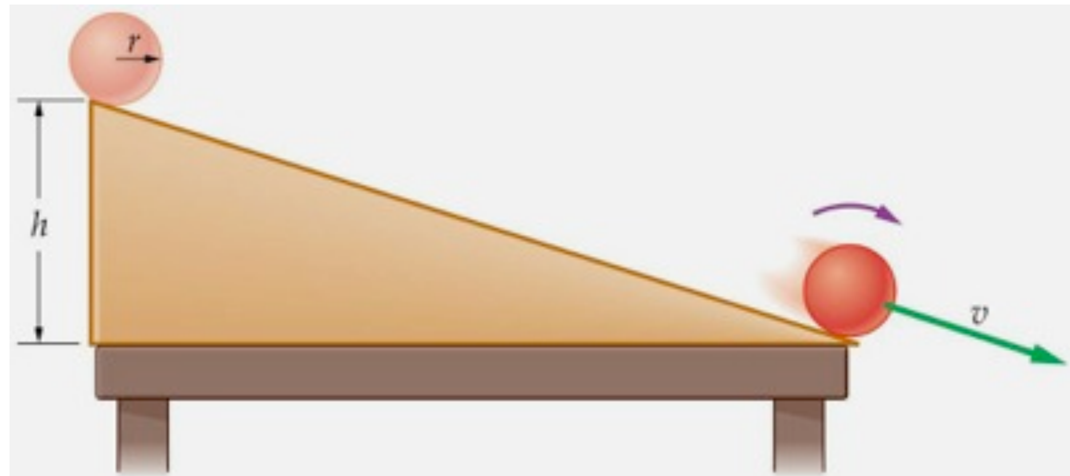
$$K_i + U_i = K_f + U_f$$

$$K = \tfrac{1}{2} m v^2 (1 + I / m r^2)$$

$$U = mgh$$

$$mgh = \tfrac{1}{2} m v^2 (1 + I / m r^2)$$

$$\boxed{v = \sqrt{2gh / (1 + I / m r^2)}}$$



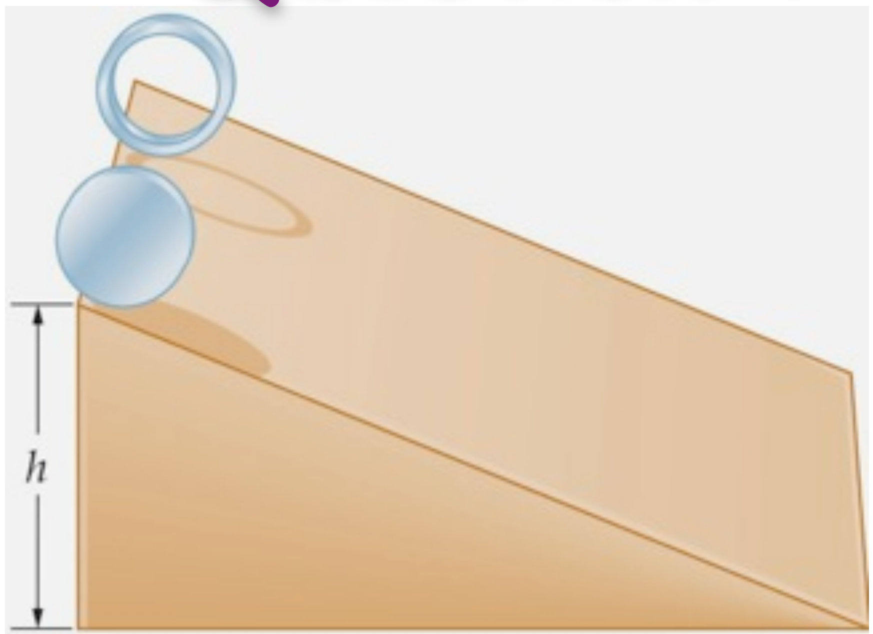
Hollow Cylinder: $I = mr^2; \quad v = \sqrt{gh}$

Solid Cylinder: $I = \tfrac{1}{2} mr^2; \quad v = \sqrt{\tfrac{4}{3} gh}$

Hollow Sphere: $I = \tfrac{2}{3} mr^2; \quad v = \sqrt{\tfrac{6}{5} gh}$

Solid Sphere: $I = \tfrac{2}{5} mr^2; \quad v = \sqrt{\tfrac{10}{7} gh}$

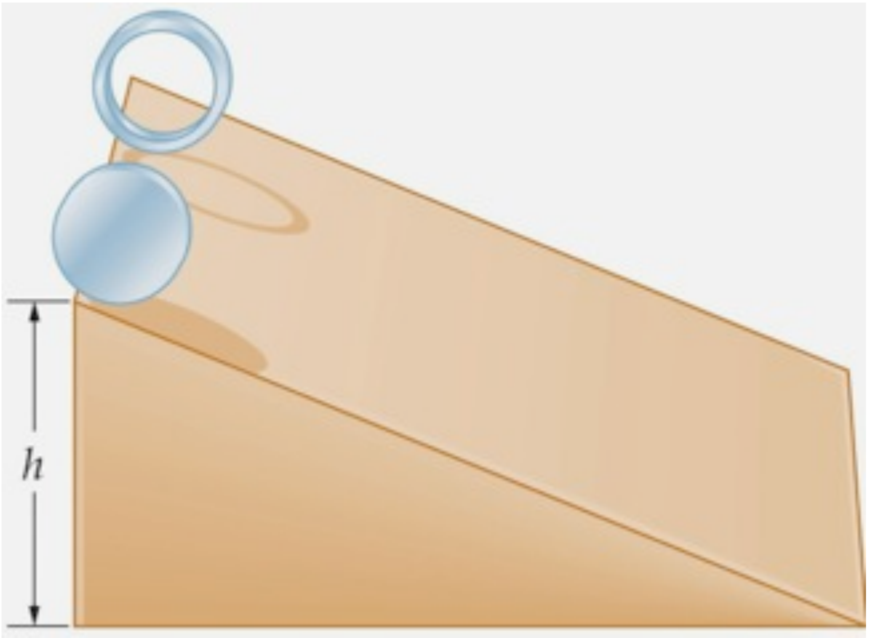
Question 2



Which of these two objects, of the same mass and radius, if released simultaneously, will reach the bottom first? Or is it a tie?

(a) Hoop; (b) Disk; (c) Tie; (d) Need to know mass and radius.

Which Object Wins the Race?



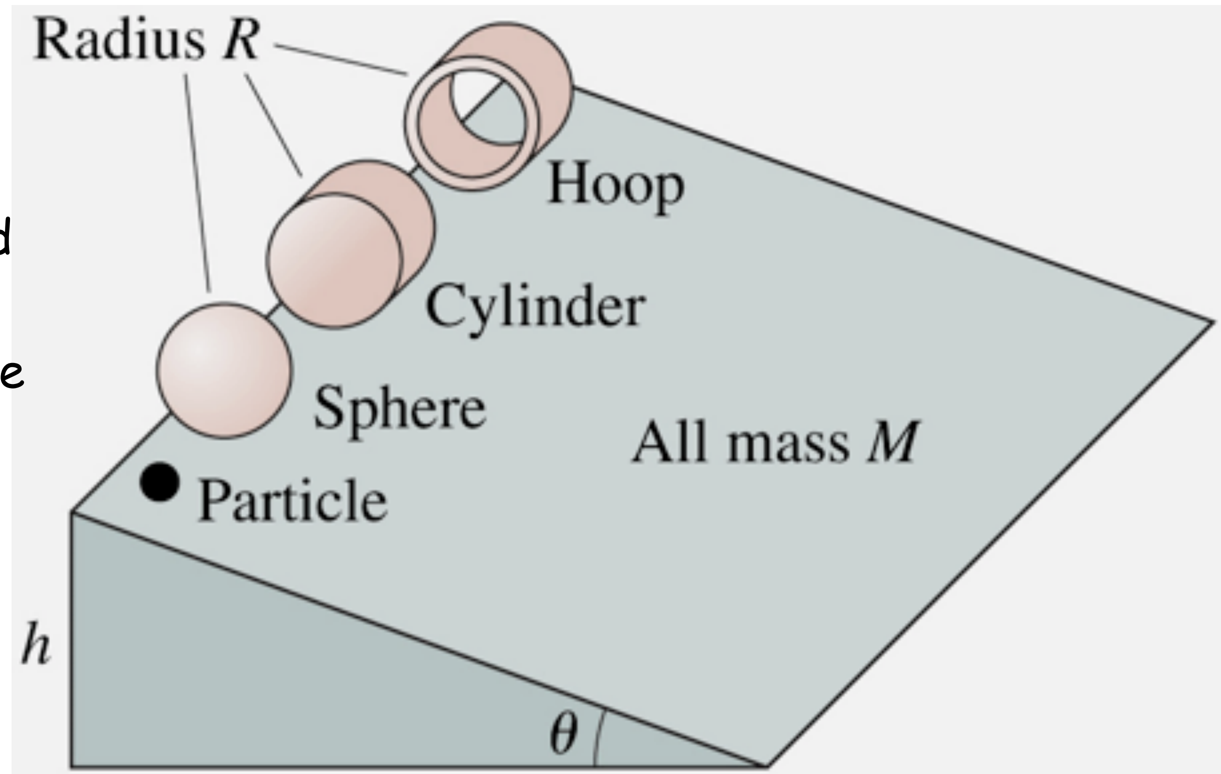
If these two objects, of the same mass and radius, are released simultaneously, the disk will reach the bottom first.

Reason: more of its gravitational potential energy becomes translational kinetic energy, and less becomes rotational.

The Great Downhill Race

A sphere, a cylinder, and a hoop, all of mass **M** and radius **R**, are released from rest and roll down a ramp of height **h** and slope θ. They are joined by a particle of mass **M** that slides down the ramp without friction.

Who wins the race? Who is the big loser?



The Winners

$$\tfrac{1}{2}I_{\mathrm{cm}}\omega^2 + \tfrac{1}{2}Mv_{\mathrm{cm}}{}^2 = Mgh$$

$$\omega = v_{\mathrm{cm}}/R \qquad\qquad I_{\mathrm{cm}} = cMR^2$$

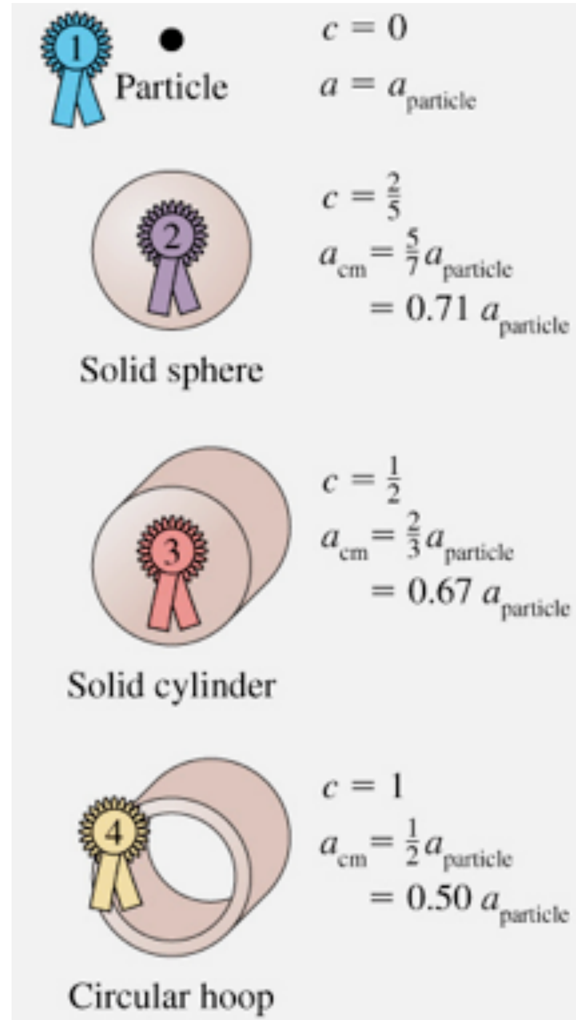
(c is a constant that depends on the object.)

$$Mgh = \tfrac{1}{2}(cMR^2)(v_{\mathrm{cm}}/R)^2 + \tfrac{1}{2}Mv_{\mathrm{cm}}{}^2$$
$$= \tfrac{1}{2}M(1+c)v_{\mathrm{cm}}{}^2$$

$$v_{\mathrm{cm}} = \sqrt{\frac{2gh}{1+c}}$$

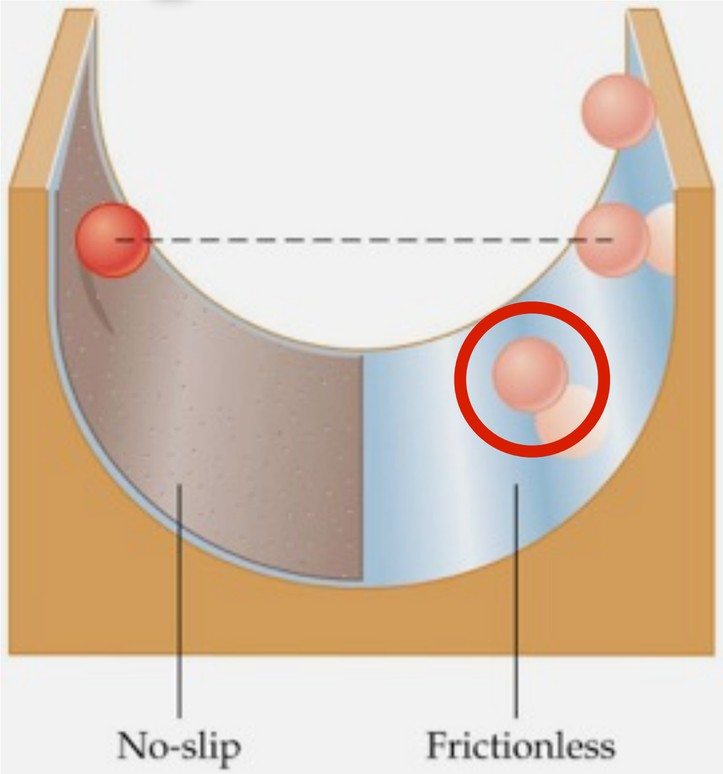
Therefore, $v_{\mathrm{particle}} > v_{\mathrm{sphere}} > v_{\mathrm{cylinder}} > v_{\mathrm{hoop}}$

and $a_{\mathrm{particle}} > a_{\mathrm{sphere}} > a_{\mathrm{cylinder}} > a_{\mathrm{hoop}}$



Compare Heights

A ball is released from rest on a no-slip (high-friction) surface, as shown. After reaching the lowest point, it begins to rise again on a frictionless surface.

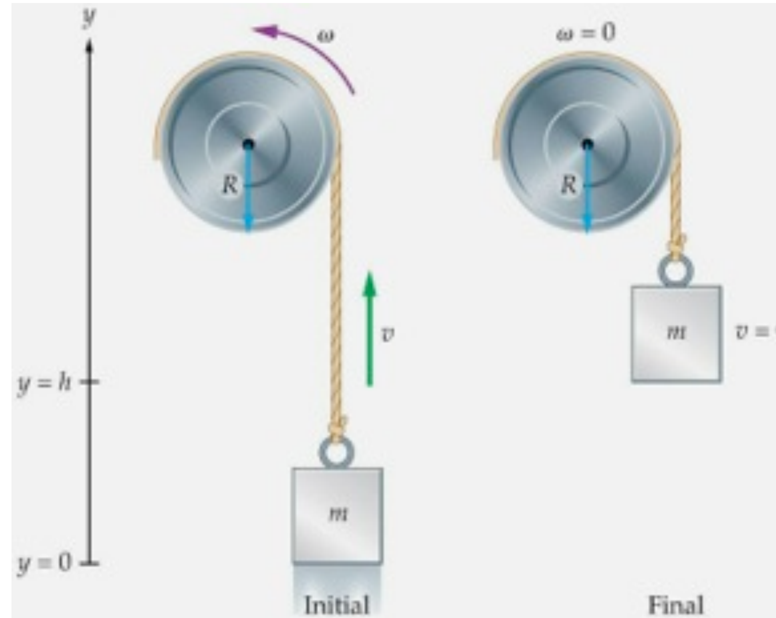


When the ball reaches its maximum height on the frictionless surface, it is higher, lower, or the same height as its release point?

The ball is not spinning when released, but will be spinning when it reaches maximum height on the other side, so less of its energy will be in the form of gravitational potential energy. Therefore, it will reach a **lower** height.

Example: Spinning Wheel

A block of mass m is attached to a string that is wrapped around the circumference of a wheel of radius R and moment of inertia I, initially rotating with angular velocity ω that causes the block to rise with speed v. The wheel rotates freely about its axis and the string does not slip.



To what height h does the block rise?

$$E_i = E_f \qquad E_f = mgh$$

$$E_i = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I(v/R)^2 = \tfrac{1}{2}mv^2(1 + I/mR^2)$$

$$\boxed{h = \left(\frac{v^2}{2g}\right)\left(1 + \frac{I}{mR^2}\right)}$$

Example: A Flywheel-Powered Car

You are driving an experimental hybrid vehicle that is designed for use in stop-and-go traffic, in which the braking mechanism transforms the translational kinetic energy into the rotational kinetic energy of a massive flywheel. The 100 kg flywheel is a hollow cylinder with an inner radius R_1 = 25.0 cm, an outer radius R_2 = 40 cm, and a maximum angular speed of 30,000 rpm. When driving at the minimum highway speed of 40 mi/h, air drag and rolling friction dissipate energy at 10.0 kW. On a dark and dreary night, the car runs out of gas 15 miles from home, with the flywheel spinning at maximum speed.

Is there enough energy in the flywheel for you and your nervous grandmother to make it home?

$$\omega = (30{,}000 \text{ rev/min})(2\pi \text{ rad/rev})/(60 \text{ s/min}) = 3{,}142 \text{ rad/s}$$

$$I = \tfrac{1}{2} M \left(R_1^2 + R_2^2\right) = \tfrac{1}{2}(100 \text{ kg})\left[(0.25 \text{ m})^2 + (0.40 \text{ m})^2\right] = 11.1 \text{ kg m}^2$$

$$K = \tfrac{1}{2} I\omega^2 = \tfrac{1}{2}(11.1 \text{ kg m}^2)(3{,}142 \text{ rad/s})^2 = 54.9 \text{ MJ}$$

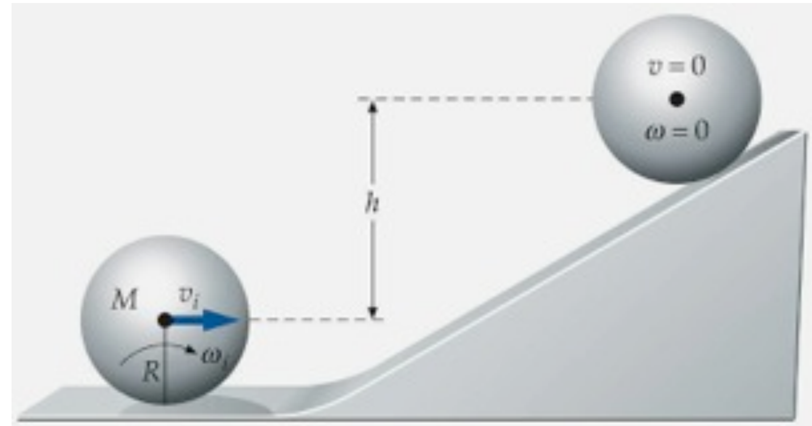
$$U = P\Delta t = (10.0 \text{ kW})(1350 \text{ s}) = 13.5 \text{ MJ}$$

$$\Delta t = \Delta x / v = (15 \text{ mi})/(40 \text{ mi/h}) = 0.375 \text{ h} = 1350 \text{ s}$$

$\boxed{U < K, \text{ so you make it home.}}$

Example: A Bowling Ball

A bowling ball that has an 11 cm radius and a 7.2 kg mass is rolling without slipping at 2.0 m/s on a horizontal ball return. It continues to roll without slipping up a hill to a height ***h*** before momentarily coming to rest and then rolling back down the hill.



Model the bowling ball as a uniform sphere and calculate ***h***.

$$W_{\text{ext}} = \Delta E_{\text{mech}} + \Delta E_{\text{therm}} \quad \Rightarrow \quad 0 = \Delta E_{\text{mech}} + 0$$

$$U_f + K_f = U_i + K_i \quad \Rightarrow \quad Mgh + 0 = 0 + \tfrac{1}{2} M v_{\text{cm}\,i}^2 + \tfrac{1}{2} I_{\text{cm}} \omega_i^2$$

$$Mgh = \tfrac{1}{2} M v_{\text{cm}\,i}^2 + \tfrac{1}{2}\left(\tfrac{2}{5} MR^2\right)\frac{v_{\text{cm}\,i}^2}{R^2} = \tfrac{7}{10} M v_{\text{cm}\,i}^2$$

$$h = \frac{7 v_{\text{cm}\,i}^2}{10g} = \frac{7(2.0\text{ m/s})^2}{10(9.8\text{ m/s}^2)} = \boxed{0.29\text{ m}}$$