Physics 170 - *Mechanics*

Lecture 19

Momentum & Impulse

Momentum – why bother?

You have already learned one quantity based on motion:

$$KE = \frac{1}{2}mv^2$$

Kinetic energy, combined with gravitational potential energy, can help you to solve many problems, and predict the motions of bodies in many circumstances.

So – why bother with another quantity based on motion?

COLLISIONS!

It's very important to be able to figure out what's going to happen when objects collide ... and simple energy methods usually don't work.

Why does Conservation of Mechanical Energy fail? Two main reasons:

- objects crumple, bend, break & deform when subjected to strong forces. These deformations absorb energy, removing it from the

  KE + GPE

  terms...

- objects stick together sometimes after they collide – that changes the mass of each, so

  $\frac{1}{2}mv^2$

  isn't simple any more.

It would be nice to find some quantity which is conserved, even during a collision...

Conservation of Linear Momentum

When no force acts on an object,

$$\vec{a} = 0$$

$$\rightarrow \frac{d\vec{v}}{dt} = 0$$

If an object's mass m does not change, then we can add it to the equation:

$$m\frac{d\vec{v}}{dt} = 0$$

$$\frac{d}{dt}(m\vec{v}) = 0$$

This ↑ quantity is <u>conserved</u> — doesn't change with time — if there are no forces acting on the object. Let's give it a name: momentum.

$$\vec{p} \equiv m\vec{v}$$

Hmm. Linear momentum does not appear very useful so far — when considering a single object.

<u>But</u> — if several objects interact, then linear momentum <u>is</u> useful.

In step 2, when balls "whack", they exert forces on each other. These forces may be very complicated. And the momentum of each ball, alone, may change:

$$m_1\vec{v}_1 \neq m_1\vec{v}_1$$

before   after

Conservation of Linear Momentum

But – suppose that the two balls are <u>isolated</u> : no net external force acts upon them.

$$\sum \vec{F}_{external} = 0$$

In that case, the total linear momentum of the "system",

$$\vec{P}_{tot} = m_1\vec{v}_1 + m_2\vec{v}_2$$

does remain constant :

$$\frac{d\vec{P}_{tot}}{dt} = \sum \vec{F}_{ext} = 0$$

so

$$\underbrace{m_1\vec{v}_1 + m_2\vec{v}_2}_{\text{before}} = \underbrace{m_1\vec{v}_1 + m_2\vec{v}_2}_{\text{after}}$$

Linear Momentum

There is a quantity which remain the same even when objects crumple or fold, it's the linear momentum.

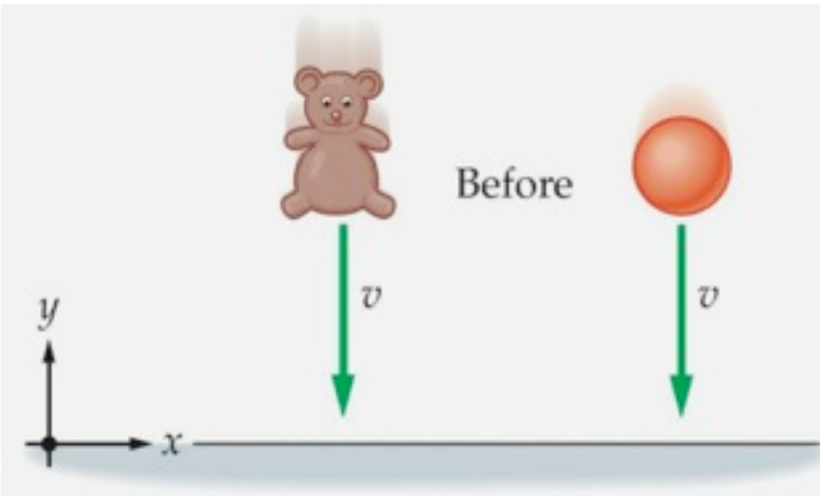
Definition of Linear Momentum, $\vec{p}$

$$\vec{p} = m\vec{v}$$

SI unit: $kg \cdot m/s$

Momentum is a vector; its direction is the same as the direction of the velocity.

Change in Momentum



Change in momentum: $\Delta p = p_{after} - p_{before}$

Teddy Bear: $\Delta p = 0-(-mv) = mv$

Bouncing Ball: $\Delta p = mv-(-mv) = 2mv$

Momentum & Newton's Second Law

Newton's second law, as we wrote it before, is:

$$\sum \vec{F} = m\vec{a} = m\frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta(m\vec{v})}{\Delta t} = \frac{\Delta\vec{p}}{\Delta t}$$

is only valid for objects that have **constant mass**.

Here is a more general form in terms of momentum, which is also useful when the mass is changing:

Newton's Second Law

$$\sum \vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

Problem Solving Strategy Momentum

Picture: Determine that the net external force ΣF_{ext} (or $\Sigma F_{\text{ext x}}$) on the system is negligible for some time interval.

Solve:

1. Draw a sketch showing the system before and after the time interval. Include coordinate axes and label the initial and final velocity vectors.
2. Equate the initial momentum to the final momentum and express this as a vector equation (or one or more scalar equations involving x, y, and z components.)
3. Substitute the given information into the equation(s) and solve for the quantity or quantities of interest.

Check: Make sure you include any minus signs that accompany velocity components, because momentum can have either sign.

Impulse

To change the momentum of a body [usually changing its speed or direction] a force must apply.

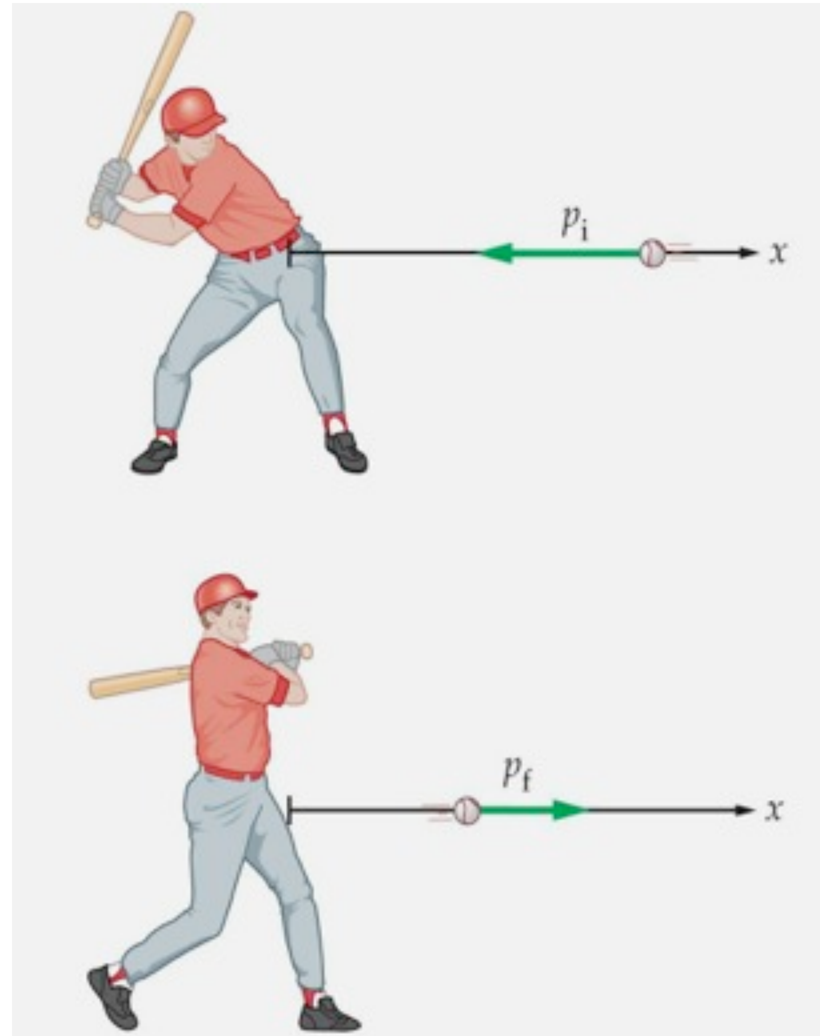
The same change in momentum may be produced by:

- large force for a short time,

- small force for a long time.

The amount by which an object's momentum changes is proportional to the product:

(force) X (time) = Impulse



Impulse

Definition of Impulse, $\vec{I}$

$$\vec{I} = \vec{F}_{av}\Delta t$$

SI unit: $N \cdot s = kg \cdot m/s$

Impulse is a vector, in the same direction as the average force.

A large impulse causes a big change in a object's momentum

Impulse

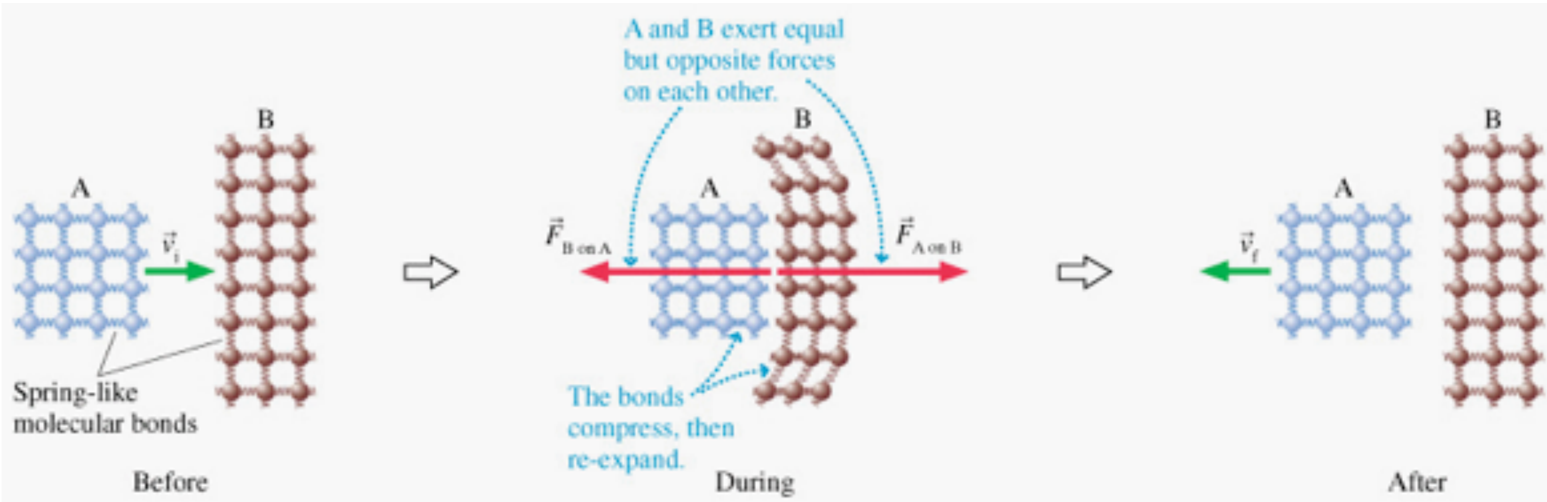
We can rewrite $\vec{F}_{av} = \frac{\Delta \vec{p}}{\Delta t}$

as $\vec{F}_{av} \Delta t = \Delta \vec{p}$

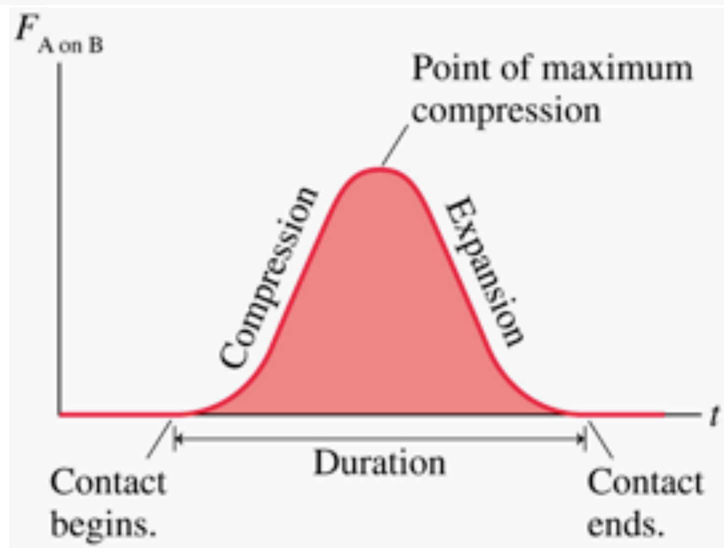
So we see that $\vec{I} = \vec{F}_{av} \Delta t = \Delta \vec{p}$

The impulse **I** is equal to the change in momentum $\Delta \mathbf{p}$.

Momentum and Impulse



Microscopic view of a “bounce”.



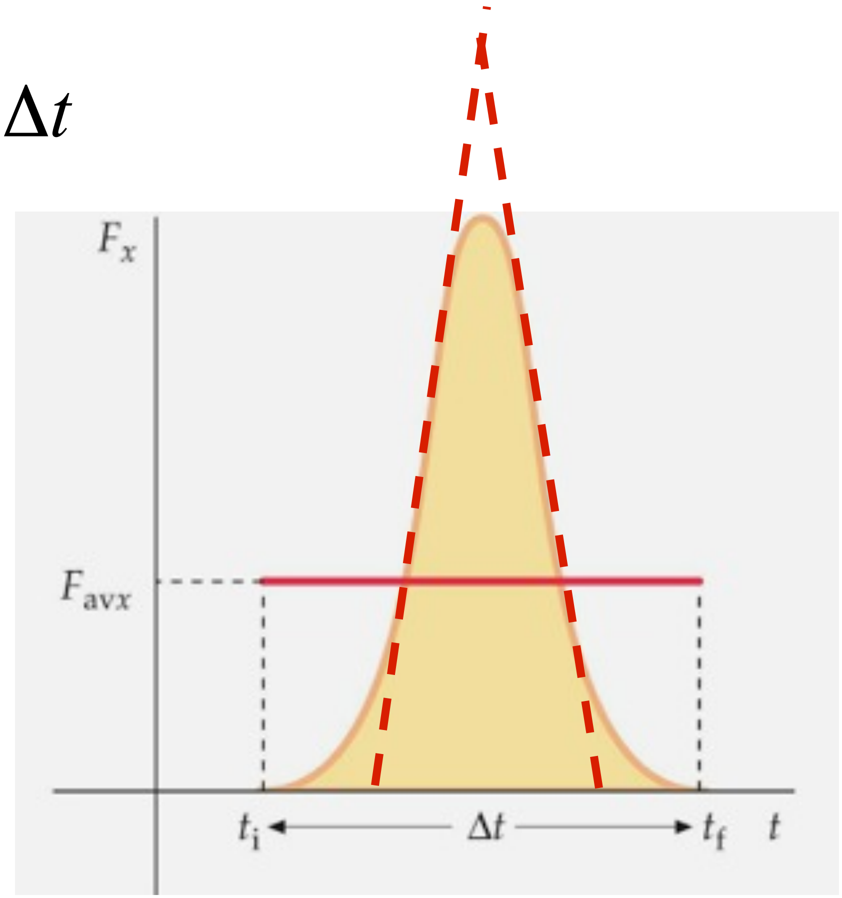
Profile of the force during a collision.

Impulse and Average Force

Definition of Impulse: $\vec{I} \equiv \vec{F}_{av} \Delta t$

$$\vec{I}_{\text{net}} = \Delta\vec{p}$$

$$\vec{I}_{\text{ext}} = \vec{F}_{\text{av ext}} \Delta t = \Delta\vec{P}_{\text{sys}}$$



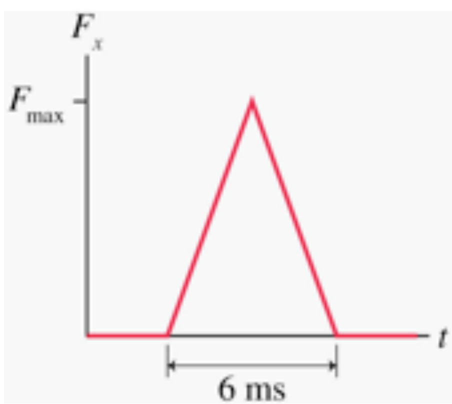
$\vec{I} = \vec{F}_{av} \Delta t$ = area under F vs. t curve

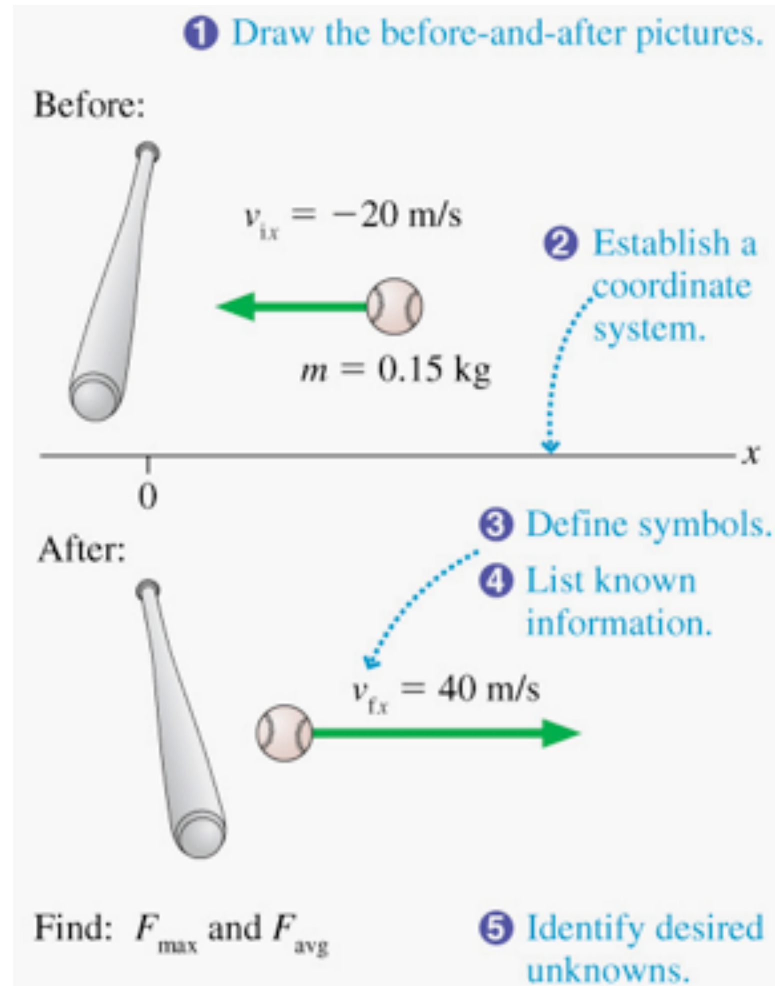
Example: Hitting a Baseball (1)

A 150 g baseball is thrown at a speed of 20 m/s. It is hit straight back to the pitcher at a speed of 40 m/s. The interaction force is as shown here.

What is the maximum force F_{max} that the bat exerts on the ball?

What is the average force F_{av} that the bat exerts on the ball?

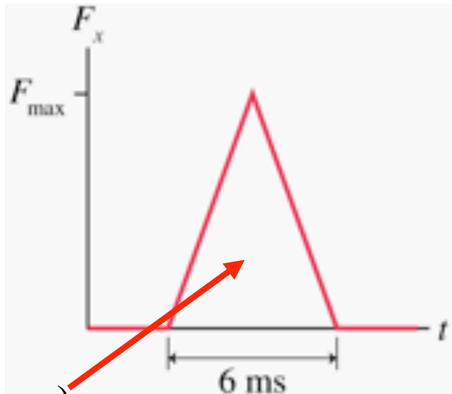




Example: Hitting a Baseball (2)

Use the impulse approximation:

Neglect all other forces on ball during the brief duration of the collision.

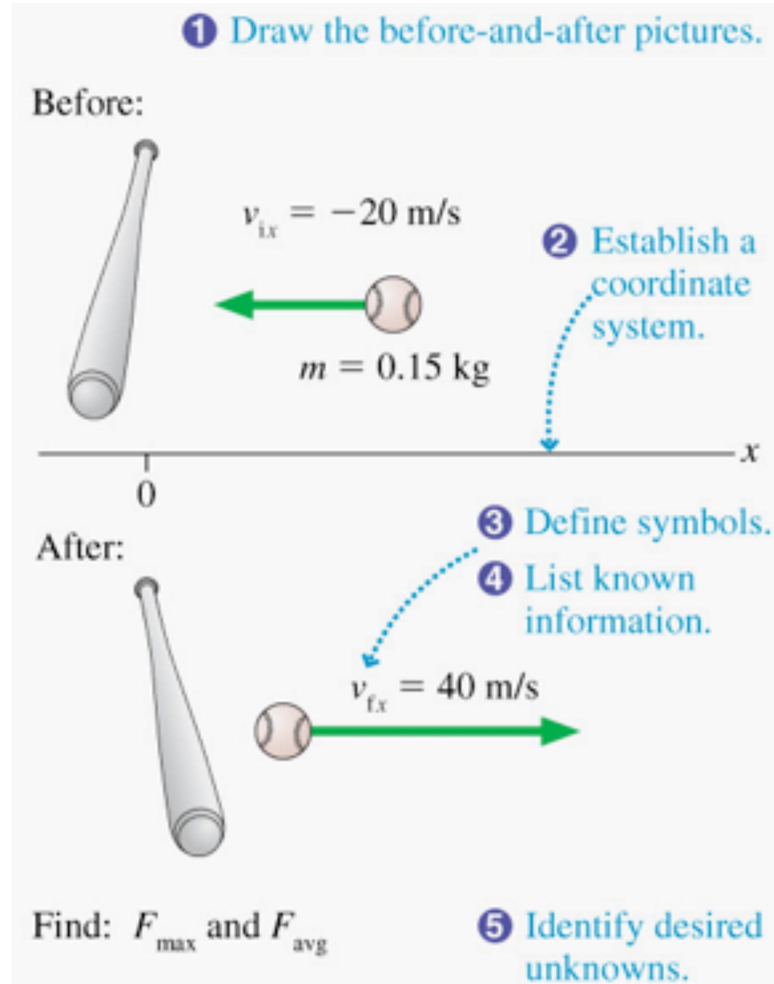


$$\Delta p_x = I_x$$
$$= \text{(area under force curve)}$$
$$= \tfrac{1}{2} F_{max}(6 \text{ ms}) = F_{max}(.003 \text{ s})$$

$$\Delta p_x = mv_{fx} - mv_{ix} = m(v_{fx} - v_{ix})$$
$$= (0.15 \text{ kg})(40 \text{ m/s} + 20 \text{ m/s})$$
$$= 9.0 \text{ kg m/s}$$

Therefore, $F_{max} = (9.0 \text{ kg m/s})/(.003 \text{ s}) = 3{,}000 \text{ N}$

$$F_{av} = \frac{\Delta p_x}{\Delta t} = \frac{(9.0 \text{ kg m/s})}{(.006 \text{ s})} = 1{,}500 \text{ N}$$



Problem Solving Strategy
Impulse

Picture: To estimate the average force F_{av}, we first estimate the impulse I of the force. Assuming other forces are negligible, the impulse of the force is the net impulse, which is equal to the change in momentum, i.e., the mass times the change in velocity. An estimate of the velocity change Δv can be made from estimates of the collision time Δt and displacement Δr.

Solve:

1. Calculate (or estimate) the impulse I and the time Δt.

This estimate assumes that during the collision, the collision force is very large compared to all other forces on the object. This procedure works **only** if the displacement during collision can be determined.

2. Draw a sketch showing before and after positions of the object. Add coordinate axes and label velocities and displacement.
3. Calculate the momentum change during the collision. $(I=\Delta p=m\Delta v)$
4. Use $F_{av}=I/\Delta t$ to calculate the average force.

Check: Average force is a vector, and should be in the same direction as Δv.

Conservation of Linear Momentum

Momentum: $\vec{p} \equiv m\vec{v}$

$$\frac{\Delta\vec{p}}{\Delta t} = \frac{\Delta(m\vec{v})}{\Delta t} = m\frac{\Delta\vec{v}}{\Delta t} = m\vec{a}$$

$$\vec{F}_{\text{net}} = \frac{\Delta\vec{p}}{\Delta t}$$

$$\vec{P}_{\text{sys}} = \sum_i m_i\vec{v}_i = \sum_i \vec{p}_i = M\vec{v}_{\text{cm}}$$

$$\sum_i \vec{F}_{\text{ext}} = \vec{F}_{\text{net ext}} = \frac{\Delta\vec{P}_{\text{sys}}}{\Delta t} = M\frac{\Delta\vec{v}_{\text{cm}}}{\Delta t} = M\vec{a}_{\text{cm}}$$

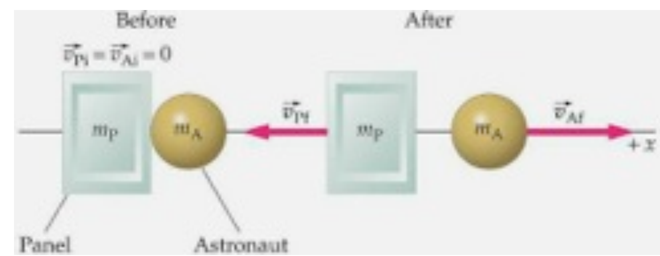
$$\text{If } \sum \vec{F}_{\text{ext}} = 0, \text{ then } \vec{P}_{\text{sys}} = \sum m_i\vec{v}_i = M\vec{v}_{\text{cm}} = \text{constant}$$

Conservation of Momentum

Example: A Space Repair

During repair of the Hubble Space Telescope, an astronaut replaces a damaged solar panel during a spacewalk. Pushing the detached panel away into space, he is propelled in the opposite direction. The astronaut's mass is 60 kg, and the panel's mass is 80 kg. Both astronaut and panel are initially at rest relative to the telescope, until the astronaut gives the panel a shove, giving it a velocity of 0.30 m/s relative to the telescope.

Assuming his tether is slack, what is his velocity relative to the telescope?



$$\sum \vec{F}_{\text{ext}} = 0 = \frac{\Delta \vec{P}_{\text{sys}}}{\Delta t}, \text{ so } \vec{P}_{\text{sys}} = \text{constant}$$

$$m_P \vec{v}_{Pf} + m_A \vec{v}_{Af} = m_P \vec{v}_{Pi} + m_A \vec{v}_{Ai} = 0 \qquad m_P \vec{v}_{Pf} = -m_A \vec{v}_{Af}$$

$$\vec{v}_{Af} = -\frac{m_P}{m_A} \vec{v}_{Pf} = -\frac{(80 \text{ kg})}{(60 \text{ kg})}(-0.30 \text{ m/s})\hat{x} = (0.40 \text{ m/s})\hat{x}$$