Geoneutrino sources and fluxes:
A systematic approach
to their uncertainties and correlations

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Based on work in progress with:
Gianluigi Fogli, Antonio Palazzo, Anna Maria Rotunno
Congratulations to the KamLAND collaboration!
Outline:

- Covariances and their importance
- Geo Neutrino Source Model: general
- Geo Neutrino Source Model: some details
- Local reservoirs and related problems
- Summary and conclusions

Apologies for missing references to many relevant works in geo- and particle physics (will appear in forthcoming paper)
Covariances and their importance
Not an easy task, even for subsets of elements - like (U, Th, K)

Previous relevant work on (U, Th, K) uncertainties by the Fiorentini et al. group focused mainly on spread of published estimates + mass balance constraints

However, inclusion of correlations, independent reassessment of uncertainties, and discussion of related problems, are also desirable for several reasons

GERM Goal

Geochemically defined Earth reservoirs ultimately have to be reconciled with a physical definition and uncertainties of data need to be discussed. We recognize that these uncertainties will be large but we note that they often result from fluctuations in modal components, such as quartz in the continental crust or olivine in the upper mantle. In order to describe accurately the compositional diversity and to preserve information on covariances between elemental concentrations, a correlation matrix should be associated with each reservoir estimate.
Covariance analyses ubiquitous in neutrino physics
(solar, atmospheric, big-bang nucleosynthesis neutrinos...)

E.g., correlation coefficients and covariance plots
for solar neutrino fluxes (Bahcall, Serenelli, Basu 2005):


<table>
<thead>
<tr>
<th>Flux</th>
<th>pp</th>
<th>pep</th>
<th>hep</th>
<th>$^7$Be</th>
<th>$^9$Be</th>
<th>$^{13}$N</th>
<th>$^{14}$O</th>
<th>$^{17}$F</th>
</tr>
</thead>
<tbody>
<tr>
<td>pp</td>
<td>1.000</td>
<td>0.954</td>
<td>0.982</td>
<td>-0.419</td>
<td>-0.720</td>
<td>-0.349</td>
<td>-0.319</td>
<td>0.219</td>
</tr>
<tr>
<td>pep</td>
<td>0.954</td>
<td>1.000</td>
<td>0.997</td>
<td>-0.784</td>
<td>-0.730</td>
<td>-0.407</td>
<td>-0.439</td>
<td>0.369</td>
</tr>
<tr>
<td>hep</td>
<td>0.982</td>
<td>0.979</td>
<td>1.000</td>
<td>-0.992</td>
<td>-0.986</td>
<td>-0.952</td>
<td>-0.968</td>
<td>0.976</td>
</tr>
<tr>
<td>$^7$Be</td>
<td>-0.419</td>
<td>-0.780</td>
<td>-0.992</td>
<td>1.000</td>
<td>0.887</td>
<td>0.154</td>
<td>0.204</td>
<td>0.322</td>
</tr>
<tr>
<td>$^9$Be</td>
<td>-0.720</td>
<td>-0.730</td>
<td>-0.986</td>
<td>1.000</td>
<td>0.269</td>
<td>0.333</td>
<td>0.486</td>
<td></td>
</tr>
<tr>
<td>$^{13}$N</td>
<td>-0.349</td>
<td>-0.407</td>
<td>-0.992</td>
<td>1.000</td>
<td>0.991</td>
<td>0.172</td>
<td>0.219</td>
<td></td>
</tr>
<tr>
<td>$^{14}$O</td>
<td>-0.319</td>
<td>-0.369</td>
<td>-0.576</td>
<td>0.332</td>
<td>0.486</td>
<td>0.172</td>
<td>0.219</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Correlations among neutrino observables are carefully estimated and routinely included in the analysis of real or prospective data, and in testing theoretical models.
Covariances are definitely relevant in geo-neutrino physics - I
(U, Th, K) abundances within a given reservoir are typically positively correlated

E.g., Kukkonen et al 2001 (Finland)

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>U</th>
<th>Th</th>
<th>K2O</th>
<th>SiO2</th>
<th>D</th>
<th>Vp</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>0.88</td>
<td>0.90</td>
<td>0.68</td>
<td>0.44</td>
<td>-0.51</td>
<td>-0.24</td>
</tr>
<tr>
<td>U</td>
<td>1</td>
<td>0.59</td>
<td>0.50</td>
<td>0.34</td>
<td>0.43</td>
<td>-0.39</td>
<td>-0.17</td>
</tr>
<tr>
<td>Th</td>
<td>1</td>
<td>0.59</td>
<td>1</td>
<td>0.52</td>
<td>-0.87</td>
<td>-0.48</td>
<td>-0.22</td>
</tr>
<tr>
<td>K2O</td>
<td>1</td>
<td>0.52</td>
<td>1</td>
<td>1</td>
<td>0.45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SiO2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>0.52</td>
<td>1</td>
<td>0.45</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vp</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

Unfortunately, quoting correlations is not common practice in local geochemical studies (even less so for global estimates)
Covariances are definitely relevant in geo-neutrino physics - II
(U, Th, K) correlations among different reservoirs can take any value

Reasonable expectations:

\[ \rho_{ij} > 0 \] if the reservoirs are very close and “homogeneous”
(“local abundances”)

\[ \rho_{ij} < 0 \] if the reservoirs are complementary (e.g., constrained
by total mass balance)

\[ \rho_{ij} \sim 0 \] if the reservoirs are basically “decoupled” (e.g., local
fluctuations vs rest of the world)
Covariances are definitely relevant in geo-neutrino physics - III

Measured geoneutrino event rates \( (R_{\text{U}}, R_{\text{Th}}) \) are anticorrelated.
Not evident in the representation chosen by KamLAND
(\textit{here shown through our reanalysis of their data})...

... due to the use of a “ratio” on the x-axis ...
... but evident when event rates \((R_U, R_{Th})\) are used as coordinates*

Solid: our KamLAND data fit

Dashed: Adapted gaussian

\[
R_U = 12.5 \pm 48.9 \text{ TNU} \\
R_{Th} = 34.7 \pm 28.5 \text{ TNU} \\
\rho(U, Th) = -0.645
\]

Negative correlation due to higher sensitivity to \(R_U + R_{Th}\) rather than to \(R_U\) or \(R_{Th}\) separately

* 1 TNU = 1 event/year/1032 protons. Rates corresponding to 100% efficiency
An important point:
“ratio errors” can be conveniently expressed through correlations.
E.g., given $X$, $Y$, and $R=X/Y$:

\[
X = X_0 (1 \pm \sigma_X)
\]
\[
Y = Y_0 (1 \pm \sigma_Y)
\]
\[
R = R_0 (1 \pm \sigma_R)
\]

Often used in Earth sciences

\[
X = X_0 (1 \pm \sigma_X)
\]
\[
Y = Y_0 (1 \pm \sigma_Y)
\]
\[
\rho_{XY} = \frac{\sigma_X^2 + \sigma_Y^2 - \sigma_R^2}{2 \sigma_X \sigma_Y}
\]

Preferable in statistical analyses

Why?

Note: 1st-order error propagation gives

\[
\sigma_{X/Y}^2 = \sigma_X^2 + \sigma_Y^2 - 2 \rho_{XY} \sigma_X \sigma_Y
\]
1) Ratio of two Gauss distributions is actually a Cauchy distribution with infinite variance (not good for statistical analyses)

2) Scatter plots involving ratios may be misleading (fake covariances)

E.g. take 3 independent Gaussian variables \((X,Y,Z)\) with, say, 15% errors.

Visual correlations in
\((Y/X \text{ vs } X)\) and in
\((Y/X \text{ vs } Z/X)\)

have no physical relevance (artifacts due to non-independent coordinates)

State-of-the-art neutrino data analyses (solar, atmospheric, laboratory) do not use ratios anymore, and stick to covariance methods (or equivalent ones)
GNSM: general aspects
Our goal is to evaluate existing information on (U, Th, K) abundances, uncertainties, and spread of ratios (Th/U, K/U ...) in each Earth reservoir, so as to provide first a “reservoir correlation matrix”

\[
\begin{bmatrix}
1 & ? & ? \\
1 & ? & \\
1 & & 
\end{bmatrix}
\]

and then to integrate such pieces of information in a global matrix for all Earth reservoirs relevant to geoneutrinos.

Missing information is supplied by educated guesses (whenever possible) or by arbitrary assumptions (when unavoidable) - but always declared explicitly. [Details in a forthcoming long paper]. Plenty of room for improvements.
Assumed structure of the total correlation matrix of abundances:
(local=close to given detectors; global=rest of the world)

Basically we assume that “local” abundance fluctuations are decoupled from global abundance uncertainties (this must be true after a certain distance).

Of course, definition of what is a “local” reservoir is not innocent.
The matrix for the global reservoirs has the following block structure

First one evaluates independently blocks related to CC, OC, UM, and BSE (Bulk Silicate Earth).

Then LM is obtained by subtraction, with all errors properly propagated.

Subtraction (anti)correlates the LM abundances with the other reservoirs.

This is a statistically correct way to implement the constraint

\[
\text{BSE} = \text{CC} + \text{OC} + \text{UM} + \text{LM}
\]
Let us anticipate, e.g., our preliminary results for global reservoirs (our "Geo Neutrino Source Model", GNSM).

Similar to Fiorentini et al.

Note correlations of LM with BSE (positive) and with CC, OC, UM (negative). KamLAND "60 TW" heat bound assumes correlations =+1 (optimistic result).
Applications of the (U,Th,K) reservoir covariance matrix

Given a GNSM, i.e., a set of abundances $a_i$ for all relevant reservoirs + the covariance (error) matrix $\sigma^2_{ij} = \rho_{ij}\sigma_i\sigma_j$

one can perform:

- **Forward** propagation of uncertainties to predictions
- **Backward** update of $a_i$ and $\sigma^2_{ij}$ after observations
Forward

Several quantities of interest (total elemental mass, radiogenic heat, geoneutrino fluxes and event rates) are linear combinations of the \((U, Th, K)\) abundances with known coefficients:

\[
P = \sum_i p_i a_i, \quad Q = \sum_i q_i a_i, \quad \ldots
\]

Their “theoretical” errors and correlations (induced by uncertainties of the GNSM) are then easily computed as

\[
\sigma_P^2 = \sum_{ij} \rho_{ij} p_i p_j \sigma_i \sigma_j
\]

\[
\sigma_Q^2 = \sum_{ij} \rho_{ij} q_i q_j \sigma_i \sigma_j
\]

\[
\rho(P, Q) = \frac{\sum_{ij} \rho_{ij} p_i q_j \sigma_i \sigma_j}{\sigma_P \sigma_Q}
\]

\[
\ldots
\]
Backward
In the future, one will have measured or inferred several quantities $Q_1, Q_2, Q_3, \ldots$ which are linear functions of the $(U, Th, K)$ reservoir abundances $a_i$:

\[
Q = Fa \quad \text{with errors} \quad \sigma_Q^2 \quad \text{(in matrix form)}
\]

Note that the vector $Q$ may include not only geoneutrino event rates at different sites, but also completely independent data, like radiogenic heat productions rates inferred by observed heat flow data.*

Then the “updated” abundances and their covariance matrix, which best fit (by least squares) both the data ($Q$) and the GNSM, are given by matrix equations:

\[
a' = \left( W_a + F^T W_Q F \right)^{-1} \cdot \left( F^T W_Q Q + W_a a \right)
\]

\[
\sigma'^2 = \left( W_a + F^T W_Q F \right)^{-1} \\
\text{(} W_a = [\sigma^2]^{-1}, \ W_Q = [\sigma_Q^2]^{-1} \text{)}
\]

*E.g., Jaupart and Mareschal 2003 estimate bulk CC heat production as $0.87(1\pm0.09) \mu W/m^3$. This can be transformed into a constraint on a linear combination of $(U, Th, K)_{CC}$
Future data will have an impact on a GNSM only if (some of) the abundance errors will be appreciably reduced.

Successive updates might either fluctuate around initial central values (GNSM corroborated) or “diverge” (GNSM falsified).

The previous equations provide quantitative tools for such exercise.
GNSM construction: some details
Earth Model: \textbf{PREM} (Dziewonsky & Anderson 1981) + \textbf{Crust} $2^\circ \times 2^\circ$ (Laske et al. 2001)

"Local" reservoirs arbitrarily defined as $3 \times 3 = 9$ crust tiles around detector sites, except for Kamioka where 13 crust tiles are taken [those named as “Japanese Island arc and forearc” in the adopted crust model].
Remarks about the crust:

1) Vertical crust structure (upper, middle, lower) is relevant for geoneutrino flux estimates only within local reservoirs

2) Outside such local reservoirs, crust is “thin” as seen from a distance: UC, MC, LC can be lumped together (great simplification)

3) Mass ratios UC:MC:LC change in different CC models:

<table>
<thead>
<tr>
<th>Crust 2x2 (our default)</th>
<th>UC : MC : LC = 0.359 : 0.330 : 0.311</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rudnick &amp; Gao 2003</td>
<td>UC : MC : LC = 0.317 : 0.296 : 0.388</td>
</tr>
</tbody>
</table>

Such difference induces up to 10% systematic variations in, e.g., CC geoneutrino fluxes or heat production rates. Should be settled. [Comparable to estimated (U,Th,K) CC uncertainties in R&G 2003.]
After the Earth model and the reservoirs are defined, several quantities of interest can be expressed as linear combinations of the (U, Th, K) abundances in each i-th reservoir:

\[ a^S_i = \frac{M^S_i}{M_i} \quad (S = U, \text{ Th, K}) \]

- Total mass of (U, Th, K)
- Radiogenic heat
- Geoneutrino luminosity
- Unoscillated neutrino flux
- Unoscillated neutrino event rate
- Oscillated neutrino flux (Pee~0.57)
- Oscillated neutrino event rate

through calculable coefficients:

\[ \begin{align*}
M^S &= \sum_i M_i a^S_i , \\
H_R &= \sum_S h_S \sum_i M_i a^S_i , \\
L &= \sum_S l_S \sum_i M_i a^S_i , \\
\Phi_D^0 &= \sum_S \phi_S \sum_i f^D_i a^S_i , \\
R_D^0 &= \sum_S r_S \sum_i f^D_i a^S_i , \\
\Phi_D &= \langle P_{ee} \rangle \Phi_D^0 = \langle P_{ee} \rangle \sum_S \phi_S \sum_i f^D_i a^S_i , \\
R_D &= \langle P_{ee} \rangle R_D^0 = \langle P_{ee} \rangle \sum_S r_S \sum_i f^D_i a^S_i ,
\end{align*} \]

- Universal coefficients
- Site- and reservoir-dependent coeff.
Universal coefficients (heat, luminosity, flux, rate) - our evaluation

<table>
<thead>
<tr>
<th>S</th>
<th>$h_S$ (μW/kg)</th>
<th>$l_S$ (10^6 $\bar{\nu}_e$/kg/s)</th>
<th>$\phi_S$ (10^{12} $\bar{\nu}_e$/cm^2/s)</th>
<th>$r_S$ (10^8 TNU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>98.0^a</td>
<td>76.4^b</td>
<td>123^b</td>
<td>15.2^c</td>
</tr>
<tr>
<td>Th</td>
<td>26.3</td>
<td>16.3</td>
<td>26.1</td>
<td>1.06</td>
</tr>
<tr>
<td>K</td>
<td>34.9 × 10^{-4}</td>
<td>28.3 × 10^{-3}</td>
<td>45.4 × 10^{-3}</td>
<td>0</td>
</tr>
</tbody>
</table>

^a Contributions: $^{238}$U (95.9 %) + $^{235}$U (4.1 %).
^b Contributions: $^{238}$U (97.0 %) + $^{235}$U (3.0 %).
^c Only from $^{238}$U.

(includes independent calculation of U, Th decay spectra)
Non-universal coefficients (“geometry”) - our evaluation

\[ f_i^D \] is the fraction of geoneutrino flux at site D that would be generated by the i-th reservoir, if the abundances were uniform in the whole Earth.

Notice comparable \( f_i \)'s for local and global contributions (due to \( 1/L^2 \) law)
Such coefficients have relatively minor uncertainties, which can be neglected at this stage.

[Some systematic deviations among independent published values for universal coefficients may be attributed, e.g., to old decay or cross-section data, or to neglect of U-235]

The main task is thus reduced, as anticipated, to assess reasonable (U,Th,K) abundances (central values, errors, and covariances from ratios) based on existing geo-literature and databases.

Not a straightforward task, in general...

Famous quotes:
“Unwary readers should take warning that ordinary language undergoes modification to a high-pressure form when applied to the interior of the Earth. A few examples of equivalents follow:

High-pressure form:  Ordinary meaning:
certain             dubious
undoubtedly        perhaps
positive proof      vague suggestion
unanswerable argument  trivial objection
pure iron            uncertain mixture of all the elements”

“Half of All Three-Sigma Results Are Wrong”
... However, the history of the solar neutrino problem tells us that attempts to assess model uncertainties are worthwhile ...

Theoretical uncertainties

Summary

Is there a “solar neutrino problem?” Yes, provided the errors in the predictions and the observations are less than the discrepancy between theory and experiment. Otherwise, no; the excitement is about a false alarm. In order to decide if there is a problem or not, to determine if something new has been discovered, we must establish the “room for maneuver” in the theoretical calculations. A quantitative evaluation of the errors is at the heart of our subject. This chapter discusses the uncertainties in predicting event rates in solar neutrino experiments. The total theoretical range for a given experiment is computed using $3\sigma$ errors for all of the measured input parameters. Uncertainties for theoretical quantities are more difficult to determine; the theoretical uncertainties are sometimes estimated by comparing the results obtained by different authors.

(U, Th, K) uncertainties in BSE

Evaluation based on:

- Comparison of RLE abundances in recent BSE models
- Chondritic Th/U ratio evaluations (Rocholl & Jochum 1993, Goreva & Burnett 2001)
- Subjective inflation of K/U ratio error in BSE
  (“canonical” value from Jochum et al. 1983 unrealistically “precise”)
- Above (relative and absolute) abundance information reformatted in covariance form

Critical points:

- Uncertainty of Al (major RLE) abundance in BSE (10%)
- K/U ratio in BSE
Jochum et al. 1983: \( \text{K/U}=12,700 \) within 1.5%  
Widely quoted in geo-literature.  
One of the authors (Hofmann), 20 years later:

Similarly, Jochum et al. (1983) estimated the K/U ratio of the primitive mantle to be \( 1.27 \times 10^4 \), a value that became virtually canonical for 20 years, even though it was based on remarkably few measurements.

Indeed, value based on just 22 MORB samples, showing a uniform K/U ratio - more uniform that Th/U ratio! (both unexpected and ununderstood). From similarity with a much older (Wasserburg '64) K/U crustal estimate, bold extrapolation to whole BSE. Based also on subsequent literature, we inflate above K/U error by x10 (i.e., 15%). Reasonable?
(U, Th, K) uncertainties in (average) CC

Evaluation based on:

- Rudnick & Gao 2003: contains careful error estimates and critical discussion of previous literature, + comparison with CC heat production data (important). We only slightly inflate some errors to account for UC:MC:LC ratio uncertainties.
- Th/U, K/U, and K/Th ratio errors taken from spread in literature (typically 9%) and other constraints [Th/U (CC) > Th/U (chondrite)]
- Above (relative and absolute) abundance information reformatted in covariance form

Critical points:

- Uncertainties in lower crust poorly defined
- Differences in Crust 2x2 and R&G 2003 models for UC+MC+LC
(U, Th, K) uncertainties in UM

Evaluation based on:

- Two recent UM major+trace elemental abundance estimates (Salters & Stracke 2004, Workmann & Hart 2004) + previous literature

- Despite being based on the same PetDB database (Su and Langmuir 2003), the two 2004 papers reach different conclusions about complementarity of CC and UM

- Concordance for (U, Th, K) abundances and their ratio possible only with generous errors - finally reformatted in covariance form

Critical points:

- Complex models - difficult to evaluate their mutual (in)dependence

- More general problem of UM-LM difference (if any)
(U, Th, K) uncertainties in (average) OC

Evaluation based on:

- Scarce literature
- Assuming the same UM (errors+correl.), concordance is possible

Critical points:

- Lack of published information
- Are errors and correlations similar in UM and OC?
(U,Th,K) uncertainties in LM
Evaluation based on linear propagation of errors from previous reservoirs, assuming the linear constraints: \( \text{BSE} = \text{UM} + \text{LM} + \text{CC} + \text{OC} \)

Critical points:
- Core ? Mantle convection?

GRAND TOTAL

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<table>
<thead>
<tr>
<th>Geo-Neutrino Source Model (GNSM) for global reservoirs</th>
<th>BSE</th>
<th>CC</th>
<th>Correlation matrix</th>
<th>UM</th>
<th>LM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reser. Elem. Abundance ±1σ</td>
<td>U Th K</td>
<td>U Th K</td>
<td>U Th K</td>
<td>U Th K</td>
<td>U Th K</td>
</tr>
<tr>
<td>BSE</td>
<td>(2.9 \times 10^{-9}) ±14%</td>
<td>1</td>
<td>.956</td>
<td>+.701</td>
<td>0</td>
</tr>
<tr>
<td>Th</td>
<td>(8.1 \times 10^{-9}) ±14%</td>
<td>1</td>
<td>.648</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>K</td>
<td>(2.3 \times 10^{-8}) ±21%</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>CC</td>
<td>(1.46 \times 10^{-6}) ±17%</td>
<td>1</td>
<td>.906</td>
<td>+.906</td>
<td>0</td>
</tr>
<tr>
<td>Th</td>
<td>(6.0 \times 10^{-6}) ±10%</td>
<td>1</td>
<td>1</td>
<td>.959</td>
<td>0</td>
</tr>
<tr>
<td>K</td>
<td>(1.6 \times 10^{-2}) ±10%</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>OC</td>
<td>(1.0 \times 10^{-7}) ±30%</td>
<td>1</td>
<td>.956</td>
<td>+.868</td>
<td>0</td>
</tr>
<tr>
<td>Th</td>
<td>(2.8 \times 10^{-7}) ±30%</td>
<td>1</td>
<td>1</td>
<td>.764</td>
<td>0</td>
</tr>
<tr>
<td>K</td>
<td>(1.5 \times 10^{-7}) ±28%</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>UM</td>
<td>(3.9 \times 10^{-9}) ±30%</td>
<td>1</td>
<td>.906</td>
<td>+.868</td>
<td>0</td>
</tr>
<tr>
<td>Th</td>
<td>(1.8 \times 10^{-9}) ±35%</td>
<td>1</td>
<td>1</td>
<td>.764</td>
<td>0</td>
</tr>
<tr>
<td>K</td>
<td>(5.0 \times 10^{-9}) ±28%</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>LM</td>
<td>(1.7 \times 10^{-9}) ±27%</td>
<td>1</td>
<td>.924</td>
<td>+.692</td>
<td>0</td>
</tr>
<tr>
<td>Th</td>
<td>(8.0 \times 10^{-9}) ±27%</td>
<td>1</td>
<td>1</td>
<td>.640</td>
<td>0</td>
</tr>
<tr>
<td>K</td>
<td>(2.1 \times 10^{-8}) ±16%</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Are such abundances compatible with mantle convection uncertainties? Yes.

GNSM central values:

\[ a_{UM} < a_{LM} < a_{BSE} \]

(partial mantle convection)

However, within \(~3\) sigma, the model is consistent both with

\[ a_{LM} = a_{UM} \]

(whole-mantle convection, left panels)

and with

\[ a_{LM} = a_{BSE} \]

(decoupled LM, right panels)
Local reservoirs and related problems
The characterization of local crust reservoirs (which give a large - but not necessarily interesting - contribution to the geoneutrino signal) is not easy. Problems:

2x2 crust resolution is poor
Horizontal distribution of (U, Th, K) often not well known
Vertical distribution of (U, Th, K) largely unknown
Elemental ratios (e.g., Th/U) may locally be anomalous ...

However, at least some of these problems can be solved by merging existing information on:

3D local models of the crust (and mantle)
(Un)published rock sample databases
Surface heat flow data
Borehole data ...

Interdisciplinary approach + long-term work required
E.g., vertical abundance of heat-producing elements may be fuzzy, non-monotonic, and highly site-dependent:

**Hokkaido, Japan**
(Furukawa 1997)

**Nojima, Japan**
(Yamaguchi 2001)

**Corsica, Mediterr.**
(Verdoya 1998)

... but must be consistent with measurable heat flow data. Borehole data (when available) provide useful constraints.

Note: New borehole planned at Sudbury, [www.icdp-online.de](http://www.icdp-online.de) (Sudbury Deep Drilling Project)
Additional information, e.g., in Sudbury may come from the “tilted” crust structure, suggesting the horizontal (U,Th,K) profile as a proxy for the vertical profile:

(Schneider 1987)

Figure 3. Plan and cross-section views of the Sudbury structure, Ontario, Canada: a. May showing Highway 144 traverse from Windy Lake through Cartier (modified from Hunt and Farhar, 1977); b. Cross-section through the structure (modified from Cark and Hutchinson, 1982). Note the interpreted collar reveals a depth profile across the Archean basement in the northeast.

Figure 4. Distribution of U, Th, K, and the resulting heat production A with distance (i.e., upcrust) from the Sudbury Basin. Note the general increase from left to right in the heat production plot.

Hopefully one could get an “overconstrained” profile.

Note: large local Th/U ratio in above figure
Inter-disciplinary work desirable to assess local uncertainties. By merging available (+new?) local info, maybe a 10% accuracy goal reachable for local geoneutrino fluxes (similar to average UC error) (Enomoto et al., and Fiorentini et al. estimates for local Japanese crust are in this range).

Uncertainties should be probably increased by, say, $x2$ in middle crust, and perhaps by $x4$ in lower crust. Local oceanic crust presumably affected by a larger error (20% ?) (More difficult access, poorer sampling.)

We have attached such hypothetical errors to all local crust contributions, assuming (UC+MC+LC) abundances as in Rudnick & Gao 2003, except for Japan, where upper crust is well known (Togashi 2000). Local correlations among (U, Th, K) are taken equal to average crust. Clearly, the results can only be indicative.

Examples of outputs including local contributions in the GNSM:
1, 2, and 3-sigma contours in the Uranium-Thorium event rate plane for the KamLAND experiment.

KamLAND data (dashed)

\[ R_U = 12.5 \pm 48.9 \text{ TNU} \]
\[ R_{\text{Th}} = 34.7 \pm 28.5 \text{ TNU} \]
\[ \rho(U, \text{Th}) = -0.645 \]

GNSM (solid)

\[ R_U = 24.9 \pm 2.0 \text{ TNU} \]
\[ R_{\text{Th}} = 6.7 \pm 0.5 \text{ TNU} \]
\[ \rho(U, \text{Th}) = +0.901 \]

GNSM compatible with data at 1-sigma, but not yet constrained by such data. Need background reduction + higher statistics.
Total radiogenic heat vs. total event rate in KamLAND

**GNSM:**

\[ R_{U+Th} = 31.6 \pm 2.5 \text{ TNU} \]
\[ H_{U+Th+K} = 21.1 \pm 3.0 \text{ TW} \]
\[ \rho(R, H) = +0.858 \]

Approximately, the ellipse corresponds to the fraction of the “allowed band” estimated by Fiorentini et al. around BSE predictions.
Total event rates (including oscillations) with errors and correlations, calculated at various detector sites

<table>
<thead>
<tr>
<th>Site</th>
<th>Rate ± 1σ (TNU)</th>
<th>Correlation matrix of GNSM predictions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Kam</td>
</tr>
<tr>
<td>Kamioka</td>
<td>31.6 ± 2.5</td>
<td>1.00</td>
</tr>
<tr>
<td>Gran Sasso</td>
<td>40.6 ± 2.9</td>
<td>1.00</td>
</tr>
<tr>
<td>Sudbury</td>
<td>47.9 ± 3.2</td>
<td>1.00</td>
</tr>
<tr>
<td>Pyhasalmi</td>
<td>49.9 ± 3.5</td>
<td>1.00</td>
</tr>
<tr>
<td>Baksan</td>
<td>50.7 ± 3.4</td>
<td>1.00</td>
</tr>
<tr>
<td>Hawaii</td>
<td>13.4 ± 2.2</td>
<td></td>
</tr>
</tbody>
</table>

Note: relatively small errors depend also on assumption of local CC (OC) upper crust abundance known at 10% (20%).
Correlation of theoretical rates relevant for model testing

E.g., the combination of two experimental data might be inconsistent with model, even if each datum is consistent.

Study of “prospective data” impact: in progress.
Conclusions

Covariance analysis represents a useful “template” to embed current and future information relevant to geoneutrino physics (it is already so in other neutrino research sub-fields)

We are still far from a satisfactory approach of this kind in (U,Th,K) geochemistry, due to intrinsic difficulties (large uncertainties, incomplete data, conflicting estimates, etc.) However, at least a tentative GNSM can be constructed

Improvements, especially for local contributions, crucial to assess quantitatively the impact of future expt’l data (and of their combination) on any GNSM (including non-orthodox ones)

Significant progress will benefit from joint, inter-disciplinary work by geophysicists, geochemists, and particle physicists