Power Requirements for Earth’s Magnetic Field

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Structure of the Earth
Origin of Inner Core

Inner core grows as the core cools
Composition of Core

Addition of light elements required to explain density (popular suggestions include O, S, Si)
Phase Diagram
Physical Processes

- mantle
  - heat flux $Q$
  - contraction
  - solidification

- outer core
  - light elements
  - latent heat

- inner core
  - $r = b - db$
  - $r = c + dc$
Evolution of Core

Total Energy: \( E = U + \Omega + K + M \)

Evolution based on energy conservation:

\[
\frac{dE}{dt} = \frac{dU}{dt} + \frac{d\Omega}{dt} = \int_V \rho \varepsilon \, dV - \int_S \mathbf{q} \cdot d\mathbf{S} - \int_S P \mathbf{v} \cdot d\mathbf{S}
\]

convective fluctuations in internal U and gravitational \( \Omega \) energies are negligible

Average over convective fluctuations to define mean state (hydrostatic, adiabatic, uniform composition)
Mean State

**Hydrostatic** \[ \nabla P_0 - \rho_0 g = 0 \]

**Adiabatic** \[ \nabla S_0 = 0 \]

**Uniform composition** \[ \nabla C_0 = 0 \]

Energies \[ U(P_0,S_0,C_0) \quad \Omega(P_0,S_0,C_0) \]
Thermal State of the Earth

Temperature drop across D":

\[ \Delta T = 900 - 1900 \text{ K} \]
Heat Flow at Top of Core

- conductivity $k \sim 7-10 \, \text{W/K m}$
- temperature $\Delta T \sim 900-1900 \, \text{K}$
- layer thickness $\delta \sim 100-200 \, \text{km}$

large uncertainty in total heat flow

$$Q = - \int_s q \cdot dS$$

$$Q = 5 - 28 \, \text{TW} \quad (Q_{\text{surface}} = 44 \, \text{TW})$$

heat conducted down adiabat

$$Q_a = 5 - 6 \, \text{TW}$$
Limits on Heat Flow

high core temperature implies large CMB heat flow
Thermal Evolution

* assumes no radiogenic heat sources in core
Boundary-Layer Model

Local stability of boundary layer

$$Ra = \frac{\alpha \rho g \Delta T \delta^3}{\kappa \eta(\bar{T})}$$

Critical Rayleigh number $Ra_c \sim 10^3$

Heat Flow

$$\frac{Q(t_1)}{Q(t_2)} = \left[ \frac{\Delta T(t_1)}{\Delta T(t_2)} \right]^{4/3} \left( \frac{\eta(\bar{T}_2)}{\eta(\bar{T}_1)} \right)^{1/3}$$
Power Requirements for Dynamo

Glatzmaier & Roberts, 1996
Dynamo Power

Dissipation $\Phi$

$$\Phi = \int_V \frac{J^2}{\sigma} \, dV + \int_V 2\eta \dot{\epsilon}^2 \, dV$$

(ohmic) \hspace{1cm} (viscous)

Numerical Models

Kuang - Bloxham model \hspace{1cm} 0.1 \ TW

Glatzmaier - Roberts model \hspace{1cm} 1.0 \ TW
Work Done by Convection

Mechanical Energy Balance

\[ \Phi = \int_V \mathbf{v} \cdot (\rho \mathbf{g} - \nabla P) \, dV \]

Fluctuations about hydrostatic state \((\rho_0 \mathbf{g} - \nabla P_0) = 0\)

\[ \rho = \rho_0 + \rho' \quad S = S_0 + S' \quad C = C_0 + C' \quad P = P_0 + P' \]

Correlation of fluctuations with \(\mathbf{v}\)

\[ \Phi = \int_V \mathbf{v} \cdot \mathbf{g} \left[ S' \left( \frac{\partial \rho}{\partial S} \right)_{P,C} + C' \left( \frac{\partial \rho}{\partial C} \right)_{P,S} \right] \, dV \]

(thermal) (compositional)
Buoyancy Flux

- generation of buoyancy at the boundaries
- flux calculated by requirement that core is well mixed
Efficiency of Convection

Power can be expressed in terms of “Carnot” efficiencies

$$\Phi = (\epsilon_t + \epsilon_c) Q$$
Heat Flow Requirement

![Graph showing CMB heat flow as a function of inner-core radius with different models and estimates.]

- Nominal estimate
- Adiabatic heat flow

Parameters:
- $\Phi = 1.0 \text{ TW}$
- $\Phi = 0.5 \text{ TW}$
- $\Phi = 0.1 \text{ TW}$

Axes:
- CMB Heat Flow (TW)
- Inner-Core Radius (km)
Thermal History

* no radiogenic heat sources in the core

inner-core radius

CMB temperature

Dissipation $\Phi$ in TW

$\Phi = 0.1$  $\Phi = 0.5$  $\Phi = 1.0$

boundary layer theory
constant $Q$

$\Phi = 0.1$ TW
$\Phi = 0.5$ TW
$\Phi = 1.0$ TW
Inconsistencies

1. Current temperature estimates imply high CMB heat flow (problems during Archean)

2. Geodynamo power can be supplied with lower heat flow (incompatible with #1)

3. Geodynamo power $\Phi = 0.5$ to $1.0$ TW still yields implausible thermal history
Possible Solutions

1. Geodynamo power is low $\Phi \sim 0.1$ TW

Explanation of low heat flow

\[ q = \frac{k \Delta T}{\delta} \]

Q = 2 TW requires 11 TW of radiogenic heat source in D”
(oceanic crust or enriched partial melt?)

How realistic is $\Phi=0.1$ TW?
Possible Solutions

2. Additional heat sources in the core
   - slows cooling for prescribed Q
     - i) avoids high initial temperature
     - ii) supplies additional power to dynamo

present-day heat flow: $Q(0) = 6$ TW
Dynamo Power

![Graph showing the relationship between geodynamo power and time before present (Ga). The graph includes two lines representing different K concentrations: 150 ppm and 225 ppm. Arrows indicate rapid cooling and radiogenic warming.]
Distinguishing between Possibilities

Options 1 and 2 are not mutually exclusive

Relative importance of 1 and 2?

i) better estimates of $\Phi$ using more realistic dynamo models

ii) better understanding of structure at base of mantle

iii) partitioning of radiogenic elements in lower mantle minerals and melts

transfer of radiogenic elements over time?
Conclusions

1. Current temperature estimates yield high heat flow (Q > 6 TW)

2. Geodynamo may operate with lower heat flow
   i) $\Phi = 0.1$ TW implies $Q \sim 2$ TW
   ii) $\Phi = 1.0$ TW implies $Q \sim 4.6$ TW

3. Power requirements $\Phi > 0.5$ TW requires additional heat sources (200 ppm K is sufficient)
   -> gradual addition of heat sources is attractive
Power for the Geodynamo

Dissipation $\Phi = (\varepsilon_T + \varepsilon_c) Q$

Present-day efficiencies $\varepsilon_T \sim 0.07$ $\varepsilon_c \sim 0.16$

Convective heat flux $Q - Q_{ad}$

Adiabatic heat flux $Q_{ad} \sim 5 - 6$ TW