ARA Trigger Issues

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ARA basic model

- 8 antennas in a generic 3D arrangement
  - Assumption: H and V polarization give a unified trigger event for each position
- Typical distances between antennas ~20m
- Time of RF propagation for 20m: ~120ns:
  - Use unit of time ("binning") 12ns – reasonable for hardware (if operating at bintime period: f=83MHz – possibly 2 or 3 times faster to parallelize execution – see later

How many antennas for a trigger?

- There might be different reasons for choosing a minimum number of antennas: here consider 2:
  - Direction estimation
  - Noise rejection
- Decision quite important as it influences the complexity of the trigger design

Direction estimation

- Besides its importance for the project, it is possibly a measure of “fuzzyness” of noise rejection
- Derive a simple “reciprocal lattice” model for the angle estimation.
- Suppose:
  - There is an antenna situated exactly at the origin of a coordinate frame
  - All other antennas are in the positive octant (limitation not necessary, but makes analysis simpler).

Direction estimation - units

- To simplify the formulas, use normalized units of speed and time:
  - Speed=1 -> speed of light
    - So in terms of wave propagation time=distance (in the direction of propagation)
    - Given a cartesian frame, the three components of the wave speed form a versor $((v_x)^2 + (v_x)^2 + (v_x)^2 = 1)$
  - Unit of time = 1 “bin” (12 ns):
    - 2 antennas 20m apart are separated by 10 bins of time and space.

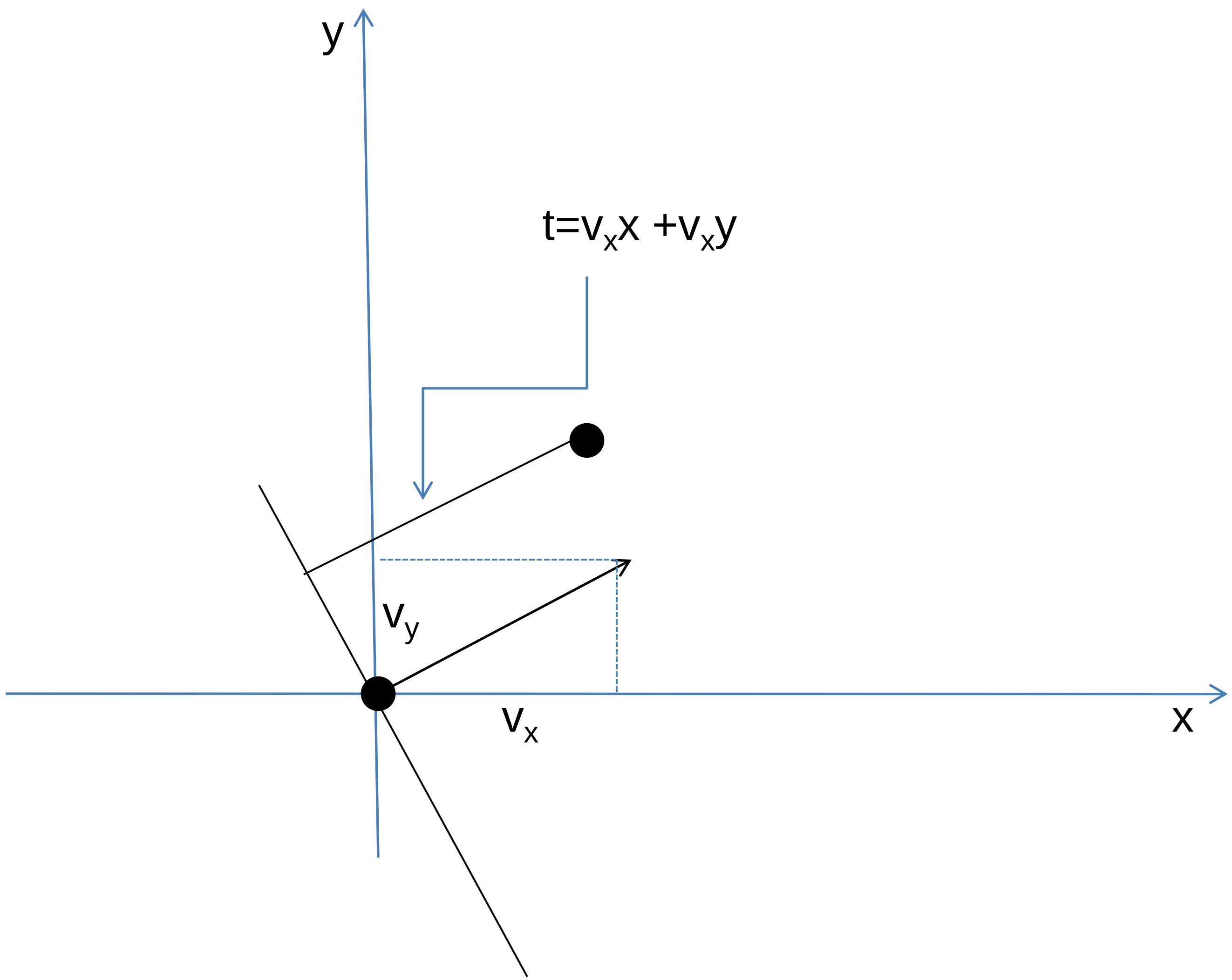
Time between triggers

- Supposing that a plane wave with versor (v_x, v_y, v_z) arrives at the origin of the coordinate frame (position of first antenna)
- The time it takes for a second antenna to trigger is equal to the distance between the two antennas in the direction perpendicular to the plane of the wave = distance of the position of the second antenna from a plane passing at the origin

Time between triggers (cont.)

- So $t= |v_x x + v_y y + v_z z| / \sqrt{(v_x)^2 + (v_y)^2 + (v_z)^2}$, where (x,y,z) is the position of the second antenna
- As v is a versor, $t= |v_x x + v_y y + v_z z|$
- Finally, as all antennas have positive x,y,z:
  - $t = v_x x + v_y y + v_z z$
  - Simple formula that allows determination of all time intervals for a given wave direction.

Time between triggers (2D example)



Estimation of maximum angle error

- In order to figure out in what conditions the maximal imprecision of angle estimation occurs, it is necessary to calculate the maximal variation of the velocity versor that will NOT cause a change of timing due to time quantization (binning).
- Given a new versor $v'=(v'_x,v'_y,v'_z)$ such that $v'=v+\Delta v$:
  - $|t'-t|<1$ (time binning) is the maximal time difference

Maximum angle error (cont.)

- But
  - $t=v_x x + v_y y + v_z z$
  - $t'=v'_x x + v'_y y + v'_z z$
- So:
  - $|t-t'|=|(v_x-v'_x)x+(v_y-v'_y)y+(v_z-v'_z)z|= |(\Delta v_x)x+(\Delta v_y)y+(\Delta v_z)z|<1$
    - 2 linear inequalities on Δv 's components
- This applies to all antennas: each extra antenna introduces 2 linear inequalities

Maximum angle error (cont.)

- Δv vs angle α between v and v'?
  - $|\Delta v|^2 = |v'-v|^2 = |v'|^2 - 2v \cdot v' + |v|^2 = 1 - 2v \cdot v' + 1 = 2(1 - |v||v'|\cos(\alpha) = 2(1-\cos(\alpha)) = 2(1-\cos(2\alpha/2)) = 2(1-(1-2\sin^2(\alpha/2))) = 4\sin^2(\alpha/2)$
  - $|\Delta v| = 2|\sin(\alpha/2)|$
  - $\alpha = 2|\arcsin(\Delta v/2)|$ (for positive α)
  - For small angles, $\alpha = \Delta v$
  - In general (due to monotonicity of arcsin) a large angle corresponds to a large Δv:
    - Can estimate maximum error using a representation of vector Δv

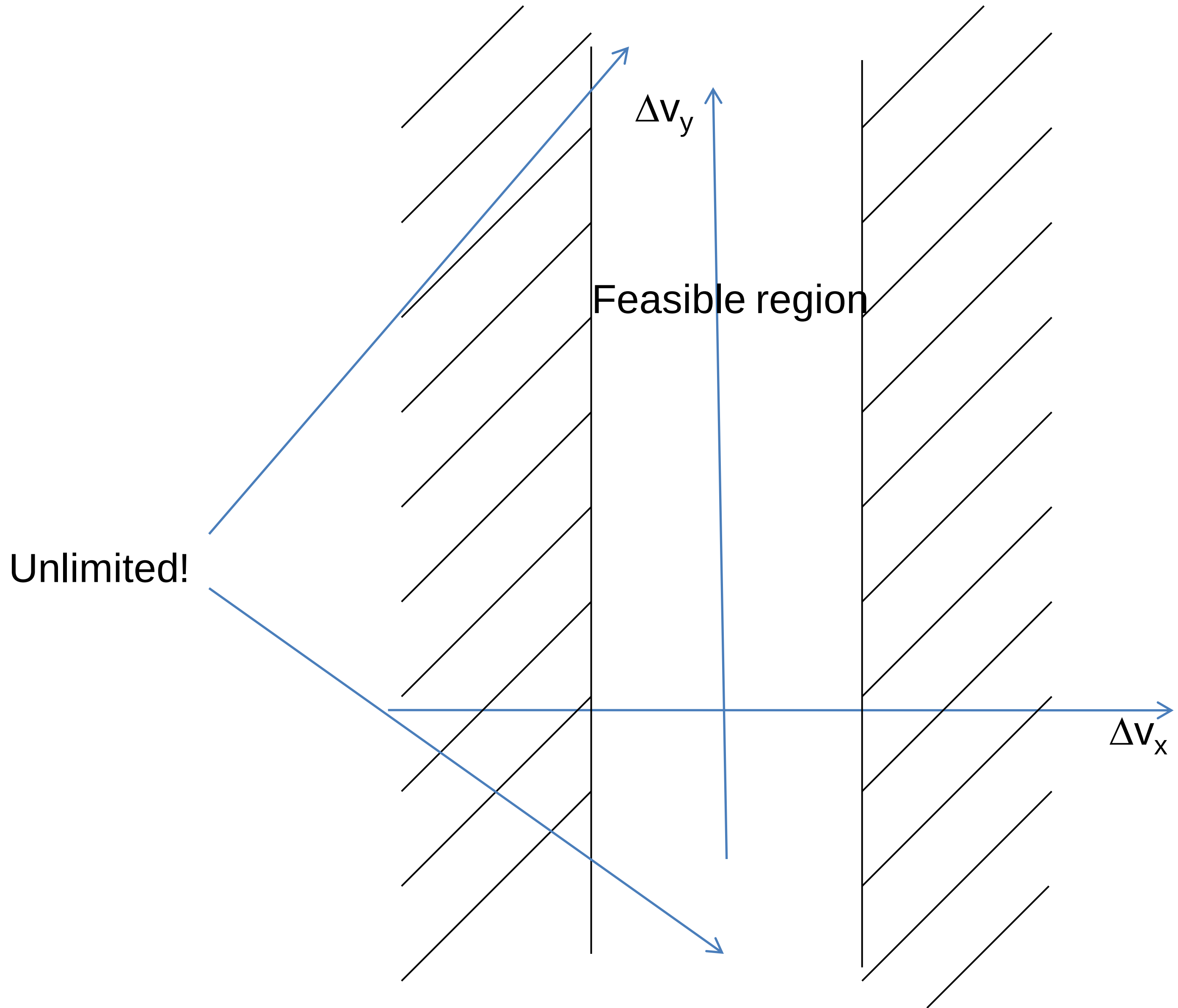
Maximum angle error (cont.)

- We also need to consider the constraint on Δv expressed as $v'=v+\Delta v$, that forces Δv to belong to a sphere of radius 1
- Maximization is equivalent to the (nonlinear) problem:
  - Maximize $|\Delta v|$ subject to
  - $|(\Delta v_x)x_1+(\Delta v_y)y_1+(\Delta v_z)z_1|<1$
  - $|(\Delta v_x)x_2+(\Delta v_y)y_2+(\Delta v_z)z_2|<1$
  - $|(\Delta v_x)x_3+(\Delta v_y)y_3+(\Delta v_z)z_3|<1$
  - "(spherical constraint)"

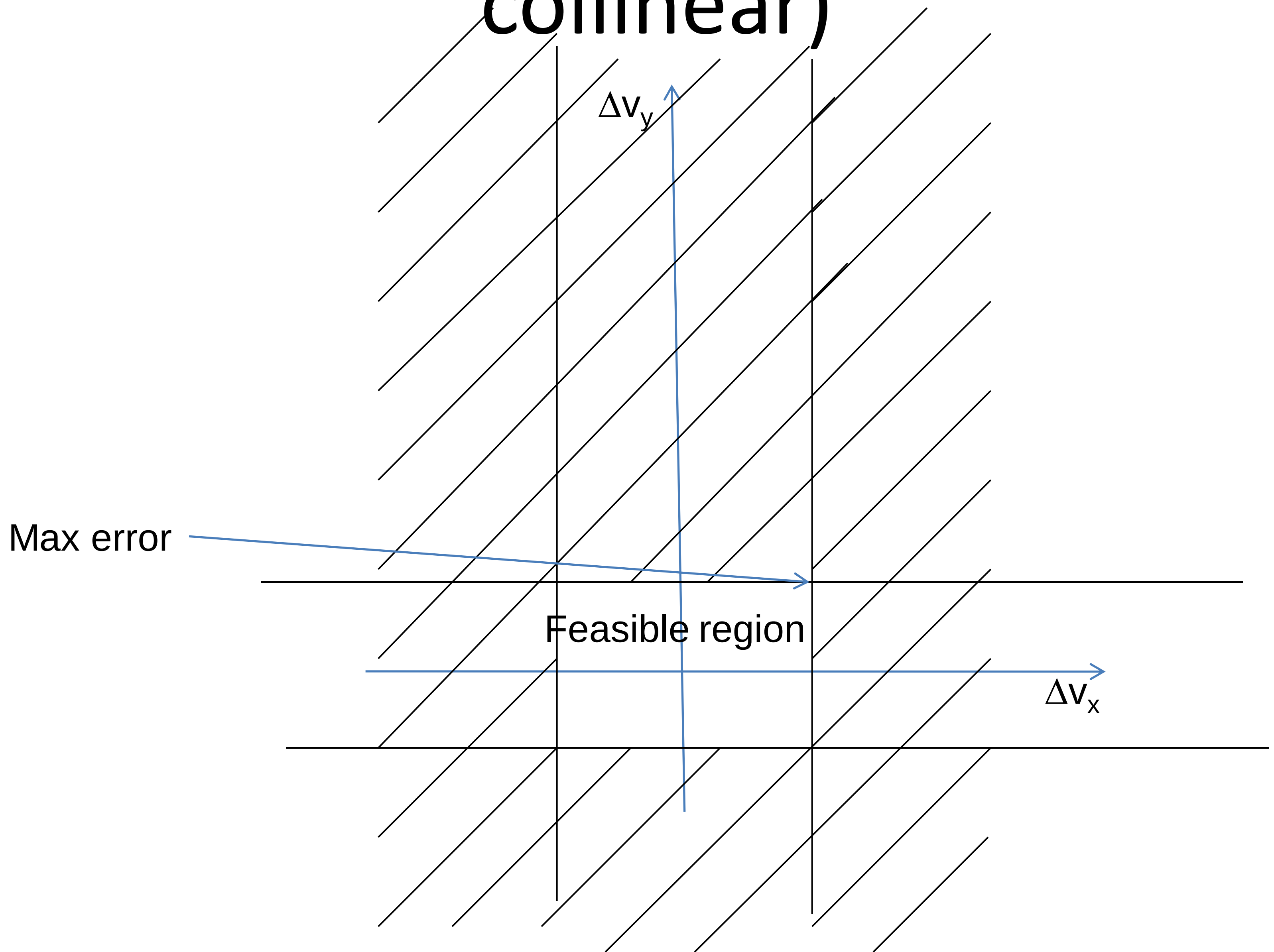
Maximum angle error (cont.)

- Disregarding for the moment the spherical constraint, the problem defines a space of solutions that is limited by couples of parallel planes each defined by an antenna:
  - In order to constrain to a finite space in dimension n we need n antennas (besides the first at the origin), not all on the same hyperplane
  - 2(+1) antennas in 2D, 3(+1) antennas in 3D!
  - 4 antennas are necessary

2D case – 2 antennas



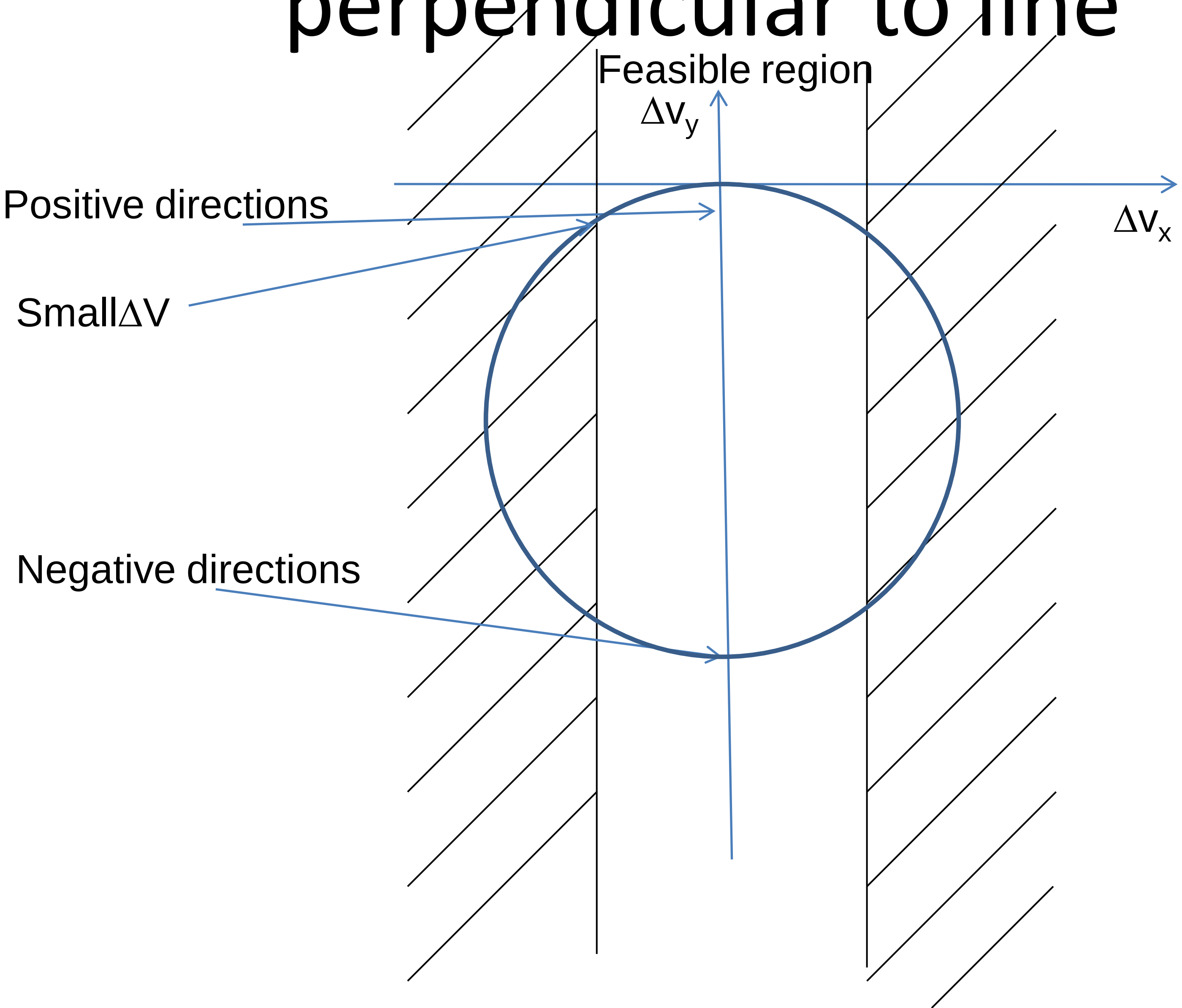
2D case – 3 antennas (non collinear)



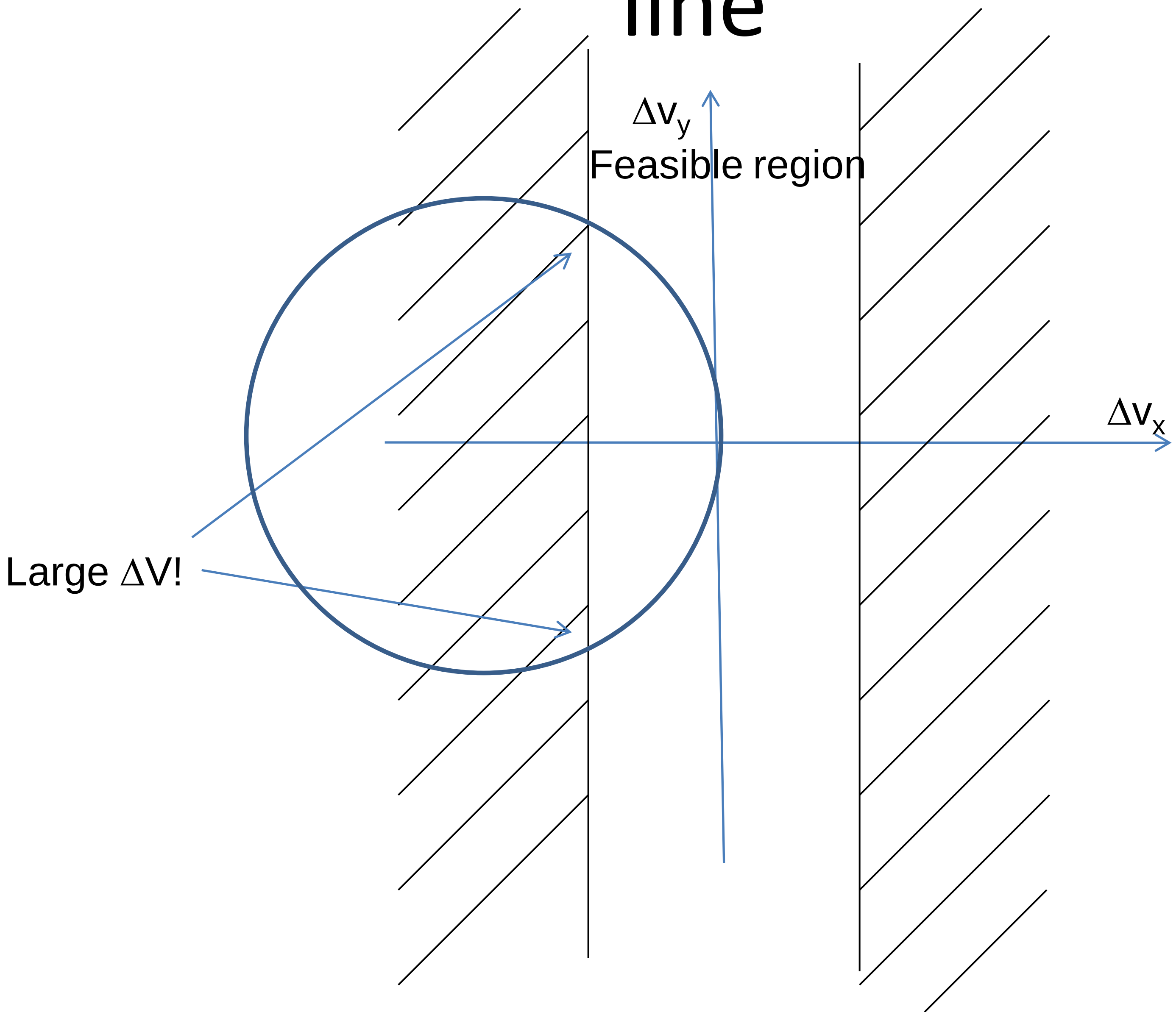
Effect of “spherical” constraint

- The spherical constaint in reality limits the possible error to the surface of the circle
- Error in fact depends on the direction of the incoming wave
- In the case of “infinite” feasible reasons:
  - The directions perpendicular to the antenna hyperplane have an uncertainty in direction (possibly not critical)
  - The directions parallel to the hyperplane have the largest error

2D case – 2 antennas – v perpendicular to line



2D case – 2 antennas – v parallel to line



3D case

- In 3D case, 3 antennas will give rise to big errors in direction measurements
- Experimentally, positive octant only:
  - With 2 antennas on x and y axis, distant 20m from first antenna in (0,0)
  - Computing all the possible patterns
  - Computing all possible angles between directions with the same pattern: error of 24 degrees!
  - Adding one more antenna on z axis: 9 degrees!
  - Adding more antennas doesn’t increase precision by much (max error 7 degrees).

Probabilistic analysis

- What is the consequence of choosing 3 or 4 antennas in terms of probability of false positives due to noise?
- In order to estimate the effect, Monte Carlo simulations (it is possible that some counting would give exact results, but I could not come up with anything reasonable…)

Monte Carlo simulation

- Given a configuration, generate all possible 8-antenna patterns, in terms of time intervals with respect to first hit
  - Use the relation $t=v_x x + v_y y + v_z z$ for each antenna for a large number of versors, then keep only one copy for each pattern
- Generate 100,000 random samples for the (#BIN)x(#ANTENNAS) combinations (in my case, 18x8 (18~10√3 – longest distance between antennas)
  - Use a probability p_{noise} independent from position or time

Monte Carlo simulation (cont.)

- For each of the random samples, evaluate if the pattern will have at least k common values (k=3,4):
  - If yes, it counts as a false positive
  - Estimate the probability of false positive $p_{noise\text{-}tot}(p_{noise},k)$

Monte Carlo results

- K=3 antennas:
  - p_{noise}=0.25: $p_{noise\text{-}tot}$=~0.999!
  - p_{noise}=0.1: $p_{noise\text{-}tot}$=~0.83!
  - p_{noise}=0.05: $p_{noise\text{-}tot}$=~0.345
- K=4 antennas:
  - p_{noise}=0.25: $p_{noise\text{-}tot}$=~0.990!
  - p_{noise}=0.1: $p_{noise\text{-}tot}$=~0.39
  - p_{noise}=0.05: $p_{noise\text{-}tot}$=~0.0595

Result discussion

- It seems that 3 antennas are too few to reduce the number of false positives, unless the noise is relatively small (~1% or so)
- Even with 4 antennas, a maximum probability of 10% is necessary
- Such numbers need to be compared to the probability of true positives

True positive analysis

- In this case, with a few hypotheses, it is possible to obtain an exact formula
- Hypotheses:
  - Every antenna is equally likely to trigger to a real wavefront
  - Such likelyhood is indicated by a p_{trig} independent of time for each timebin
- In such case, a correct global trigger occurs if at least k antennas are above threshold

True positive analysis (cont.)

- Case k=3 ($q_{trig} = 1-p_{trig}$, the numbers are binomial coefficients 8 choose x):
  - $p_{true\text{-}tot} = 1-[(q_{trig})^8+8(q_{trig})^7(p_{trig}) +28(q_{trig})^7(p_{trig})^2]$
- Case k=4:
  - $p_{true\text{-}tot} = 1-[(q_{trig})^8+8(q_{trig})^7(p_{trig}) + 28(q_{trig})^7(p_{trig})^2+56(q_{trig})^6(p_{trig})^3]$

True positive results

- K=3 antennas
  - p_{trig}=0.5: $p_{true\text{-}tot}$=~0.855
  - p_{trig}=0.2: $p_{true\text{-}tot}$=~0.20
  - p_{trig}=0.1: $p_{true\text{-}tot}$=~0.038!
- K=4 antennas:
  - p_{trig}=0.5: $p_{true\text{-}tot}$=~0.636
  - p_{trig}=0.2: $p_{true\text{-}tot}$=~0.056!
  - p_{trig}=0.1: $p_{true\text{-}tot}$=~0.0051!!

True positive result discussion

- It is clear that to obtain a decent amount of detection is necessary to have a significant amount of true positive:
  - This implies working at a threshold that guarantees a triggering probability of 20% using 3 antennas or closer to 50% for 4 antennas
- However, choosing such a low threshold increases the p_{noise} for the same SNR:
  - Optimal choice depends on expected SNR

SNR vs. p_{trig} <->p_{noise}

- How is the SNR related to the relation between p_{trig} <->p_{noise}?
- In a simplistic model, where each waveform with amplitude larger than a threshold triggers, and the amplitude of noise follows the gaussian:
  - $A_{signal} = \sqrt{(SNR)}\sigma$
  - If SNR=1, and threshold=A_{signal}:
    - p_{trig}=50%, p_{noise} =33% (tail over σ)
  - If SNR=4, and threshold=A_{signal}:
    - p_{trig}=50%, p_{noise} =5% (tail over 2σ)

SNR and optimal parameters

- Using the model of the previous slide:
  - At SNR=1:
    - With 3 antennas: $p_{true\text{-}tot}$=~0.855, but $p_{noise\text{-}tot}$>~0.999! And we are completely overwhelmed by continuous triggers
    - With 4 antennas: $p_{true\text{-}tot}$=~0.636, and $p_{noise\text{-}tot}$>~0.990! As well
  - At SNR=4:
    - With 3 antennas: $p_{true\text{-}tot}$=~0.855, but $p_{noise\text{-}tot}$~0.345! We still have to deal with frequent spurious triggers
    - With 4 antennas: $p_{true\text{-}tot}$=~0.636, and $p_{noise\text{-}tot}$ =~0.0595 and it looks like there is chance to have a decent rejection of false positives

Real life questions

- What is the real relationship with SNR (power not amplitude, frequency components, inertia of triggers….)?
- Can we deal with a continuous evaluation of all possible 4-antennas combination?
  - Some insight in the second question

Hardware cost of evaluation

- Hardware cost of the evaluation? Using
  - symmetrical antenna model discussed at the beginning
    - Total of 875 different combinations
  - Generating all the 8 choose 3 (or 4) combinations and considering only unique patterns:
    - 2316 patterns with 3 antennas
    - 19721 patterns with 4 antennas
  - Optimizing with SIS for FPGAs: the “complete” case (8 antennas) costs 755 5-input functions – 13 levels of logic reasonable. 1 OOM more?

Hardware plan

- Possibilities:
  - Window-based Evaluation: after any trigger, wait for MaxDistance bins and store the array, then do pattern matching:
    - Advantage: MaxDistance time to take decision (but is it ok – are we not losing data?)
    - Disadvantage: large LUT/compression/pattern matcher (144 inputs or more)
  - Clock by clock decision on 3 inputs: 4 triggers ordered in time, 0^{th} dropped:
    - Advantage from symmetry of station (conflict with other objectives?)
    - Requires non-trivial control FSM to choose/drop 4-uples