Total Internal Reflection

Total internal reflection occurs when $\theta > \theta_c$ and provides 100% reflection. This has better efficiency than silvered mirror.

Examples of devices using Critical Angle

- Prism Binoculars
- Fiber Optics

Fiber optics is extremely important for high speed Internet and digital data transfer at long distances. Many companies (Lucent) have laid fiber over long Distances to provide internet service.

Lecture 24, ACT 2: Critical Angle...

An optical fiber is surrounded by another dielectric. In case I this is water, with an index of refraction of 1.33, while in case II this is air with an index of refraction of 1.00.

Compare the critical angles for total internal reflection in these two cases

- a) $\theta_1 > \theta_{II}$
- b) $\theta_1 = \theta_{II}$
- c) $\theta_1 < \theta_{II}$

Fiber Optic Network Maps

Fiber optics has replaced copper for long distance communication and is becoming increasingly used for local communications.

Lecture 24, ACT 2: Critical Angle...

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Since $n_1 > n_2$, TIR will occur for $\theta > \theta_c$ critical angle. Snell’s law says $\sin \theta_c = n_2/n_1$.

If $n_1=1.0$, then $\theta_c$ is as small as it can be. So $\theta_1 > \theta_c$. 
Consider our EM plane wave. The E field is polarized in the Y-direction. We say this is "linearly polarized light".

Most light sources are not polarized in a particular direction. This is called unpolarized light or radiation.

**Cell Phone Problem**

32.18) A sinusoidal electromagnetic wave emitted by a cell phone has a wavelength of 35.4 cm and an electric field amplitude of 5.4 x 10^-2 V/m at a distance of 250 m from the antenna. (a) The intensity? (b) The total average power?

a.) \( I = \frac{1}{2} \varepsilon_0 \varepsilon E^2; \)

\[ I = \frac{8.85 \times 10^{-12} C^2 / \text{Nm}^2 (3 \times 10^8 \text{m/s})(5.4 \times 10^{-2} \text{V/m})}{2} \]

\[ I = 3.87 \times 10^{-7} \text{W/m}^2 \]

b.) Assume isotropic: \( I = P / A; \) \( P = 4 \text{m}^2 \) \( I = 4 \pi (250m)^2 \) \( I = 3 \text{W} \)

**EM wave: energy and intensity**

Previously, we demonstrated the energy density existed in E fields in capacitor and in B fields in inductors. We can sum these energies,

\[ u = \frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2} \mu_0 B^2 \]

Since, \( E \propto B \) and \( c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \) then \( u \) in terms of \( E \),

\[ u = \frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2} \mu_0 \left( \frac{E^2}{c^2} \right) = \frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2} \mu_0 E^2 = \varepsilon_0 E^2 \]

\[ \Rightarrow \text{EM wave has energy and can transport energy at speed } c \]

Suppose we have transverse E and B fields moving, at velocity \( c \), to the right. If the energy density is \( \text{u} = \varepsilon_0 E^2 \) and the field is moving through area \( A \) at velocity \( c \), the energy in volume \( ACT \), \( uACT \), passes through. Hence the amount of energy flow per unit time per unit surface area \( \varepsilon = \frac{\mu_0 c^2}{2} \)

\[ I = \frac{\varepsilon_0 c}{2} E^2 \]

**EM wave: Intensity**

The intensity, \( I \), is the energy flow per unit area.

In Y&F, section 32.4, the average for a sinusoidal plane wave (squared) is worked out.

\[ I = \frac{\varepsilon_0 c}{2} \]

**EM wave; energy and intensity**

**LP Intensity Reduction**

This set of two linear polarizers produces LP light. What is the final intensity?

- First LP transmits 1/2 of the unpolarized light: \( I_1 = \frac{1}{2} I \)
- Second LP projects out the E-field component parallel to the TA:
  \[ E_2 = E_1 \cos \theta \]
  \[ I_2 = I_1 \cos^2 \theta \]

**Polarization**

Long molecules absorb E-field parallel to molecule. 

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b.) Assume isotropic: \( I = P / A; \) \( P = 4 \text{m}^2 \) \( I = 4 \pi (250m)^2 \) \( I = 3 \text{W} \)
5) What percentage of the intensity gets through both polarizers?

   a) 50%
   b) 25%
   c) 0%

6) Is it possible to increase this percentage by inserting another polarizer between the original two? Explain.

**Lecture 23, ACT 1**

• Light of intensity \( I_0 \) polarized along the \( x \) direction is incident on a set of 2 linear polarizers as shown.

   1A
   
   Assuming \( \theta = 45^\circ \), what is \( I_0 \) the intensity at the exit of the 2 polarizers, in terms of \( I_0 \)?

   (a) \( I_a = \frac{1}{2} I_0 \)
   (b) \( I_a = \frac{1}{4} I_0 \)
   (c) \( I_a = 0 \)

   1B

   • What is the relation between \( I_a \) and \( I_0 \), the final intensities in the situation above when the angle \( \theta = 45^\circ \) and \( 30^\circ \), respectively?

   (a) \( I_a < I_0 \)
   (b) \( I_a = I_0 \)
   (c) \( I_a > I_0 \)

• In general, the first polarizer reduces the intensity by \( \cos^2 \theta \) while the second polarizer reduces it by an additional factor of \( \cos^2 \theta \).

• Thus, the final output intensity is given by:

   \[ I_2 = I_0 \cos^4 \theta \cos^2 \theta \]

   This has a maximum when \( \theta = 45^\circ \).

**Yes ...**

adding an intermediate polarizer will restore some light !!!

- A polarizer at an intermediate angle can change the direction of polarization so that some light is able to get through the last filter.

**Polarization by reflection**

The reflected rays are partially polarized in the horizontal plane. The transmitted rays are also partially polarized.
Polarization by reflection

For a certain angle, the Brewster angle, the reflected light is completely polarized in the horizontal plane. This occurs when the angle between the reflection and refraction rays is 90°.

\[ \tan \theta_p = \frac{n_b}{n_a} \]

Medium b

From Maxwell’s eqn, it can be shown that Brewster’s angle is given by

Light reflected on dashboard to the windshield will be polarized in the horizontal plane. Using polaroid dark glasses with a vertical axis will remove most of reflected light.

Polarization by Scattering

• Suppose unpolarized light encounters an atom and scatters (energy absorbed & reradiated).
  • What happens to the polarization of the scattered light?
    • The scattered light is preferentially polarized perpendicular to the plane of the scattering.
      » For example, assume the incident unpolarized light is moving in the z-direction.
      » Scattered light observed along the x-direction (scattering plane = x-z) will be polarized along the y-direction.
      » Scattered light observed along the y-direction (scattering plane = y-z) will be polarized along the x-direction.

Electric field lines from oscillating dipole

full computer simulation - a snapshot in time

Dipole radiation pattern

• Oscillating electric dipole generates e-m radiation that is linearly polarized in the direction of the dipole
• Radiation pattern is doughnut shaped & outward traveling
  » zero amplitude above and below dipole
  » maximum amplitude in-plane

Applications

• Sunglasses
  • The reflection off a horizontal surface (e.g., water, the hood of a car, etc.) is strongly polarized. Which way?
  • A perpendicular polarizer can preferentially reduce this glare.

• Polarized sky
  • The same argument applies to light scattered off the sky:

Polarizing filters important in photography!
Dispersion: $n = n(\omega)$

The index of refraction depends on frequency, due to the presence of resonant transition lines. For example, ultraviolet absorption bands in glass cause a rising index of refraction in the visible, i.e., $n(\text{higher } \omega) > n(\text{lower } \omega)$:

$n_{\text{ultraviolet}} = 1.52 \quad n_{\text{blue}} = 1.53$

Rainbows

Why is the sky blue?
- Light from Sun scatters off of air particles—"Rayleigh scattering".
  - Rayleigh scattering is wavelength-dependent.
  - Shorter wavelengths (blue end of the visible spectrum) scatter more.

- This is also why sunsets are red!
  - At sunset, the light has to travel through more of the atmosphere.
  - If longer wavelengths (red and orange) scatter less...
  - The more air sunlight travels through, the redder it will appear.
  - This effect is more pronounced if there are more particles in the atmosphere (e.g., sulfur aerosols from industrial pollution).