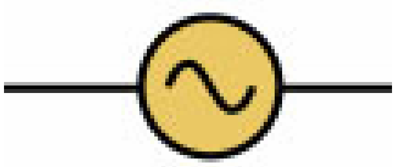
Alternating Currents (Chap 31.1)

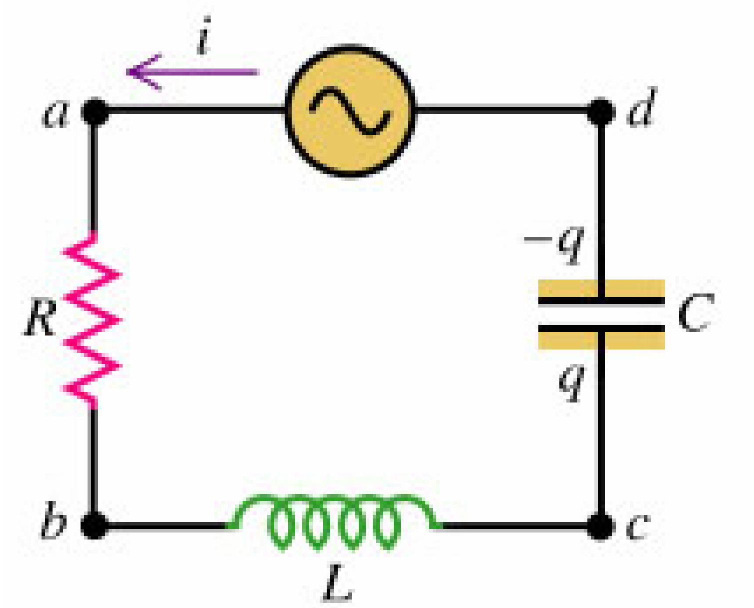
In this chapter we study circuits where the battery is replaced by a sinusoidal voltage or current source.

$$v(t) = V_0 \cos(\omega t) \quad \text{or} \quad i(t) = I_0 \cos(\omega t)$$

The circuit symbol is,



An example of an LRC circuit connected to sinusoidal source is,



Important:
I(t) is same throughout – just like DC case.

Alternating Currents (Chap 31.1)

Since the currents & voltages are sinusoidal, their values change over time and the averages are zero.

A more useful description of sinusoidal currents and voltages are given by considering the average of the square of this quantities.

We define the RMS (root mean square), which is the square root of the average of ,

$$i^2(t) = \left(I_0 \cos(\omega t)\right)^2$$

$$\left\langle i^2(t) \right\rangle = \left\langle \left(I_0 \cos(\omega t)\right)^2 \right\rangle = I_0^2 \frac{1}{2}\left(1 + \left\langle \cos(2\omega t) \right\rangle\right) = \frac{I_0^2}{2}$$

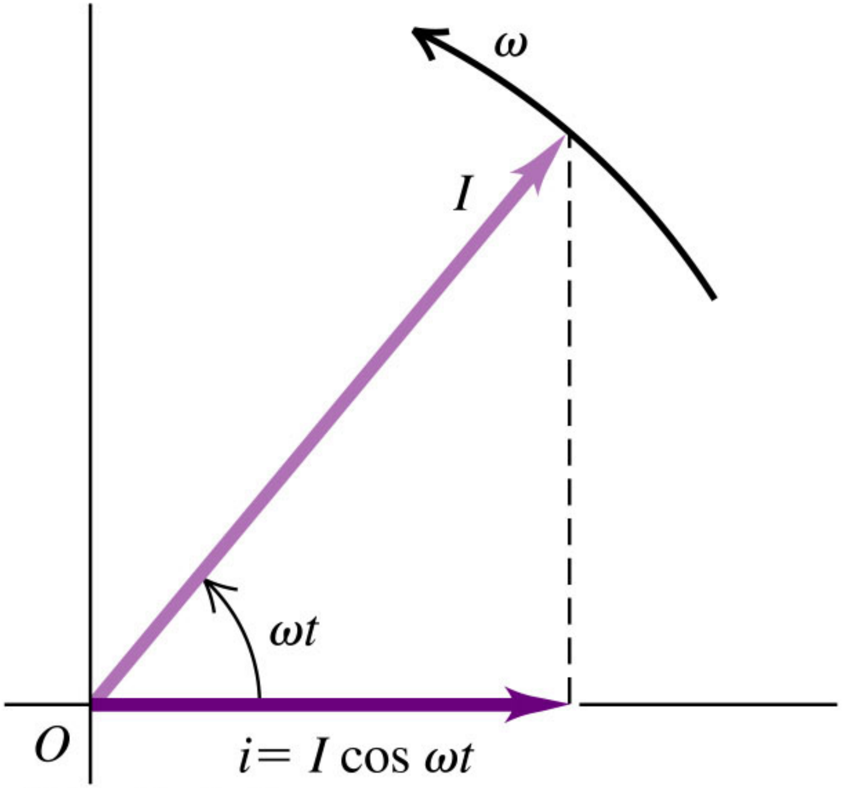
$$I_{RMS} = \sqrt{\left\langle i^2(t) \right\rangle} = \frac{I_0}{\sqrt{2}}$$

Alternating Currents ; Phasors

A convenient method to describe currents and voltages in AC circuits is "Phasors". Since currents and voltages in circuits with capacitors & inductors have different phase relations, we introduce a phasor diagram. For a current, $i = I\cos(\omega t)$

We can represented this by a vector rotating about the origin. The angle of the vector is given by ωt and the magnitude of the current is its projection on the X-axis.

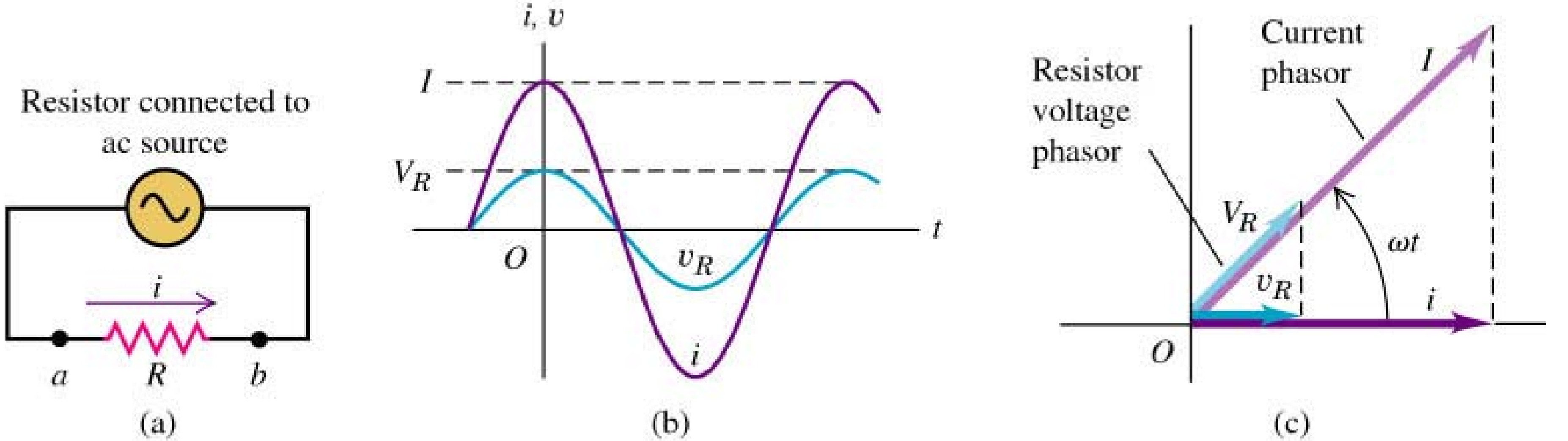
If we plot simultaneously currents & voltages of different components we can display different phases .



Note this method is equivalent to imaginary numbers approach where we take the real part (x-axis projection) for the magnitude

“Beam me up Scotty – It ate my phasor!”

Alternating Currents: Resistor in AC circuit



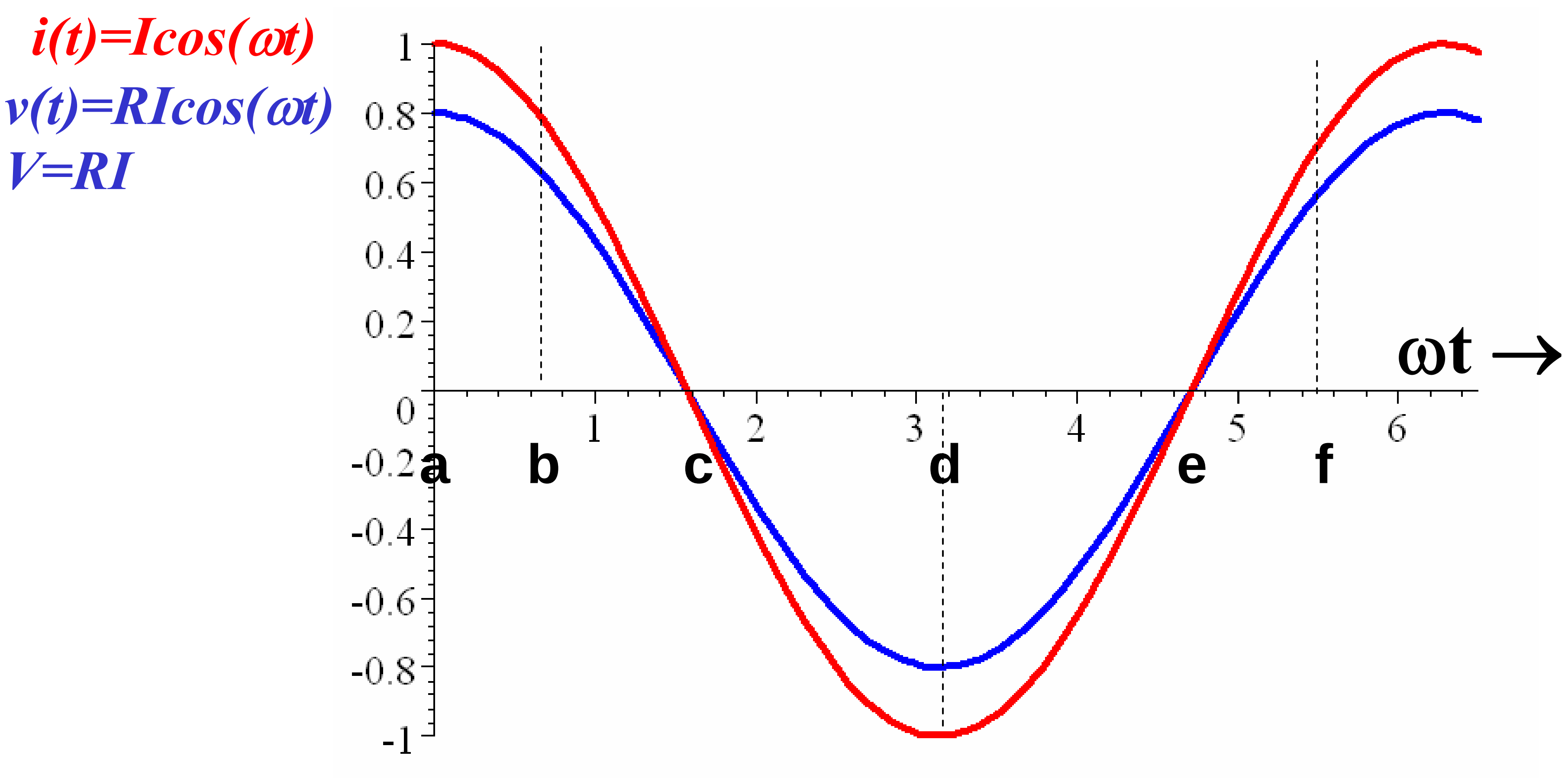
A resistor connected to an AC source will have the voltage, v_R, and the current across the resistor has the same phase. We can draw the current phasor and the voltage phasor with the same angle.

$$v_R = V_R \cos(\omega t) = iR = I\cos(\omega t)R$$

and $V_R = IR$ (just like DC case)

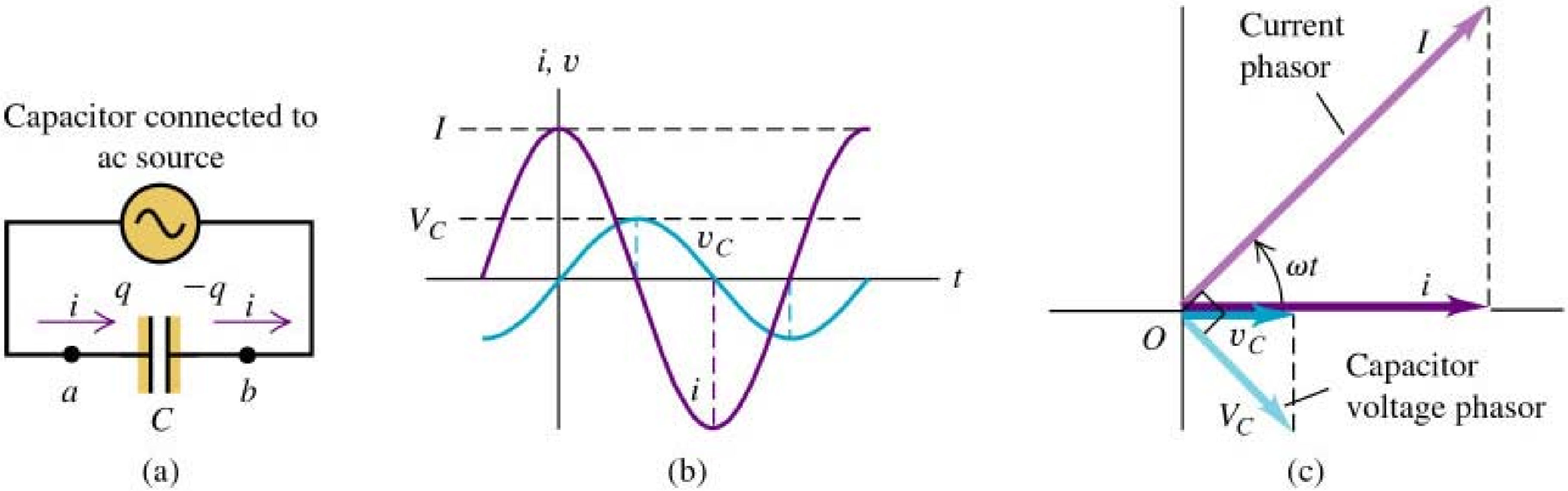
Phasors are rotating 2 dimensional vectors

Resistor in AC circuit; i & v versus ωt



Note: Voltage is in phase with current

Alternating Currents: Capacitor in AC circuit



A capacitor connected to an AC current source will have the voltage lagging behind the current by 90 degree. We can draw the current phasor and the voltage phasor <u>behind</u> the current by 90 degrees.

$$i = \frac{dq}{dt} = I\cos(\omega t)$$

Find voltage: $v = \frac{q}{C} = \frac{1}{C}\int i dt = \frac{I}{\omega C}\sin(\omega t) = V_{MAX}\sin(\omega t)$ $\quad q = \frac{I}{\omega}\sin(\omega t)$

$$V_{MAX} = I\left(\frac{1}{\omega C}\right)$$

Alternating Currents ; Capacitor in AC circuit

We define the capacitive reactance, X_C, as $$X_C = \frac{1}{\omega C}$$

$$V_{cap}^{MAX} = \frac{I}{\omega C} = I\left(\frac{1}{\omega C}\right) = I\ X_C$$

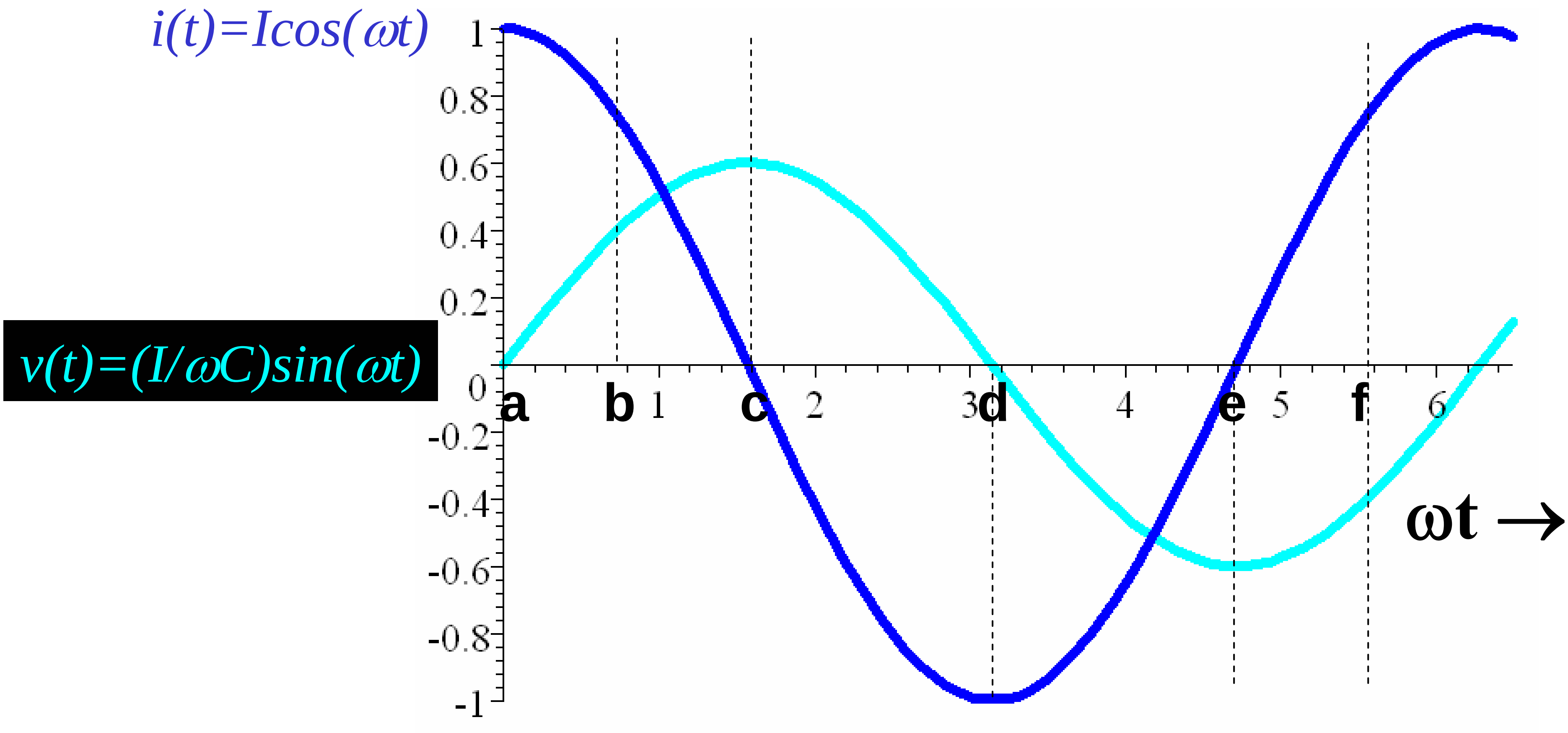
Like: $V_R = IR$

We stated that voltage lags by 90 deg., so equivalent solution is

$$v = \frac{I}{\omega C}\cos(\omega t - 90) = \frac{I}{\omega C}[\cos\omega t \cos 90 + \sin\omega t \sin 90]$$

$$= \frac{I}{\omega C}\sin\omega t$$

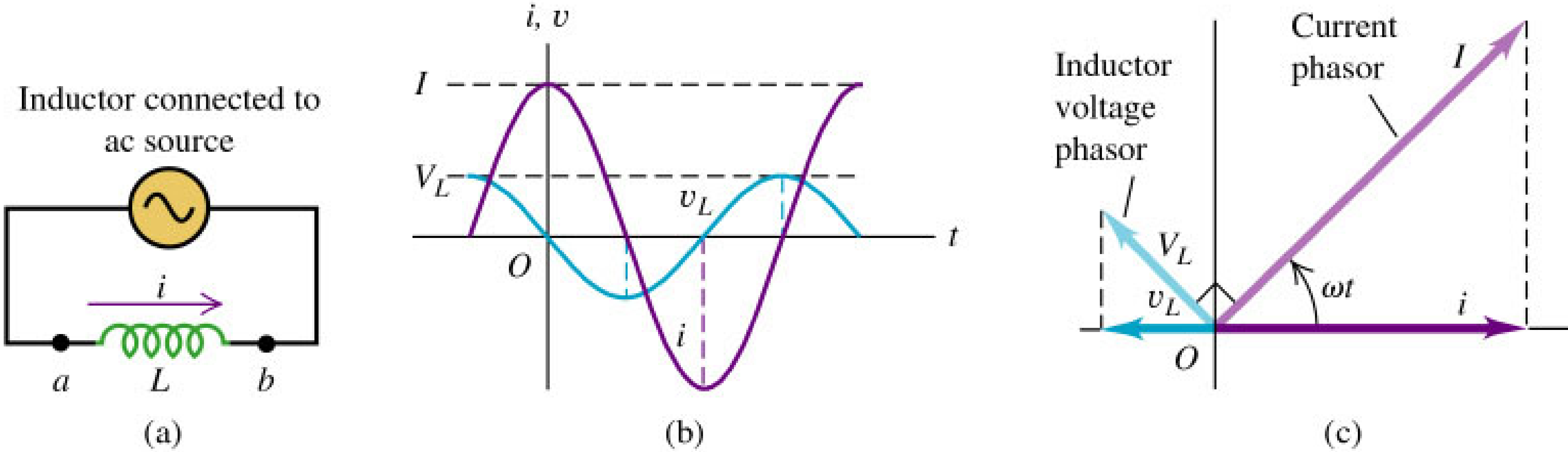
Capacitor in AC circuit; i & v versus ωt



Note voltage lags 90 deg. Behind current

$$V(t)=(I/\omega C)\sin(\omega t)= (I/\omega C)\cos(\omega t-\pi/2)$$

Alternating Currents: Inductor in AC circuit



An inductor connected to an AC current source will have the voltage leading ahead the current by 90 degree. We can draw the current phasor and the voltage phasor <u>ahead</u> the current by 90 degrees.

$$i = I\cos(\omega t) \quad \text{and} \quad V = L\frac{di}{dt} = -IL\omega\sin(\omega t) = -V_{MAX}\sin(\omega t)$$

$$V_{MAX} = IL\omega$$

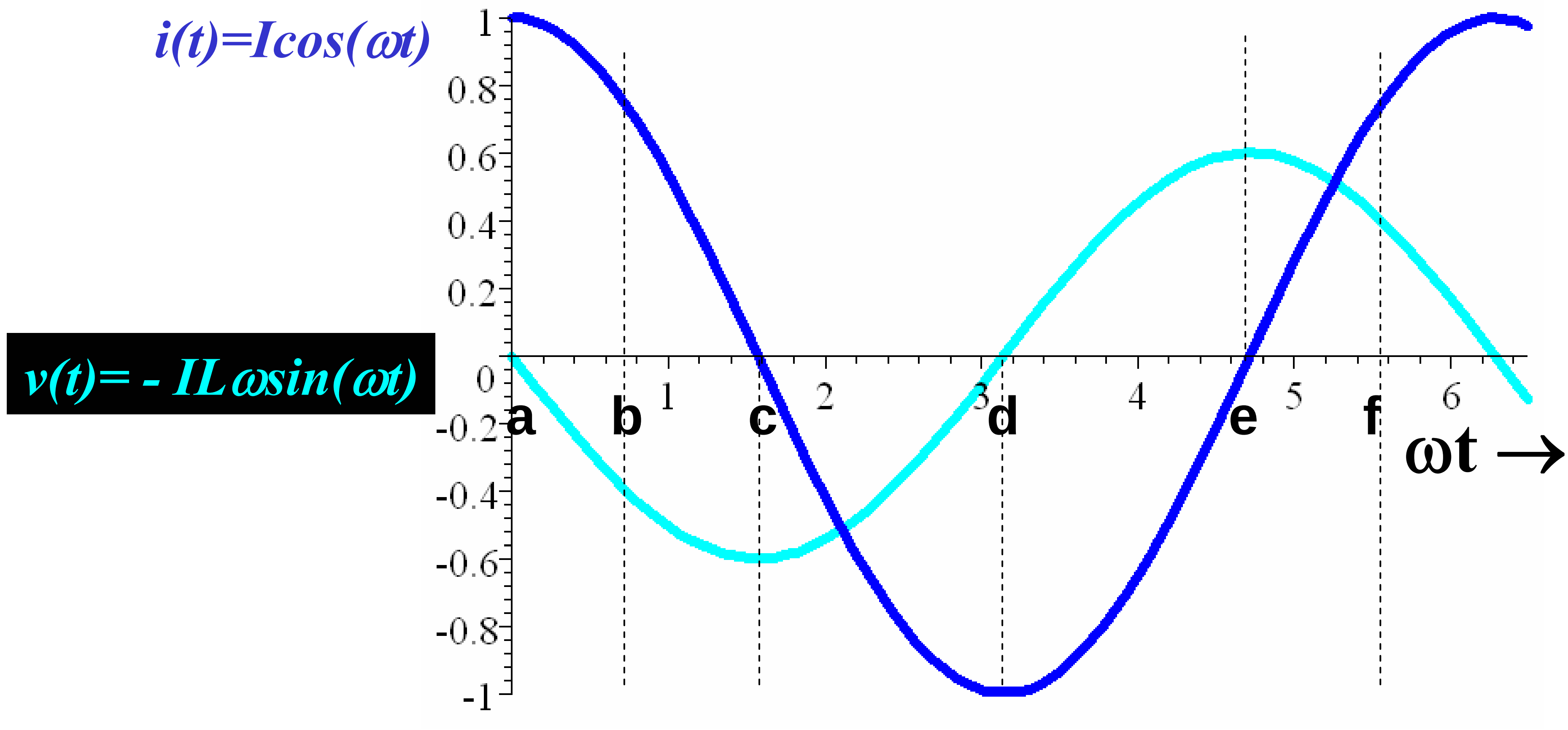
$$\cos(\omega t + 90) = -\sin(\omega t)$$

Define inductive reactance, X_L, as $X_L = \omega L$

$$V_{ind}^{MAX} = I\omega L = I(\omega L) = I\ X_L$$

Like: $V_R = IR$

Inductor in AC circuit; i & v versus ωt



Draw phasor diagram for each point
Note voltage leads 90 deg. ahead current

$$v(t) = -IL\omega\sin(\omega t) = IL\omega\cos(\omega t + \pi/2)$$

What is *reactance*?

You can think of it as a frequency-dependent resistance.

$$X_C = \frac{1}{\omega C}$$

For high ω, $\chi_C \sim 0$

- Capacitor looks like a wire ("short")

For low ω, $\chi_C \rightarrow \infty$

- Capacitor looks like a break

For low ω, $\chi_L \sim 0$

- Inductor looks like a wire ("short")

$$X_L = \omega L$$

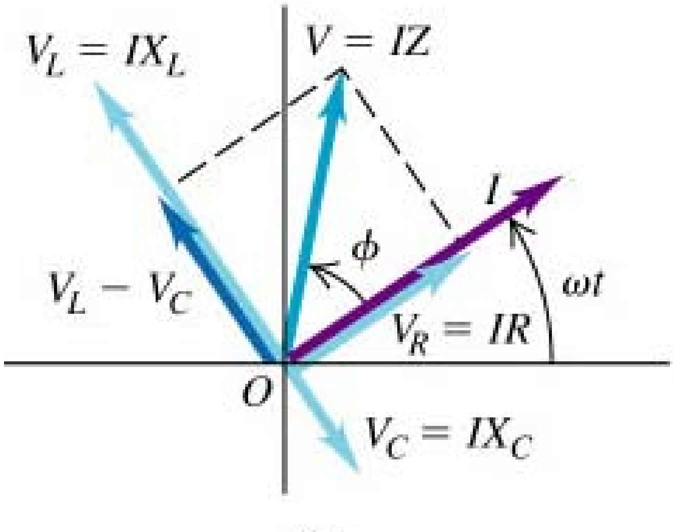
For high ω, $\chi_L \rightarrow \infty$

- Inductor looks like a break

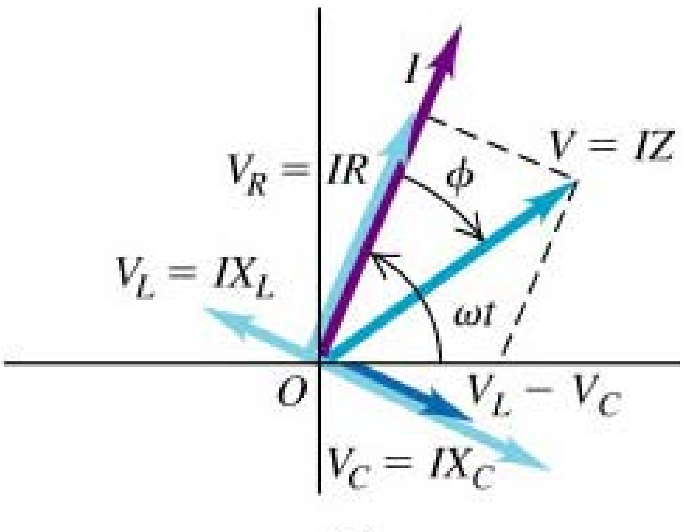
(inductors resist change in current)

$$(\text{"}X_R\text{"} = R)$$

Alternating Currents: LRC circuit



(b)



(c)

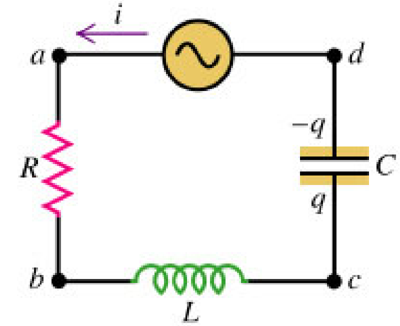


Figure (b) has $X_L > X_C$ and (c) has $X_L < X_C$.

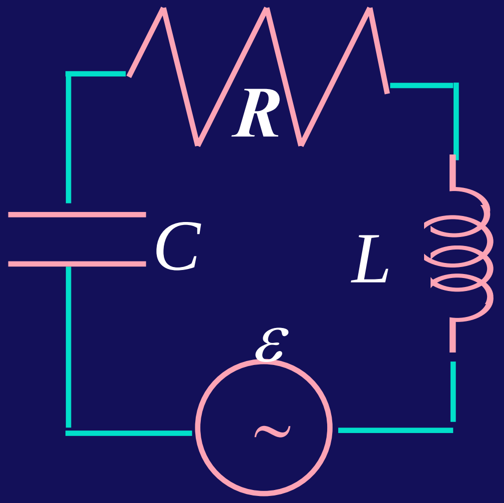
Using Phasors, we can construct the phasor diagram for an LRC Circuit. This is similar to 2-D vector addition. We add the phasors of the resistor, the inductor, and the capacitor. The inductor phasor is +90 and the capacitor phasor is -90 relative to the resistor phasor.

Adding the three phasors vectorially, yields the voltage sum of the resistor, inductor, and capacitor, which must be the same as the voltage of the AC source. Kirchoff's voltage law holds for AC circuits.

Also V_R and I are in phase.

Phasors

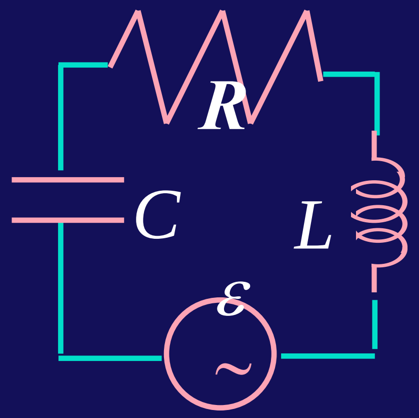
Problem: Given $i(t) = I\cos(\omega t)$,
find V_R, V_L, V_C, I_R, I_L, I_C



Strategy:

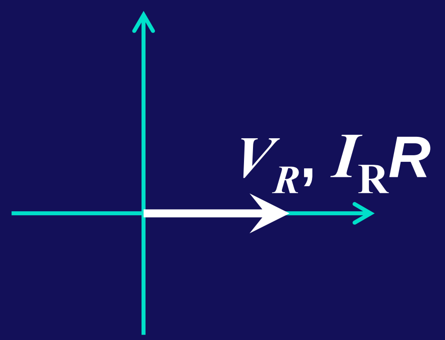
We will use Kirchhoff's voltage law that the (phasor) sum of the voltages V_R, V_C, and V_L must equal ε.

Phasors, cont.

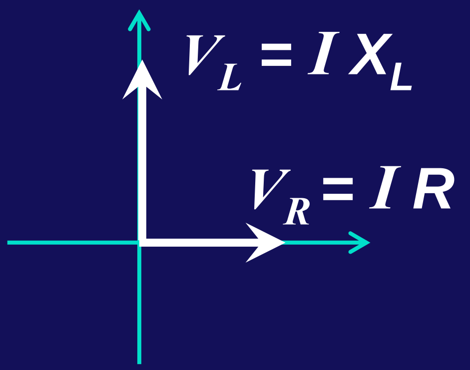


Problem: Given $i(t) = I\cos(\omega t)$,
find V_R, V_L, V_C, I_R, I_L, I_C

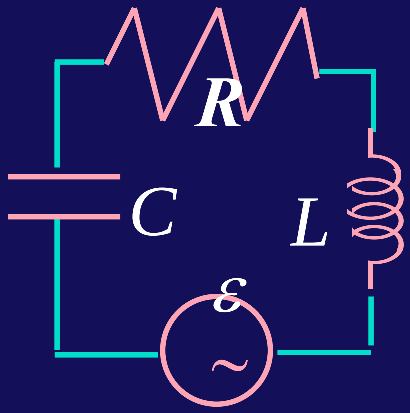
1. Draw V_R phasor along x-axis (this direction is chosen for convenience). Note that since $V_R = I_R R$, this is also the direction of the current phasor i_R. Because of Kirchhoff's current law, $I_L = I_C = I_R \equiv I$ (i.e., the same current flows through each element).



2. Next draw the phasor for V_L. Draw V_L 90° further *counterclockwise*. The length of the V_L phasor is $I_L X_L = I\omega L$

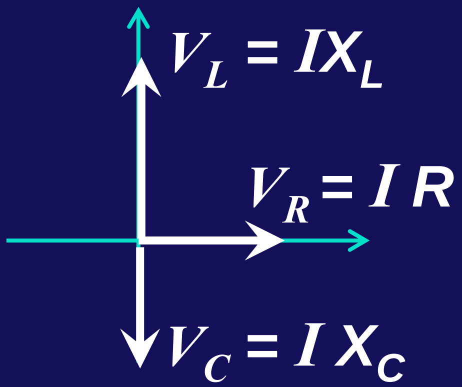


Phasors, cont.



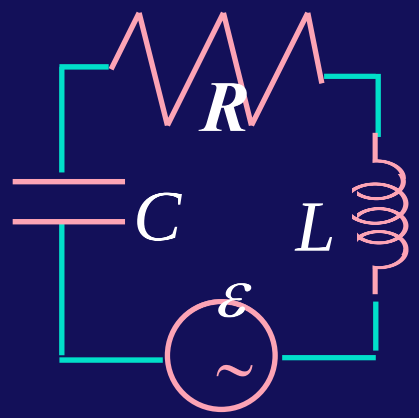
Problem: Given i(t) = $I\cos(\omega t)$,
find V_R, V_L, V_C, I_R, I_L, I_C

3. Draw V_C 90° further clockwise. The length of the V_C phasor is $I_C X_C = I/\omega C$

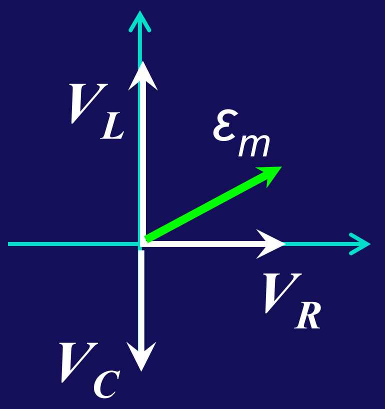


The lengths of the phasors depend on R, L, C, and ω.
The relative orientation of the V_R, V_L, and V_C phasors is ***always*** the way we have drawn it. Memorize it!

Phasors, cont.



Problem: Given $i(t) = I\cos(\omega t)$,
find $V_R, V_L, V_C, I_R, I_L, I_C$



- The phasors for V_R, V_L, and V_C are added like *vectors* to give the drive voltage $V_R + V_L + V_C = \varepsilon_m$:
  - From this diagram we can now easily calculate quantities of interest, like the net current I , the maximum voltage across any of the elements, and the *phase* between the current the drive voltage (and thus the power).

Voltage *V(t)* across AC source

$$V_R = IR$$
$$V_L = IX_L$$
$$V_C = IX_C$$

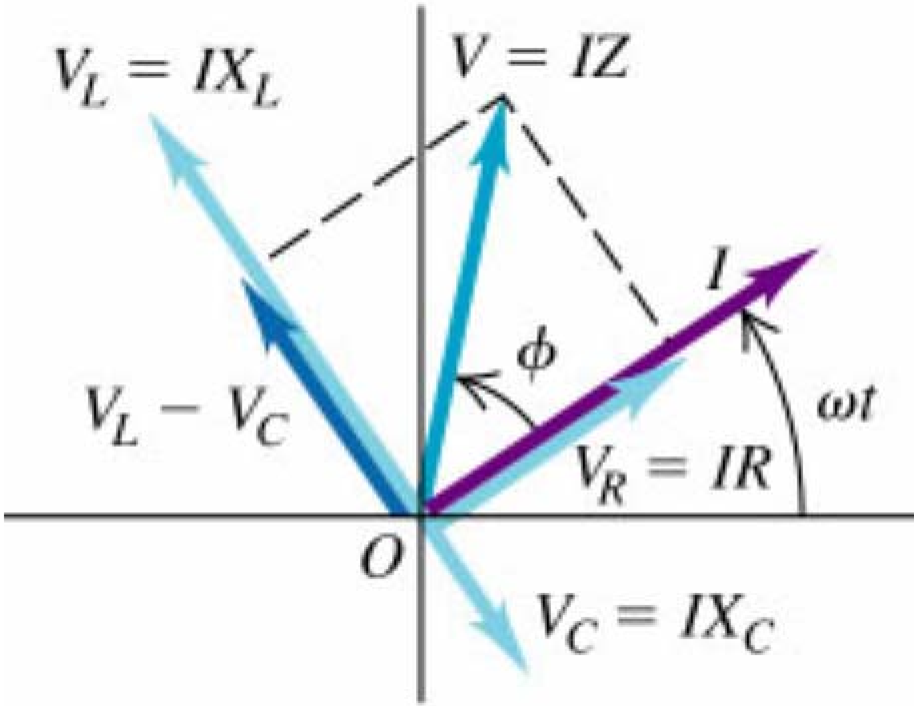
$$v(t) = \sqrt{(V_R)^2 + (V_L - V_C)^2}\cos(\omega t + \phi)$$
$$= \sqrt{(IR)^2 + (IX_L - IX_C)^2}\cos(\omega t + \phi)$$
$$= I\sqrt{(R)^2 + (X_L - X_C)^2}\cos(\omega t + \phi) = IZ\cos(\omega t + \phi)$$
$$Z = \sqrt{(R)^2 + (X_L - X_C)^2}$$ Z is called "impedance"

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{\omega L - 1/\omega C}{R}$$

Also:

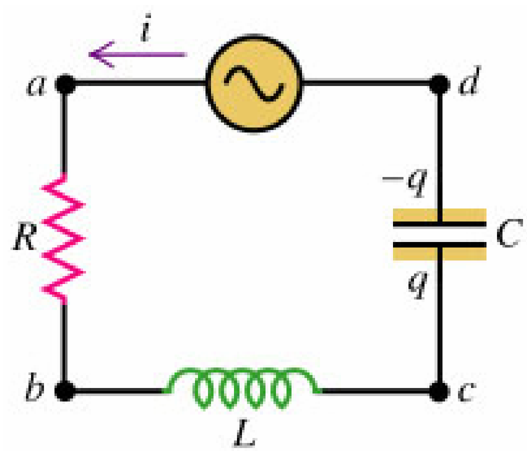
$$V = \varepsilon_m = V^{MAX} = I\ Z$$ Like: $V_R = IR$

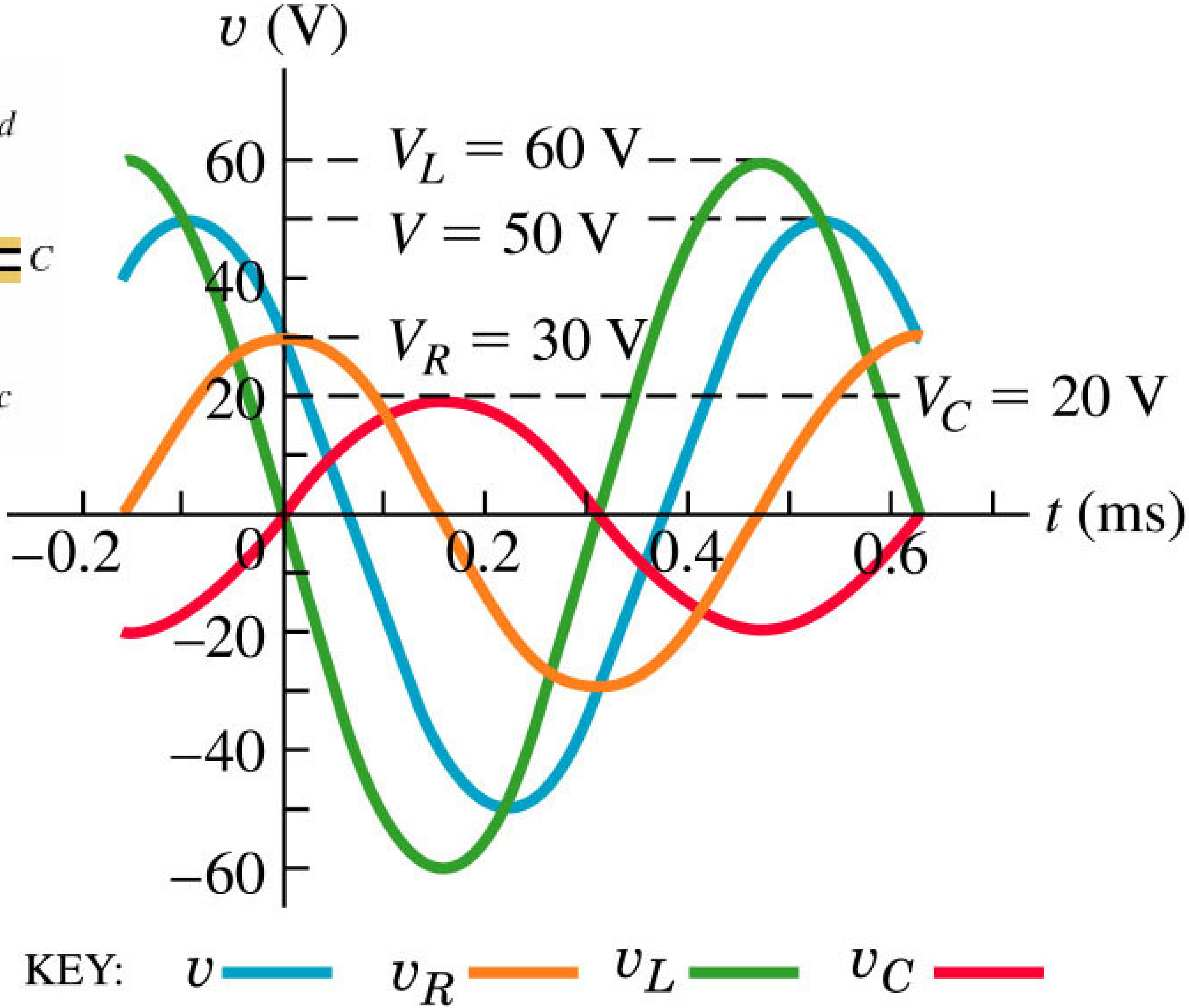
$$V_{rms} = I_{rms} Z$$
$$i = I\cos\omega t$$
$$v = V\cos(\omega t + \phi)$$



Alternating Currents: LRC circuit, Fig. 31.15

Y&F Example 31.5

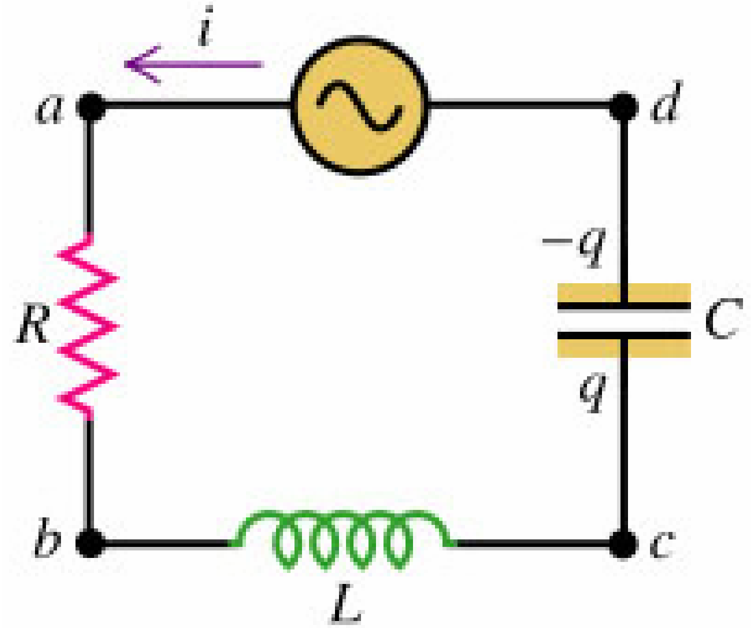




V=50v
ω=10000rad/s
R=300ohm
L=60mH
C=0.5μC

LRC series circuit; Summary of instantaneous Current and voltages

$$V_R = IR$$
$$V_L = IX_L$$
$$V_C = IX_C$$



$$i(t) = I\cos(\omega t)$$
$$v_R(t) = IR\cos(\omega t)$$
$$v_C(t) = IX_C\cos(\omega t - 90) = I\frac{1}{\omega C}\cos(\omega t - 90)$$
$$v_L(t) = IX_L\cos(\omega t + 90) = I\omega L\cos(\omega t + 90)$$
$$v_{ad}(t) = I\sqrt{(X_R)^2 + (X_L - X_C)^2}\cos(\omega t + \phi)$$
$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{\omega L - 1/\omega C}{R}$$