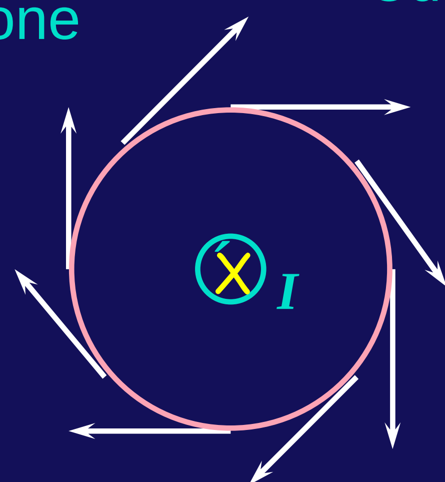
Ampere's Law

"High symmetry"

$$\oint \vec{B} \bullet d\vec{l} = \mu_0 I$$

Integral around a path …
hopefully a simple one

Current "enclosed" by that path



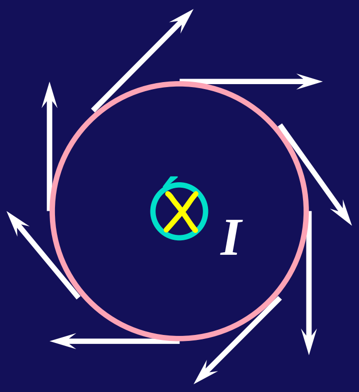
Calculation of Magnetic Field

- Two Ways to calculate
  - – Biot-Savart Law ("Brute force")

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2}$$

  - – Ampere's Law ("High symmetry")

$$\oint \vec{B} \bullet d\vec{l} = \mu_0 I$$



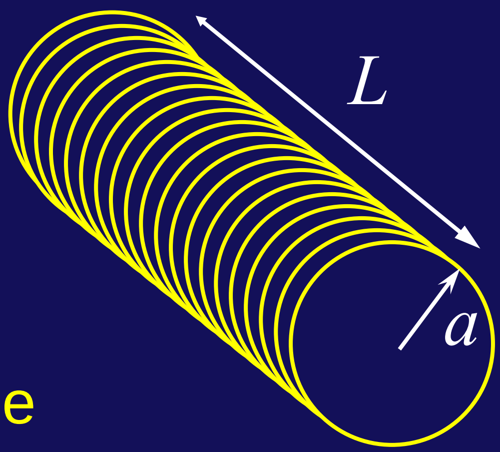
–AMPERIAN SURFACE/LOOP

These are the analogous equations

B Field of an ideal Solenoid

- A nearly constant magnetic field is often produced by a solenoid.

- A solenoid is defined by a current i flowing through a wire that is wrapped n *turns per unit length* on a cylinder of radius a and length L.



- To correctly calculate the B-field, we should use Biot-Savart, and add up the field from the different loops.

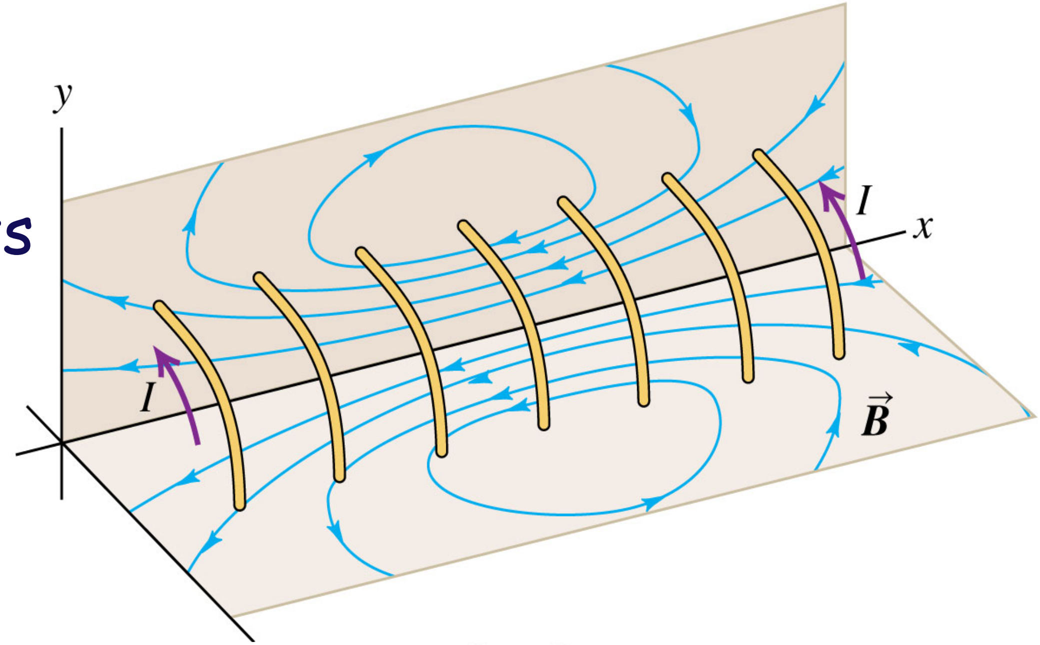
- If $a << L$, the B field is to first order contained within the solenoid, in the axial direction, and of constant magnitude. In this limit, we can calculate the field using Ampere's Law.

Ideal Solenoid

B Field of an ideal Solenoid

B field of real solenoid:

As coil gets longer (l >> a) and turns/length gets greater, field outside gets smaller, and field inside becomes more uniform and stronger.



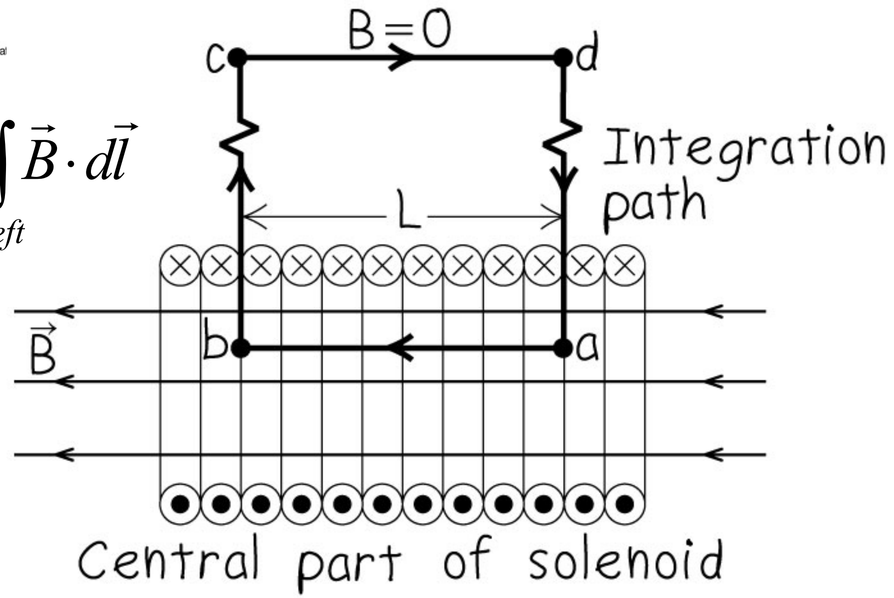
B field of ideal solenoid:

$$\int \vec{B}\cdot d\vec{l} = \int_{top} \vec{B}\cdot d\vec{l} + \int_{right} \vec{B}\cdot d\vec{l} + \int_{bottom} \vec{B}\cdot d\vec{l} + \int_{left} \vec{B}\cdot d\vec{l}$$

$$\int \vec{B}\cdot d\vec{l} = 0 + 0 + \int B dl \cos(0) + 0$$

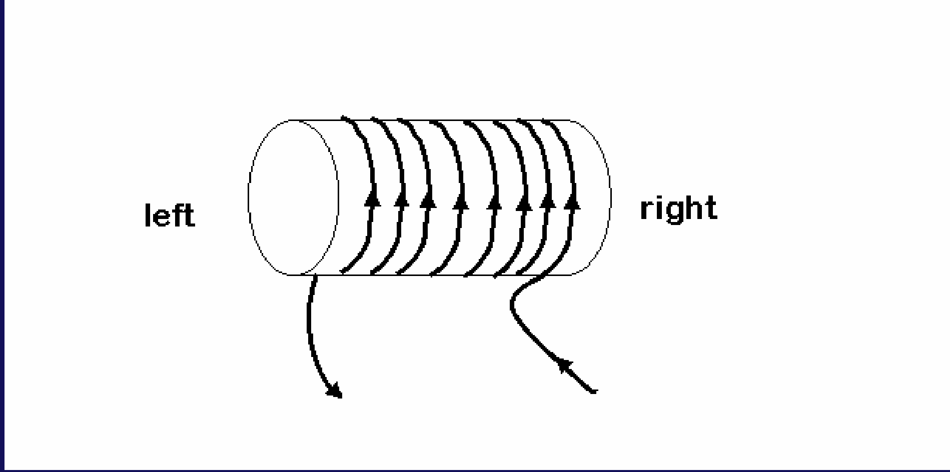
$$= B\int dl = BL = \mu_0 I_{enc} = \mu_0 nLI$$

$$\therefore\ B = \mu_0 nI$$



Preflight 15:

A current carrying wire is wrapped around an iron core, forming an electro-magnet.



6) Which direction does the magnetic field point inside the iron core?

a) left   b) right   c) up   d) down

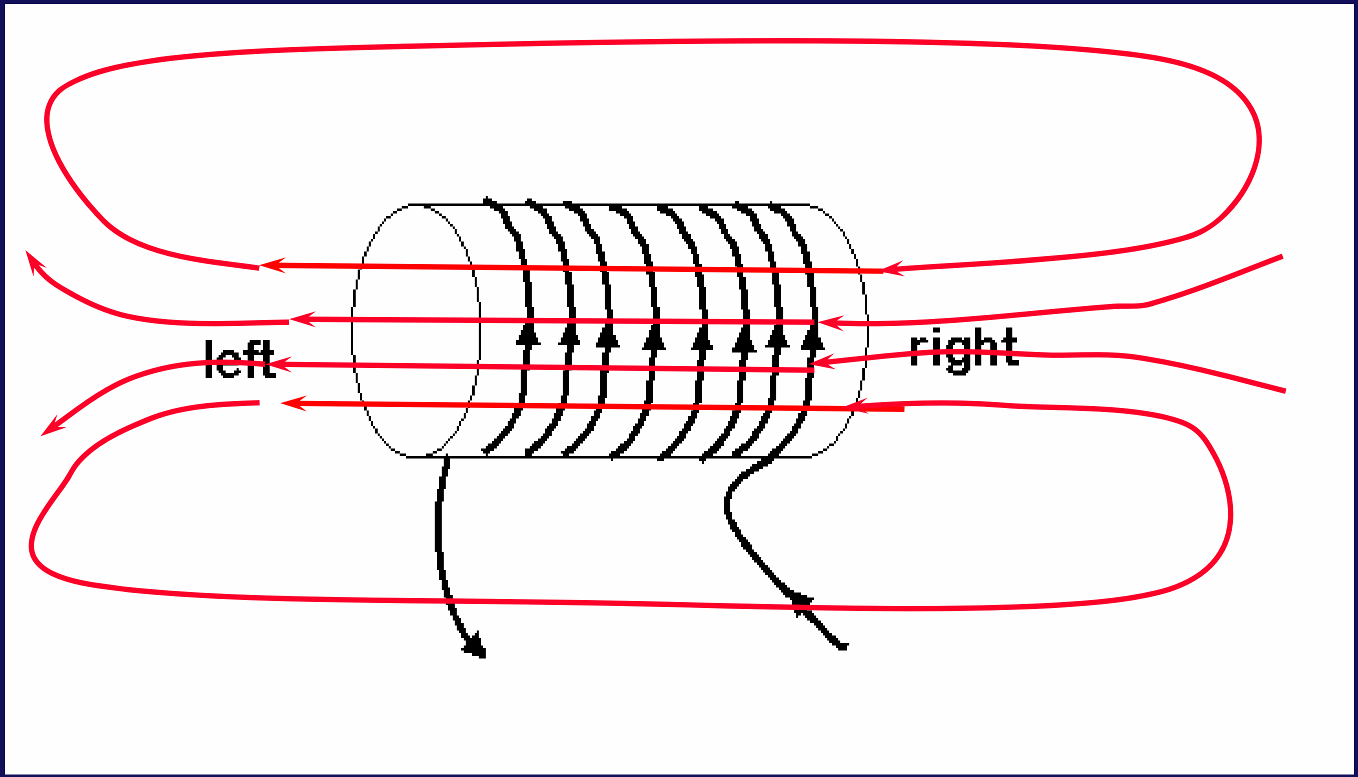
e) out of the screen   f) into the screen

7) Which side of the solenoid should be labeled as the magnetic north pole?

a) right

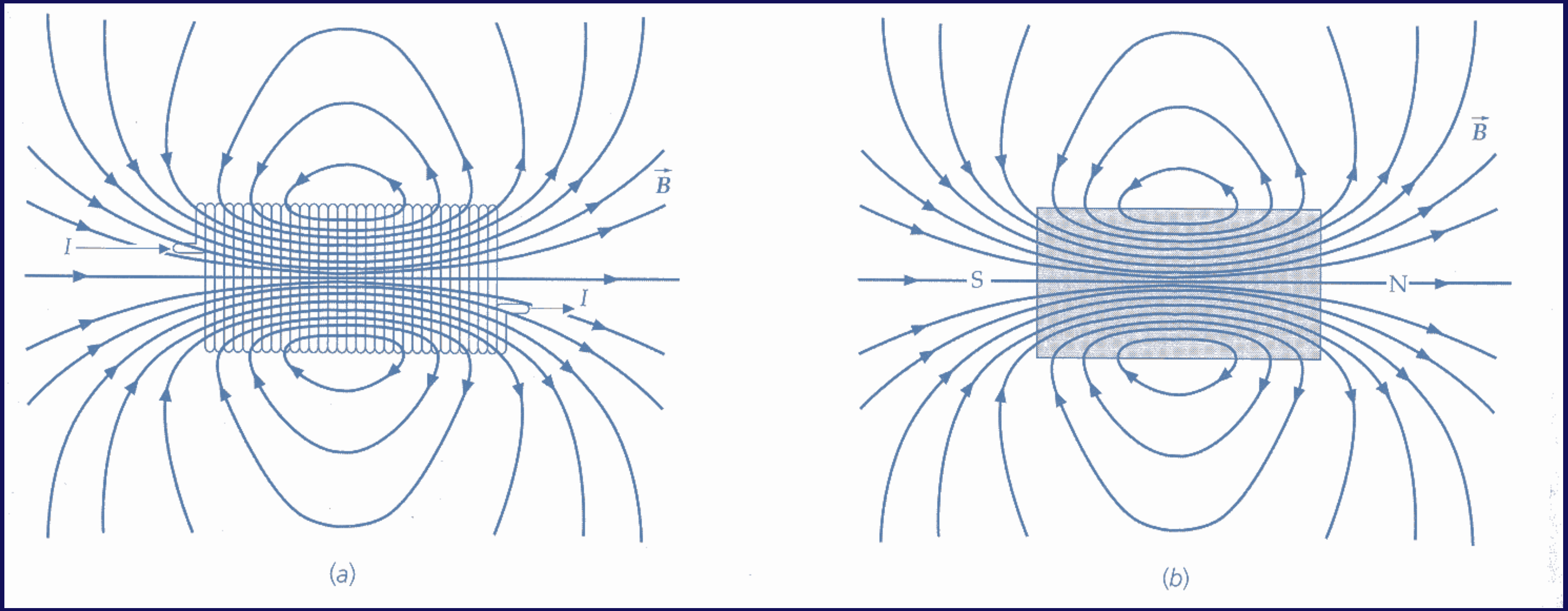
b) left

Use the wrap rule to find the B-field: wrap your fingers in the direction of the current, the B field points in the direction of the thumb (to the left). Since the field lines leave the left end of solenoid, the left end is the north pole.



Solenoids

The magnetic field of a solenoid is essentially identical to that of a bar magnet.

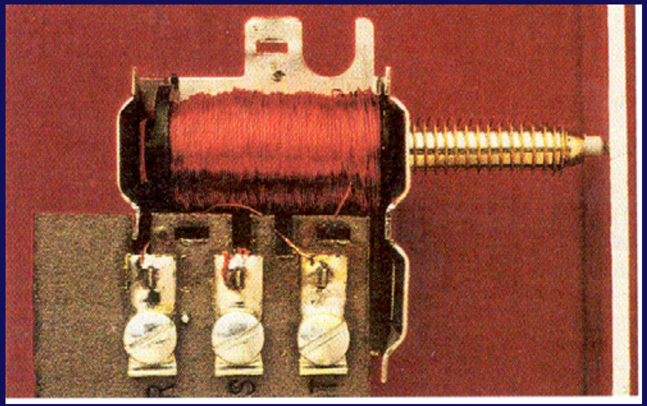


The big difference is that we can turn the solenoid *on* and ***off*** ! It attracts/repels other permanent magnets; it attracts ferromagnets, etc.

Solenoid Applications

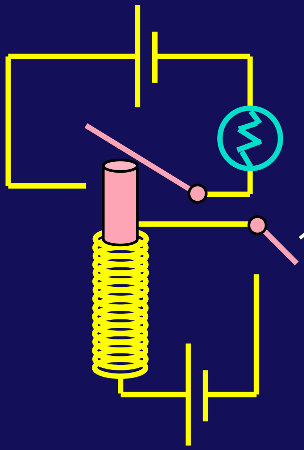
Digital [on/off]:

- Doorbells



Magnet off → plunger held in place by spring
Magnet on → plunger expelled → strikes bell

- Power door locks
- Magnetic cranes
- Electronic Switch "relay"



Close switch
→ current
→ magnetic field pulls in plunger
→ closes larger circuit

Advantage:

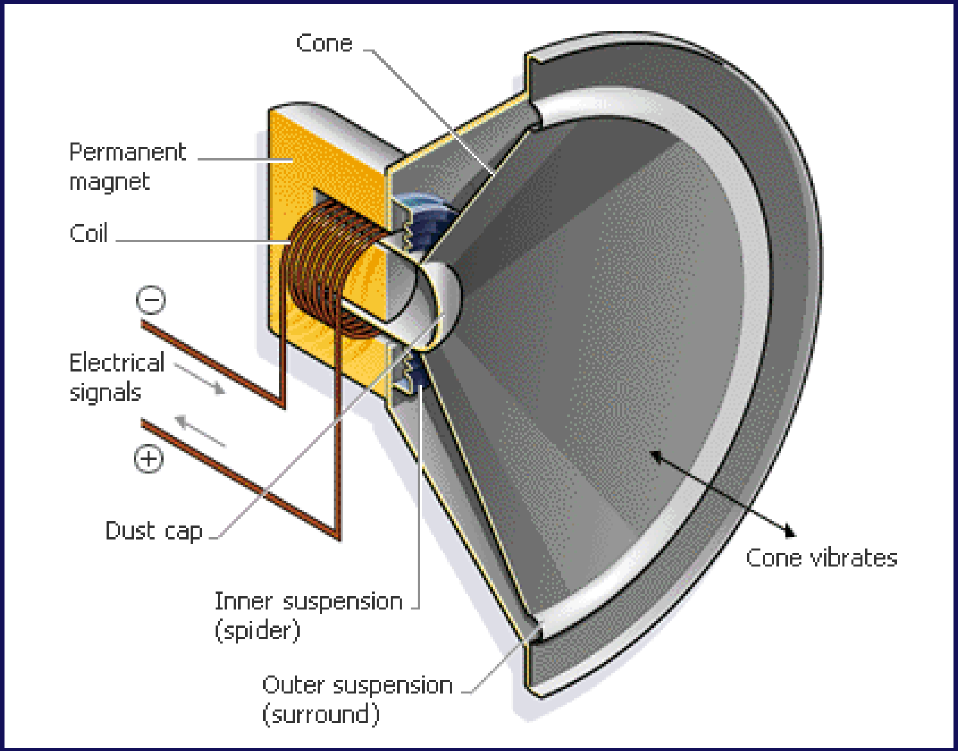
A small current can be used to switch a much larger one

– Starter in washer/dryer, car ignition, …

Solenoid Applications

Analog (deflection $\propto$ ***I***):

- Variable A/C valves
- Speakers

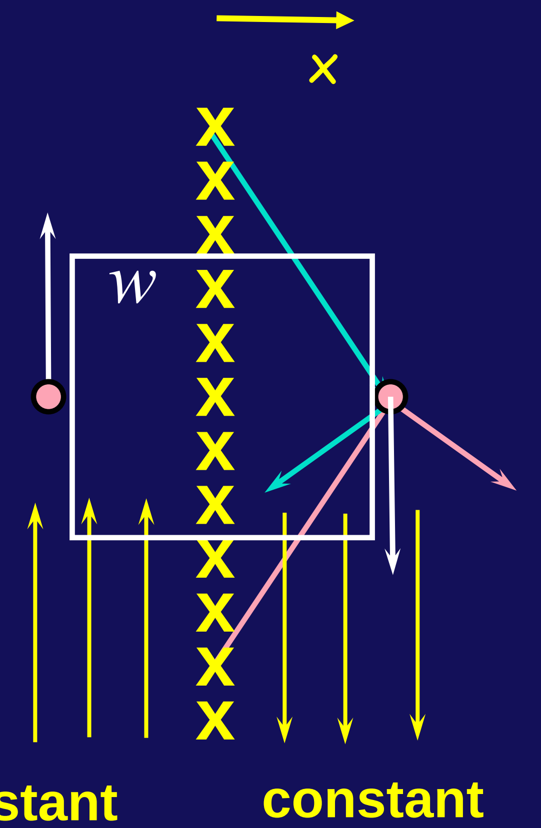


Solenoids are everywhere!

In fact, a typical car has over 20 solenoids!

B Field of ∞ Current Sheet

- Consider an ∞ sheet of current described by n wires/length each carrying current i into the screen as shown. Calculate the B field.



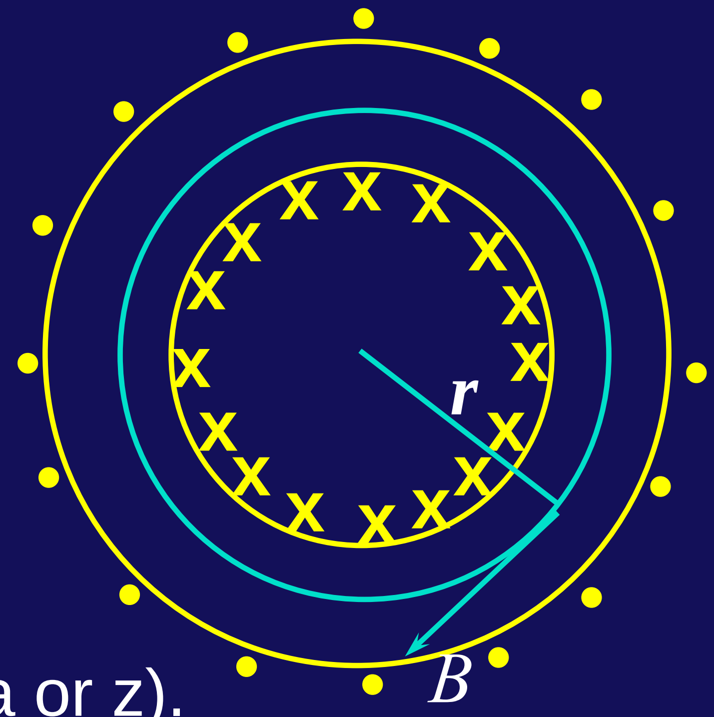
- What is the direction of the field?
    - Symmetry ⇒ vertical direction
- What variables does B depend on? *Maybe x?*

- Calculate using Ampere's law for a square of side w centered on sheet:

- $$\oint \vec{B} \bullet d\vec{l} = Bw + 0 + Bw + 0 = 2Bw$$
- $$I = nwi$$

therefore, $\oint \vec{B} \bullet d\vec{l} = \mu_0 I \Rightarrow$ $$B = \frac{\mu_0 ni}{2}$$

Toroid



- Toroid defined by N *total* turns with current i.
- $B=0$ outside toroid! (Consider integrating B on circle outside toroid)
  - Direction? tangent to circle.
  - Magnitude depends on? r (not theta or z).
  - To find *B* inside, consider circle of radius *r*, centered at the center of the toroid.

$$\oint \vec{B} \bullet d\vec{l} = B(2\pi r) \qquad I = Ni$$

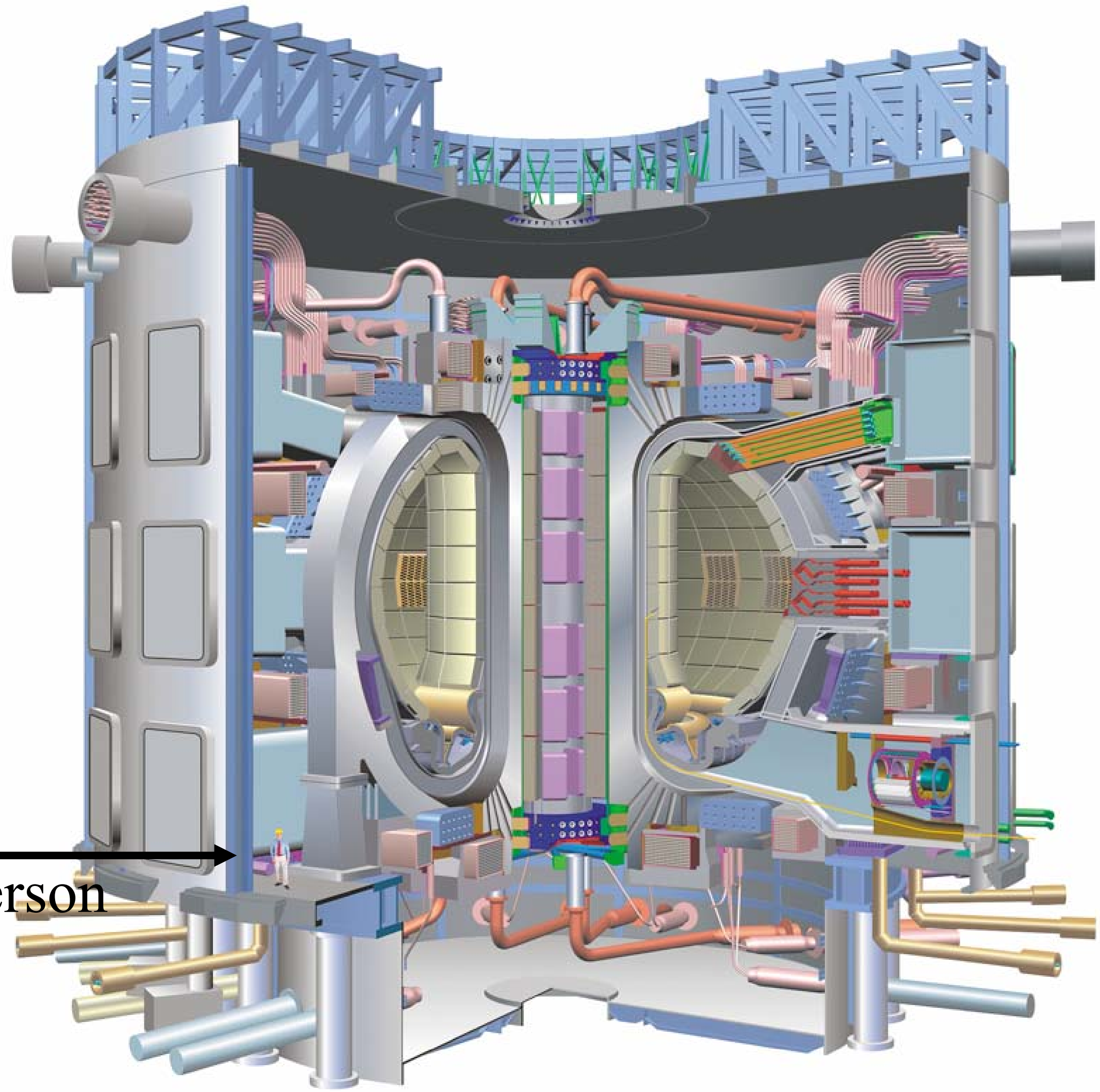
Apply Ampere's Law: $\oint \vec{B} \bullet d\vec{l} = \mu_0 I \Rightarrow$

$$B = \frac{\mu_0 N i}{2\pi r}$$

ITER Tokamak; giant toroid for fusion

A joint US-Europe-Japan project to be built in southern France by 2019. A toroidal field contains the hot plasma. Fusion should provide clean power. ITER is prototype for future machines and is supposed to produce 500MW of power. Construction began in 2007. Many delays and budget overruns. Building costs now $50 B or 10 X original estimate.

$${}^{2}_{1}D + {}^{3}_{1}T \rightarrow {}^{4}_{2}He + {}^{1}_{0}n + 17.6\,MeV$$



Summary

Example B-field Calculations:

- **Inside a Long Straight Wire** $B = \frac{\mu_0 I}{2\pi} \frac{r}{a^2}$
- **Infinite Current Sheet** $B = \frac{\mu_0 n i}{2}$
- **Solenoid** $B = \mu_0 n i$
- **Toroid** $B = \frac{\mu_0 N i}{2\pi r}$
- **Circular Loop** $B_z \approx \frac{\mu_0 i R^2}{2z^3}$