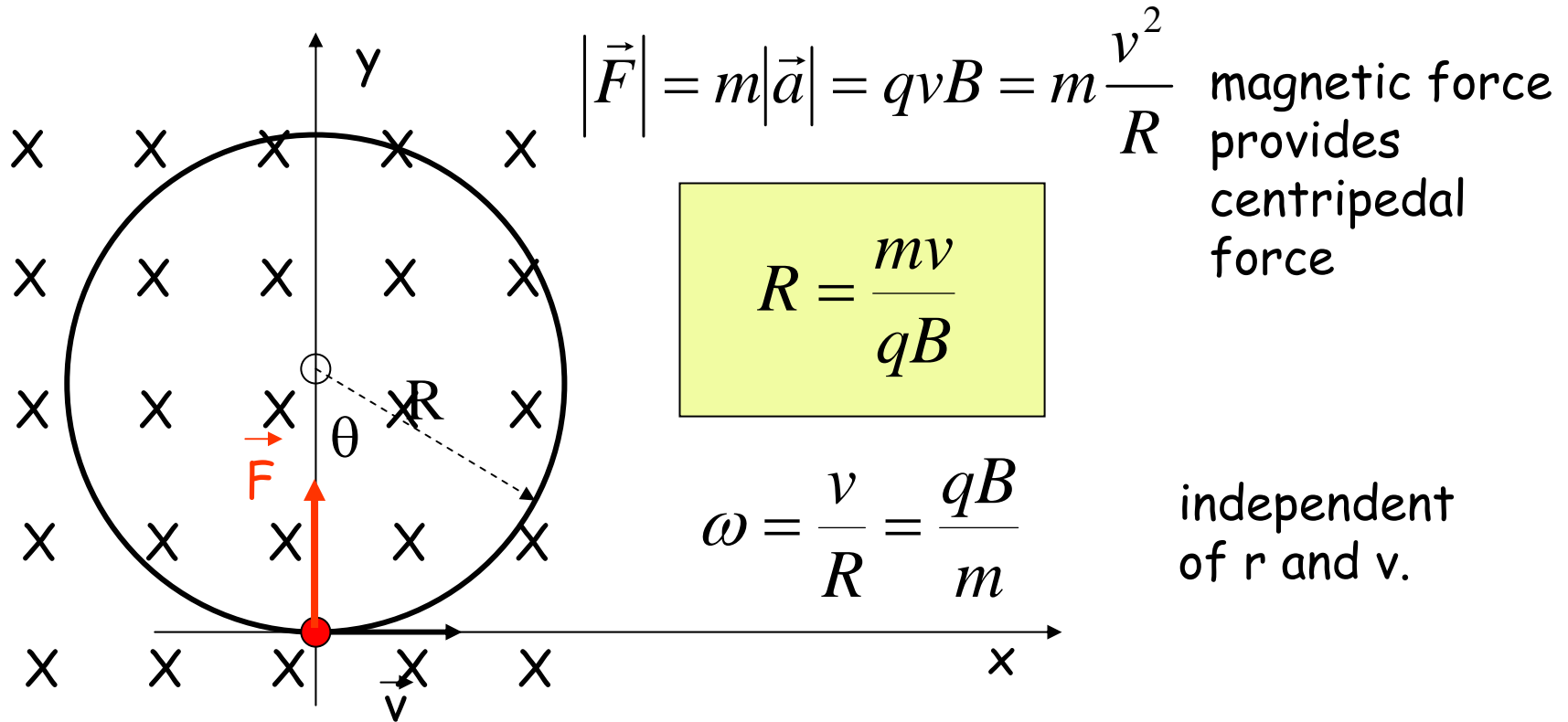


Charged particle motion in a constant B field

- velocity in plane \perp to B.

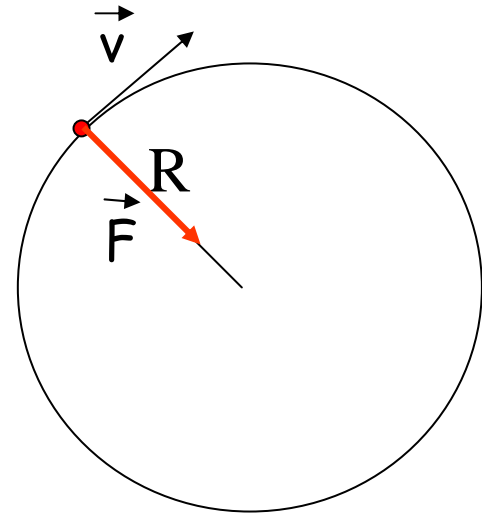
Suppose we have a magnetic field given by $\vec{B} = -B_0 \hat{z}$ and a particle starts out at the origin moving in the +x direction. The particle will move in a circle with the radius R and angular velocity ω :



Review of Circular Motion

If the force F is perpendicular to the velocity, the particle experiences circular motion:

$$|\vec{F}| = m|\vec{a}| = \frac{mv^2}{R}$$



Force F is always perpendicular to velocity.
 F does no work.

Problem:

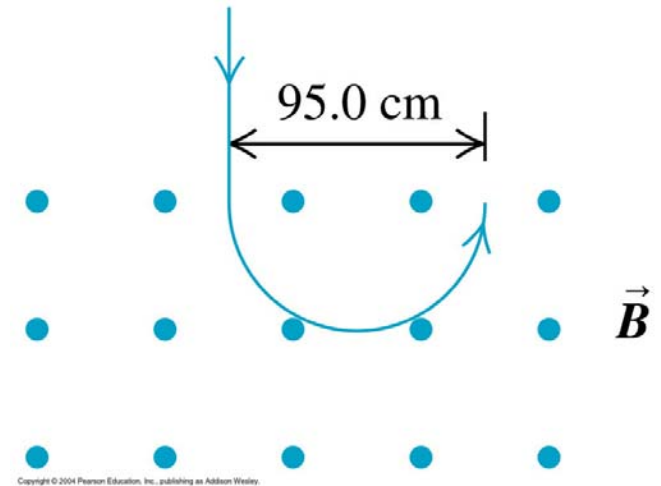
In an experiment with cosmic rays, a vertical beam of particles that have a charge magnitude of $3e$ and a mass 12 times the proton mass enters a uniform horizontal Magnetic field of 0.250T and is bent in a semicircle of 95cm . Find the speed of the particle. Is charge positive or negative?

$$R = \frac{mv}{qB}$$

$$v = \frac{q}{m} BR$$

$$v = \frac{3(1.6 \times 10^{-19} \text{C})}{12(1.67 \times 10^{-27} \text{Kg})} (0.25\text{T})(0.475\text{m})$$

$$v = 2.84 \times 10^6 \text{ m/s}$$



Motion in a constant magnetic field

If the velocity of the charge particle is perpendicular to the B field the motion is circle with radius $R=mv/qB$.

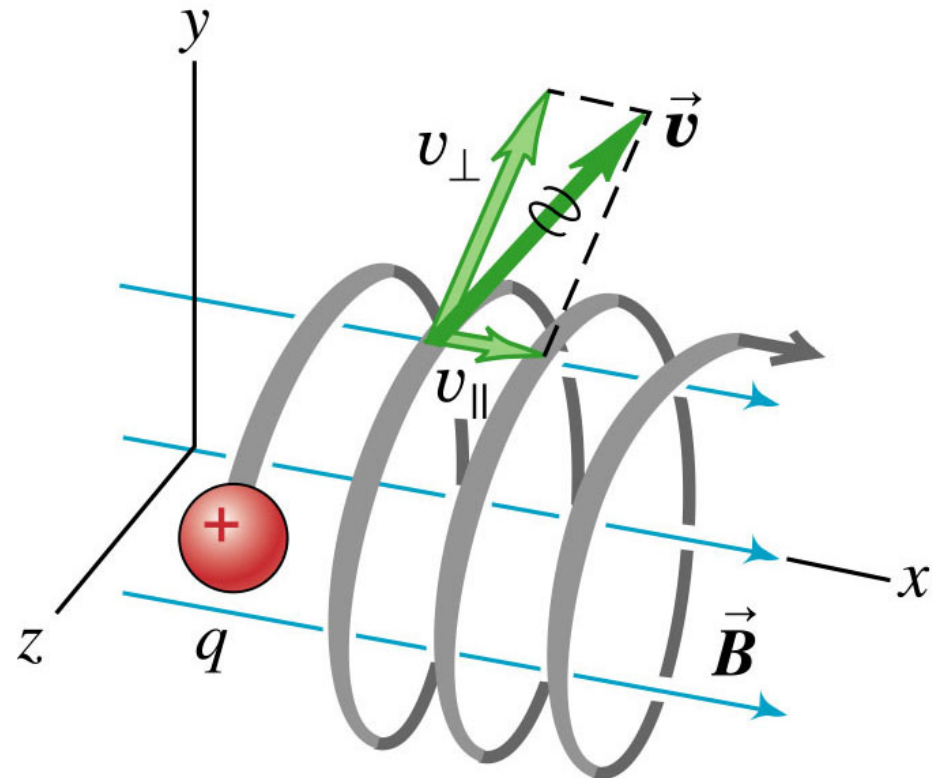
If the velocity is not perpendicular, the motion is a helix. In this case we break up the velocity into components perpendicular v_{\perp} and parallel to the field v_{\parallel} .

$$\vec{F} = q\vec{v}_{\parallel} \times \vec{B} = 0$$

Therefore have uniform motion in B direction.

Path looks like extended spring, v_{\parallel} constant along the B direction and v_{\perp} moves in a circle.

$$R \text{ (of helix)} = mv_{\perp} / qB$$



Applications

Electric/Magnetic Velocity Selector

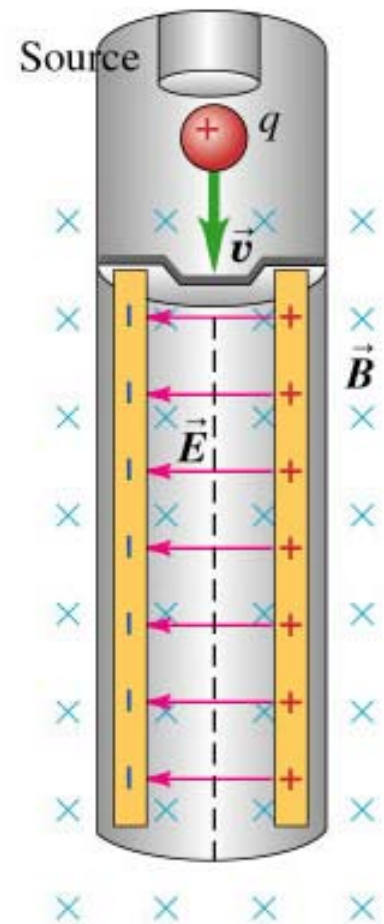
A charged particle enters a region with perpendicular electric and magnetic fields. The electric and magnetic forces will cancel if the velocity is just right. Particles with this velocity will go through undeflected. Others will be deflected.

$$\vec{F}_E = q\vec{E} = -\vec{F}_m = -q\vec{v} \times \vec{B}$$

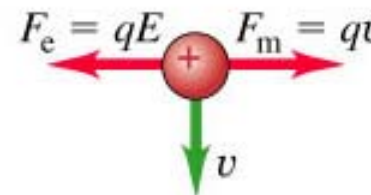
$$q|\vec{E}| = q|\vec{v}||\vec{B}|\sin\theta = q|\vec{v}||\vec{B}|1$$

$$|\vec{v}| = \frac{|\vec{E}|}{|\vec{B}|}$$

Choosing a particular E/B ratio will select the desired velocity. Example, $E=1,000\text{V/m}$ and $B=0.001\text{T}$ selects $v = 1,000,000\text{ m/s}$



(a)



(b)

Mass Spectrometer (Chemistry)

Measure m to identify substances:

1.) Ionize atoms by hitting them with accelerated electrons.

2.) Accelerate ions through known potential V .

$q=e$ if singly ionized.

$$U=qV$$

3.) Velocity select.

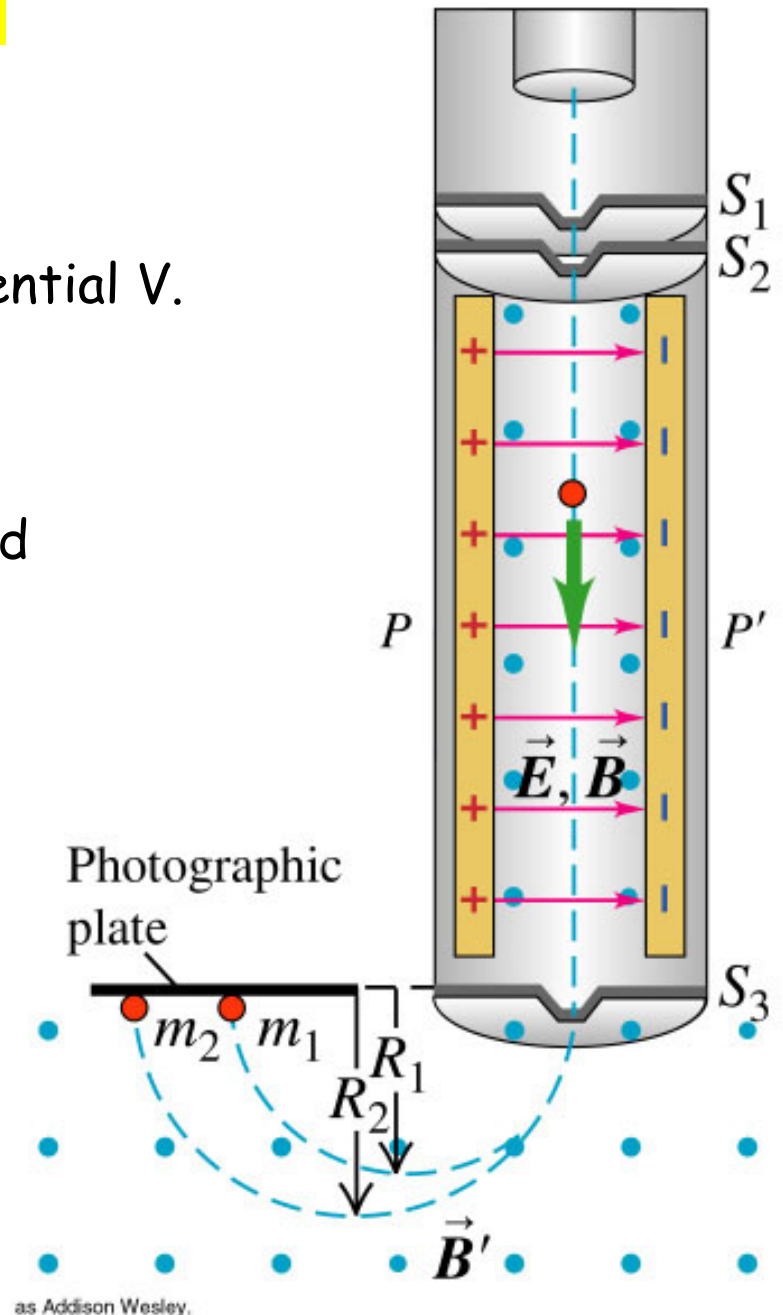
3.) Pass ions through known B field and measure R .

$$R = \frac{mv}{qB'}$$

$$v = \frac{qB'}{mR}; \quad v^2 = \frac{q^2 B'^2}{m^2 R^2}$$

$$\frac{1}{2}mv^2 = qV$$

$$m = \frac{qB'^2 R^2}{2V}$$



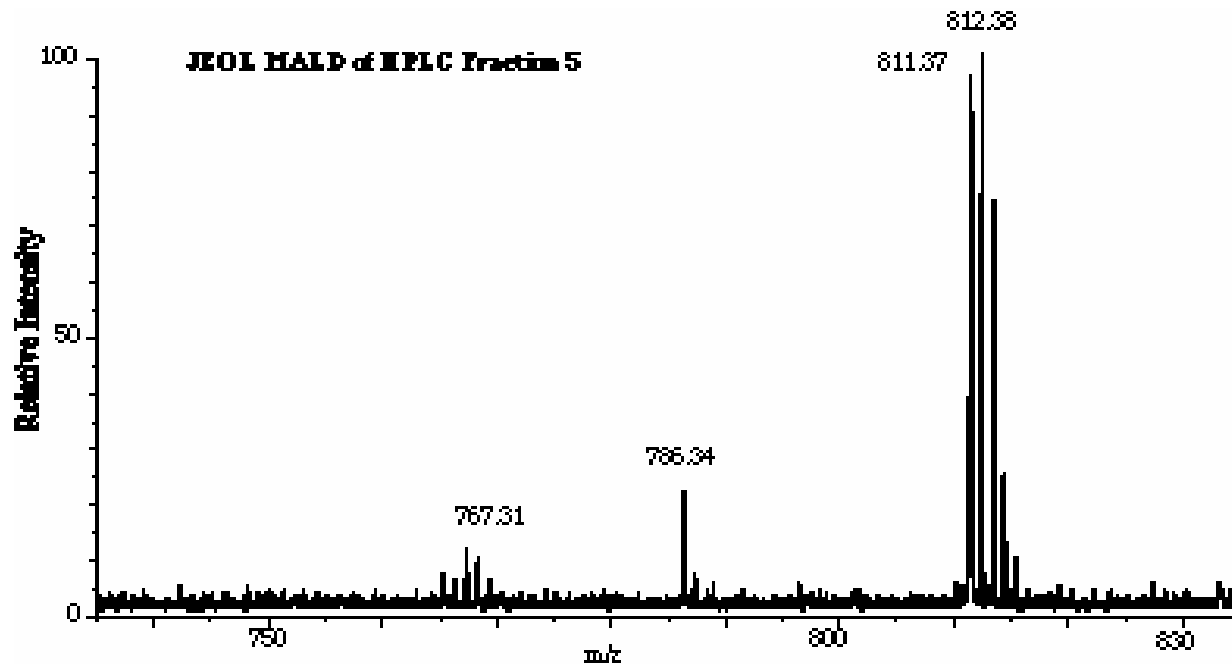
Mass Spectrometer (Chemistry)

Applications:

Paleoceanography: Determine relative abundances of isotopes (they decay at different rates → geological age)

Space exploration: Determine what's on the moon, Mars, etc.

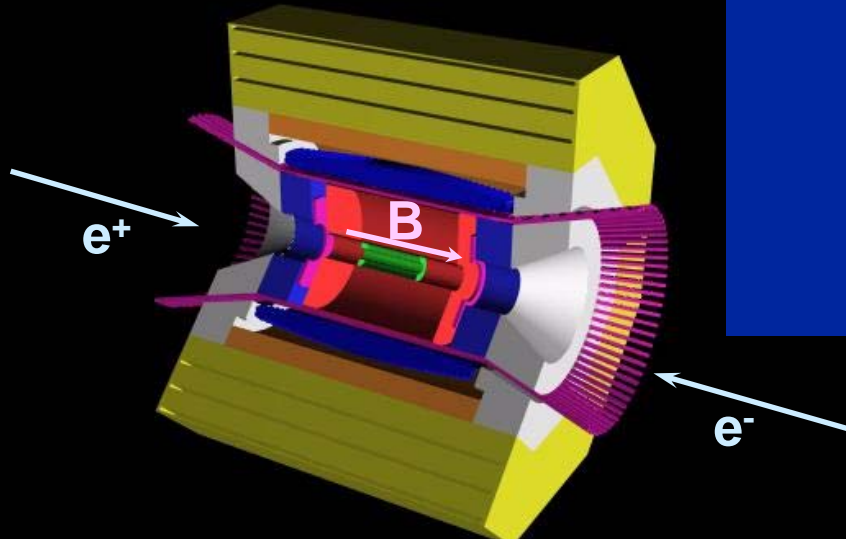
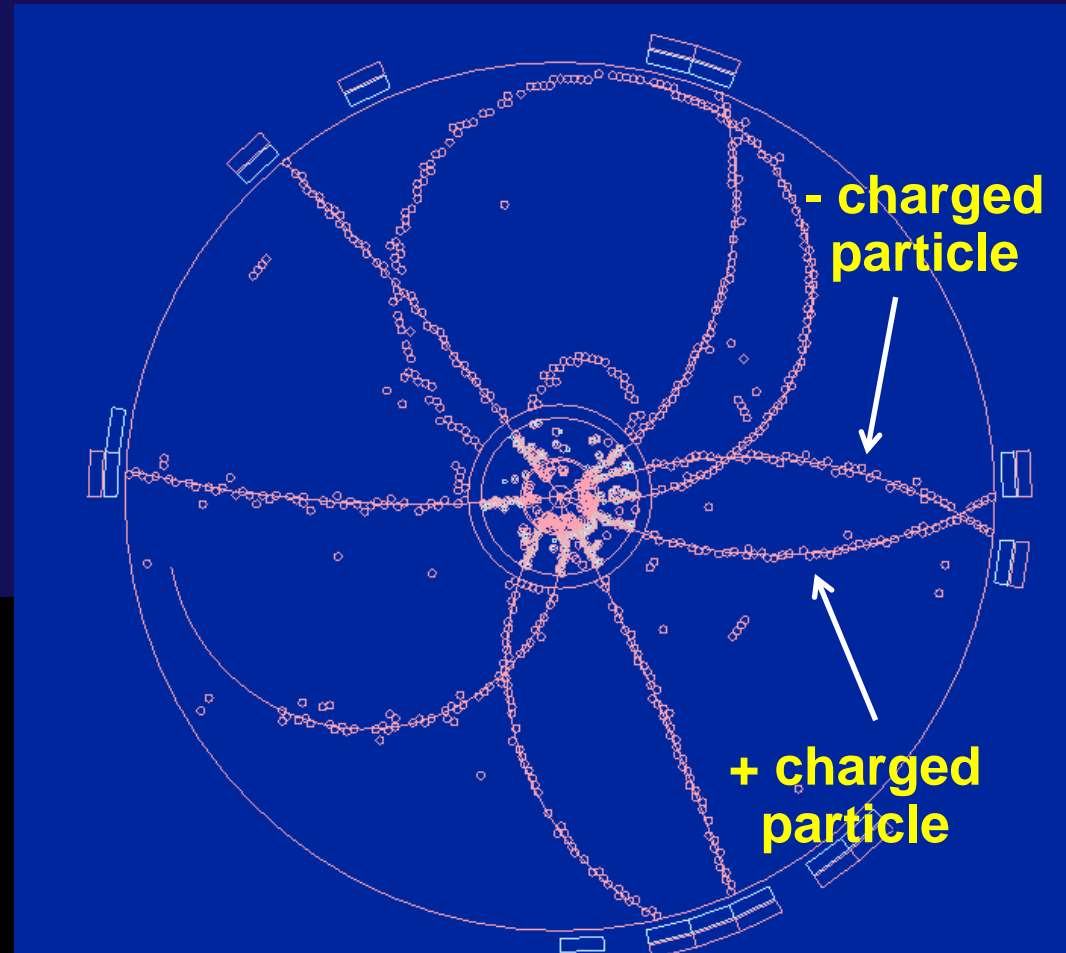
Detect chemical and biol. weapons (nerve gas, anthrax, etc.).



Yet another example

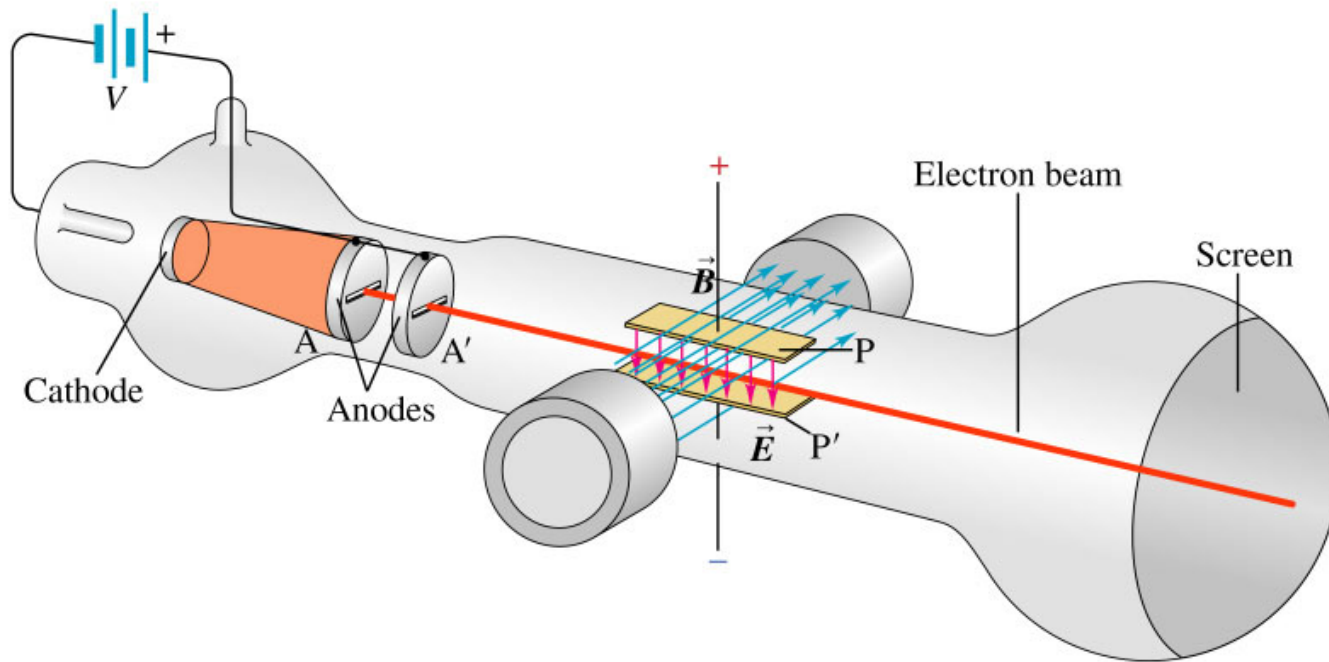
$$R = \frac{mv}{qB} = \frac{p}{qB}$$

- Measuring curvature of charged particle in magnetic field is usual method for determining momentum of particle in modern experiments: e.g.



End view: B into screen

Thompson's charge/mass ratio of electron measurement



$$\frac{1}{2}mv^2 = eV$$

$$v = \sqrt{\frac{2eV}{m}}$$

$$\frac{E}{B} = v = \sqrt{\frac{2eV}{m}}$$

$$\frac{e}{m} = \frac{E^2}{2VB^2}$$

Found single value of e/m
independent of cathode material.
This led to discovery of electrons.

$$e/m = 1.7588 \times 10^{11} \text{ C/kg}$$

Nobel prize in 1906

Cyclotron, the first "Atom Smasher"

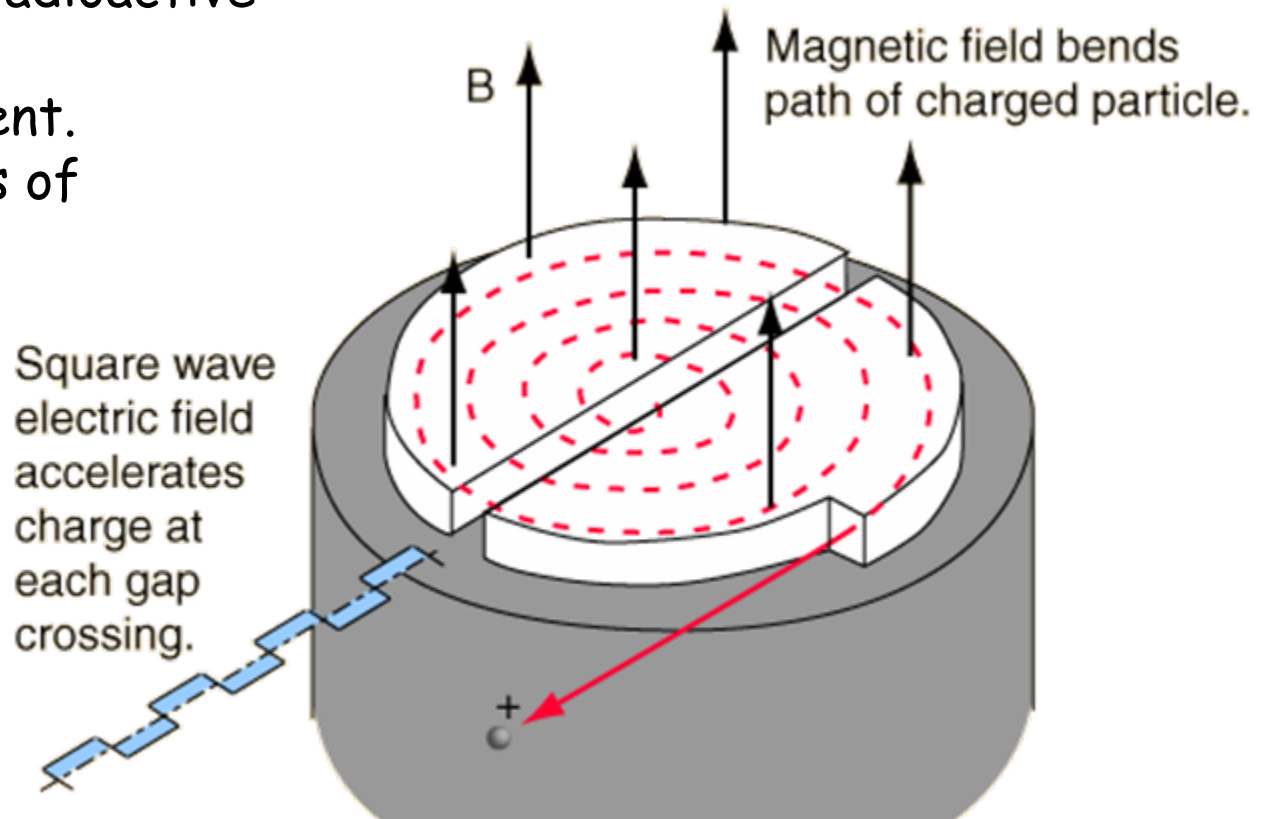
Accelerates protons and deuterons to high energies to bombard atomic nuclei.

Allows:

- study of nuclear physics.
- production of radioactive isotopes
- cancer treatment.

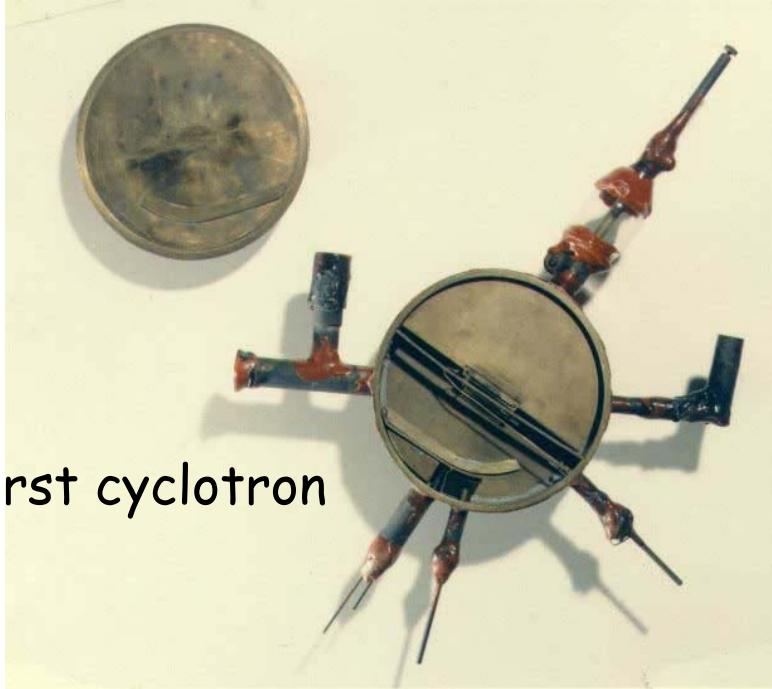
There are thousands of cyclotrons in use.

E. O. Lawrence and M. S. Livingston, invented particle accelerator, called cyclotron. Won 1939 Nobel Prize.



Cyclotron

Ernest Lawrence, South Dakota, 1922



first cyclotron

Led to giant accelerators
near Chicago.



Cyclotron

Electromagnet causes particles to move in a "circle".
Oscillating voltage between Dees accelerates particles each time particle crosses gap.

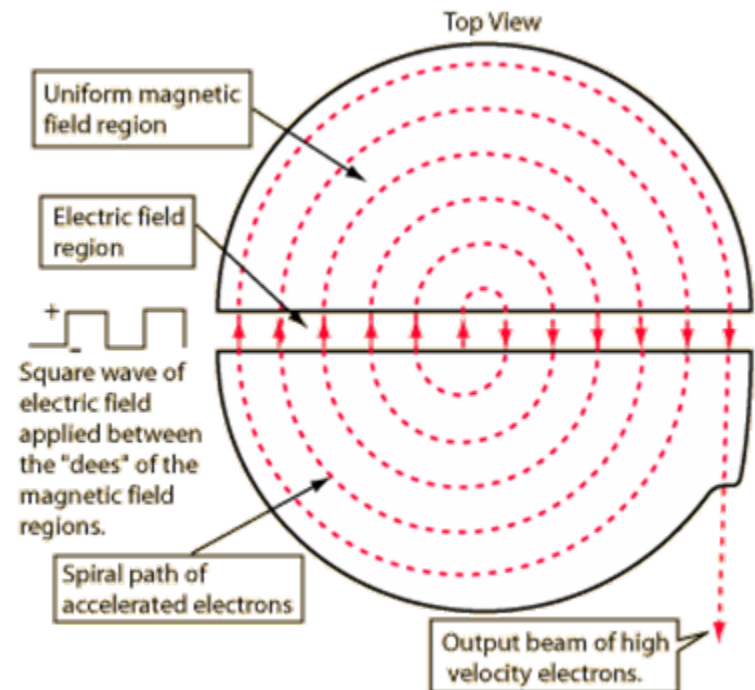
Very important to operation of cyclotron - frequency of electric field does NOT depend on v .

$$\omega = \frac{qB}{m}; \quad f = \frac{1}{T} = \frac{qB}{2\pi m}$$

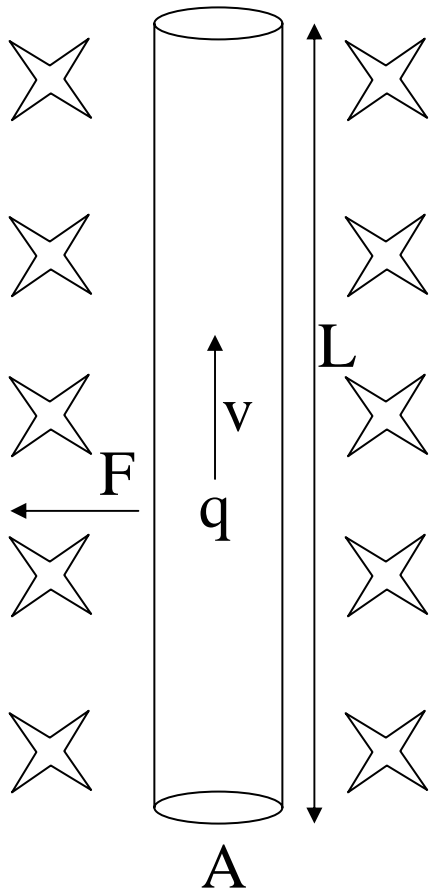
Maximum energy determined by size of cyclotron. Limit imposed by size and cost of magnet.

$$v_{\max} = \frac{qBR_{\max}}{m}$$

$$K_{\max} = \frac{1}{2}mv_{\max}^2 = \frac{q^2 B^2}{2m} R_{\max}^2$$



Magnetic force on current carrying wire



Suppose we have a straight wire with current where charges q are moving upwards and there is a B field pointing into this slide. There is a $\vec{F} = q\vec{v} \times \vec{B}$ force tending to push the wire to the left.

Recall that current is $I = n q v A$ (Y&F eqn 25.2) with n =#charge/vol, v =velocity and A =area. In a length, L , the #charges = $n L A$, so the total force magnitude is,

$$F = (nLA)qvB = (nqvA)LB = ILB$$

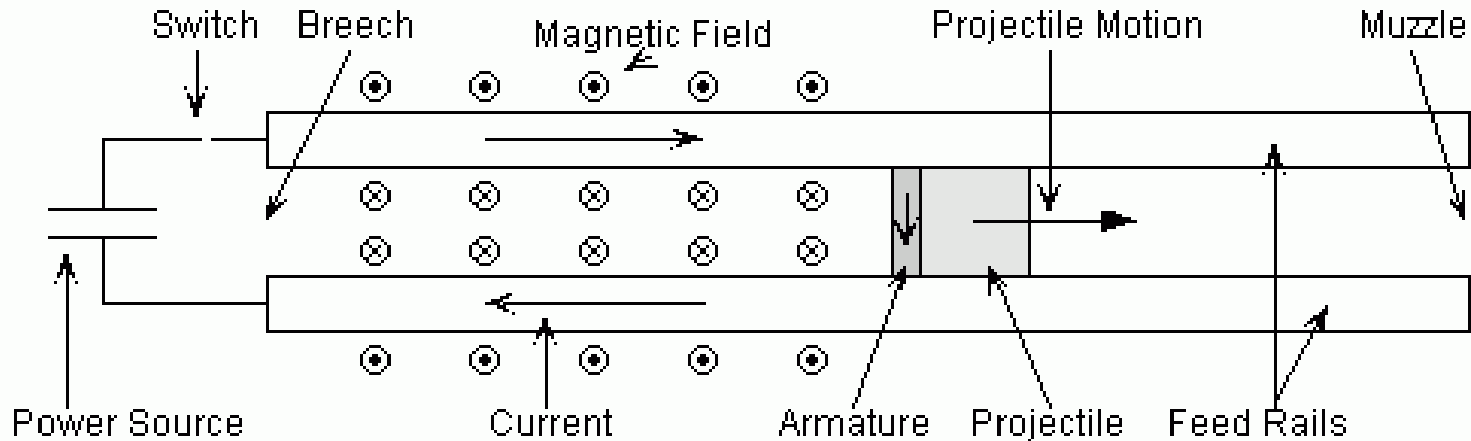
If we consider a small line segment, dl , we can write the vector force eqn. as,

$$d\vec{F} = Id\vec{l} \times \vec{B}$$

More general.

Rail Gun Example

Rail Gun



Suppose we have a movable projectile between bars supplying current flow and a uniform perpendicular magnetic field. Assume, $B=4\text{T}$, $I=5000\text{A}$, projectile width= 10cm and mass of bar= 0.1kg .

A) What is the force on the projectile?

B) How fast will be the be moving after it moves a distance 1m ?