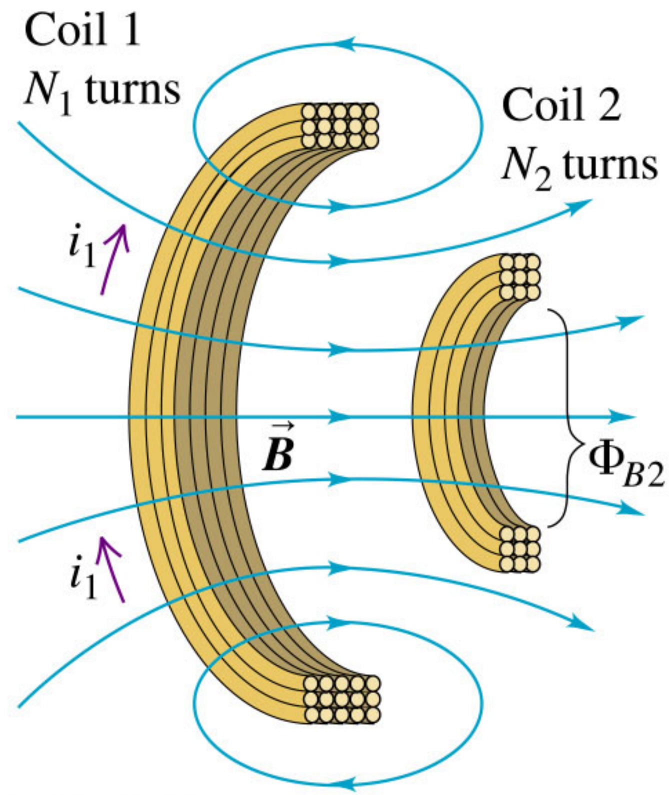
Mutual Inductance

If we have a constant current i_1 in coil 1, a constant magnetic field is created and this produces a constant magnetic flux in coil 2. Since the Φ_{B2} is constant, there NO induced current in coil 2.



If current i_1 is time varying, then the Φ_{B2} flux is varying and this induces an emf ε_2 in coil 2, the emf is

$$\varepsilon_2 = -N_2 \frac{d\Phi_{B2}}{dt}$$

We introduce a ratio, called **mutual inductance**, of flux in coil 2 divided by the current in coil 1.

$$M_{21} = \frac{N_2 \Phi_{B2}}{i_1}$$

Mutual Inductance

mutual inductance, $M_{21} = \dfrac{N_2 \Phi_{B2}}{i_1}$, can now be used in Faraday's eqn

$$M_{21}\ i_1 = N_2\ \Phi_{B2}$$

$$M_{21}\frac{di_1}{dt} = N_2\frac{d\Phi_{B2}}{dt} = -\varepsilon_2; \quad \varepsilon_2 = -M_{21}\frac{di_1}{dt}$$

We can also the varying current i_2 which creates a changing flux Φ_{B1} in coil 1 and induces an emf ε_1. This is given by a similar eqn.

$$\varepsilon_1 = -M_{12}\frac{di_2}{dt}$$

It can be shown (*we do not prove here*) that, $M_{12} = M_{21} = M$

The units of mutual inductance is $T \cdot m^2/A$ = Weber/A = Henry

(after the Joseph Henry, who missed Faraday's Law)

Mutual Inductance

The induced emf,

$$\varepsilon_1 = -M\frac{di_2}{dt}$$

has the following features;

- The induced emf opposes the magnetic flux change
- The induced emf increases if the currents changes very fast
- The induced emf depends on M, which depends only the geometry of the two coils and not the current.
- For a few simple cases, we can calculate M, but usually it is just measured.

Problem 30.1

Two coils have mutual inductance of 3.25×10^{-4} H. The current in the first coil increases at a uniform rate of 830 A/s .

A) What is the magnitude of induced emf in the 2^{nd} coil? Is it constant?

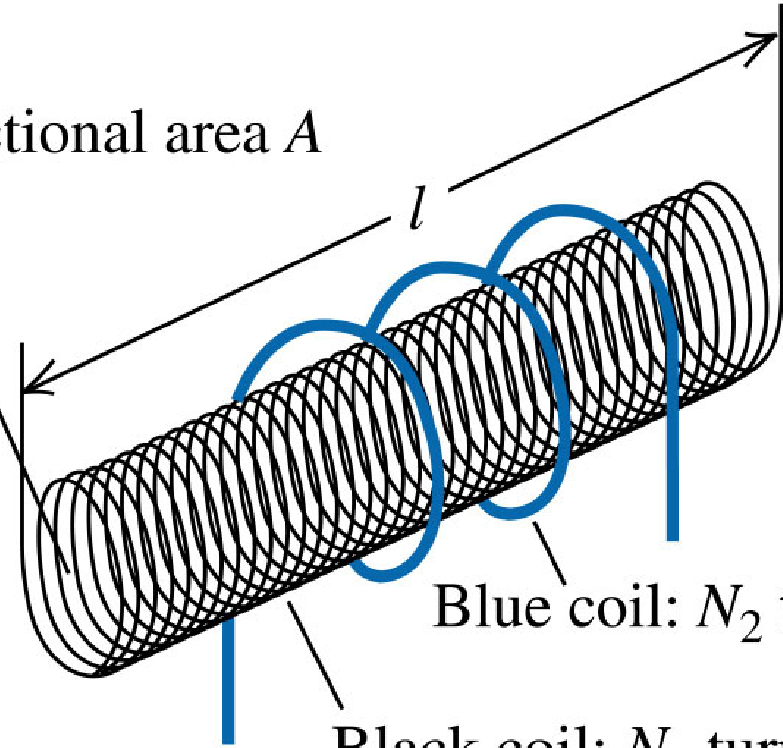
B) suppose that the current is instead in the 2^{nd} coil, what is the magnitude of the induced emf in the 1^{st} coil?

$$\varepsilon_2 = -M\frac{di_1}{dt}$$

$$= -(3.25\times10^{-4}\,H)(830\frac{A}{s}) = -\,0.27V$$

$$\varepsilon_1 = -M\frac{di_2}{dt} = -0.27V$$

Tesla Coil Example



Magnetic field due to coil 1 is

$$B_1 = \mu_0 n_1 i_1 = \mu_0 N_1 i_1 / l$$

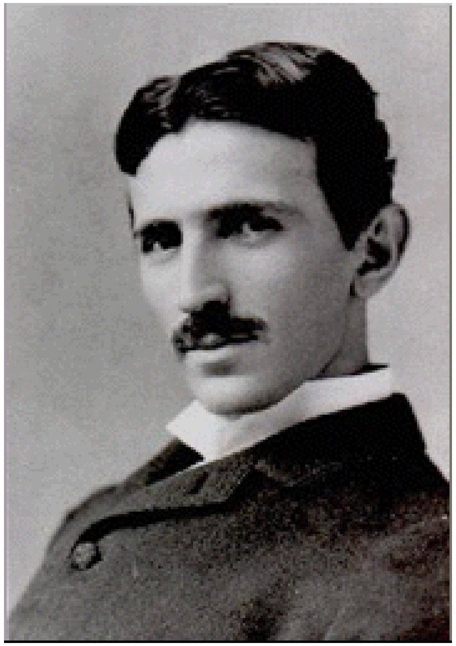
Mutual inductance is,

$$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_2 B_1 A}{i_1}$$

$$= \frac{N_2 \mu_0 N_1 i_1 A}{i_1 l} = \frac{\mu_0 N_1 N_2 A}{l}$$

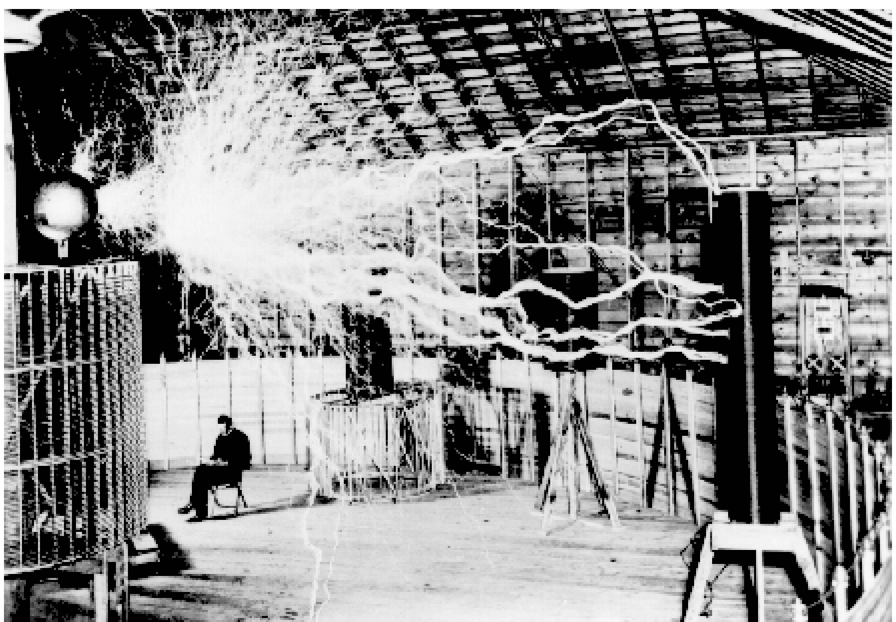
The induced emf in coil1 from coil2 is

$$\varepsilon_1 = -M \frac{di_2}{dt} = -\frac{\mu_0 N_1 N_2 A}{l} \frac{di_2}{dt}$$



Nicolai Tesla (1856-1943)

Born in Croatia, graduated from University of Prague. Arrived in New York with 4cents and went to work for Edison. Tesla invented polyphase alternating-current system, induction motor, alternating-current power transmission, Tesla coil transformer, wireless communication, radio, and fluorescent lights. He set up a Tesla coil in

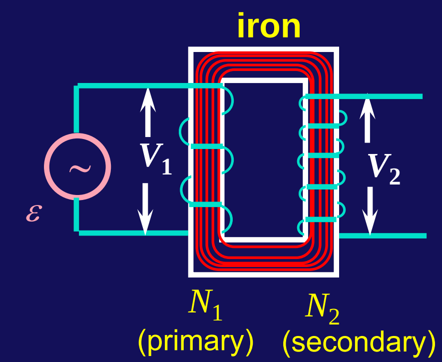


Colorado Springs in 1899, below is a photo of this lab. He lighted lamps 40Km away. He also claimed to receive messages from another planet!! In honor of his contributions to electromagnetic phenomena, the Magnetic field intensity was named in units of “Tesla”

Applications of Mutual Inductance

- Transformers
  - Change one AC voltage into another

$$V_2 = \frac{N_2}{N_1} V_1$$

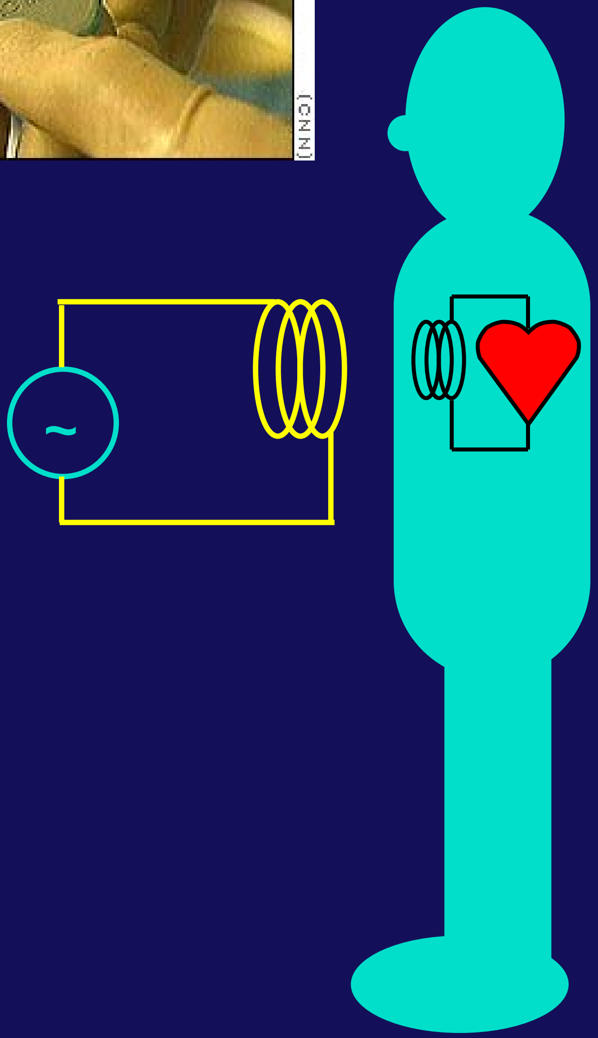


- Airport Metal Detectors
  - Pulsed current → pulsed magnetic field → Induces emf in metal
  - Ferromagnetic metals "draw in" more B → larger mutual inductance → larger emf
  - Emf → current (how much, how long it lasts, depends on the resistivity of the material)
  - Decaying current produces decaying magnetic field → induces current in receiver coils
  - Magnitude & duration of signal depends on the composition and geometry of the metal object.

Applications of Mutual Inductance

- Pacemakers
  - It's not easy to change the battery!
  - Instead, use an external AC supply.

  - Alternating current
    - → alternating $\boldsymbol{B}$
    - → alternating Φ_B inside "wearer"
    - → induces AC current to power pacemaker



Applications of Mutual Inductance

Upon leaving the hospital with his new pacemaker, Herman realizes that batteries are NOT included.

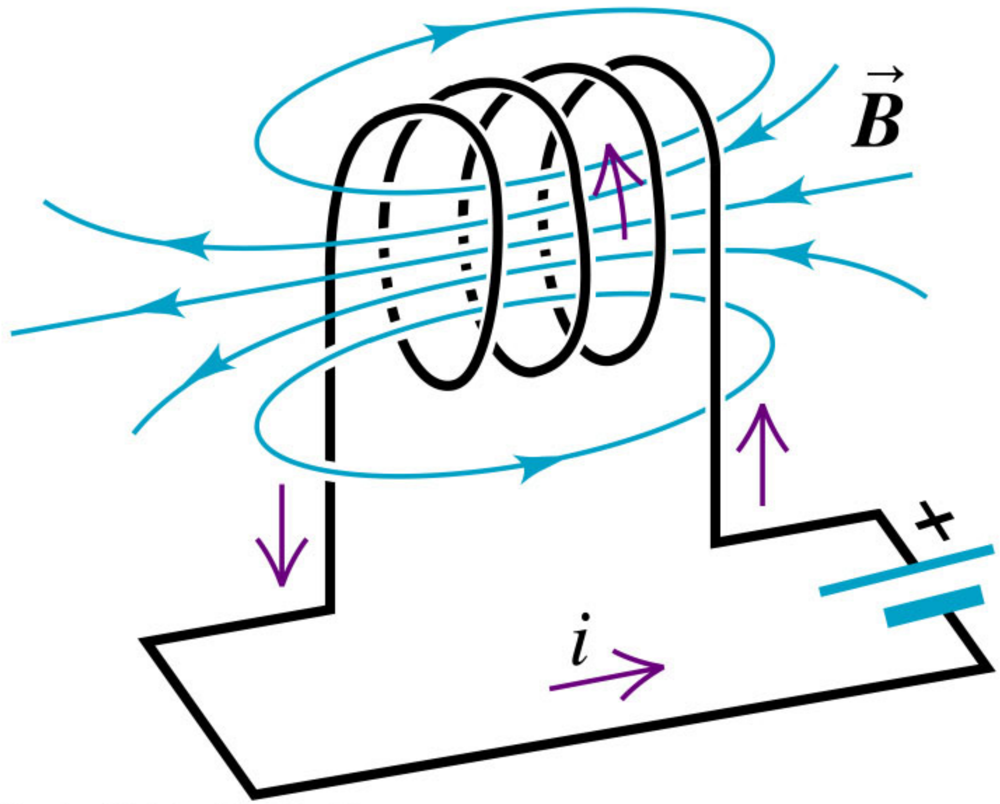
Self Inductance

We previously considered induction between 2 coils. Now we consider the situation where a single isolated coil induces emf on itself. This is Called "back emf" and if the current changes, there is a self induced emf that opposes the change in current. We form the same ratio, now called Self-Inductance, *L*,

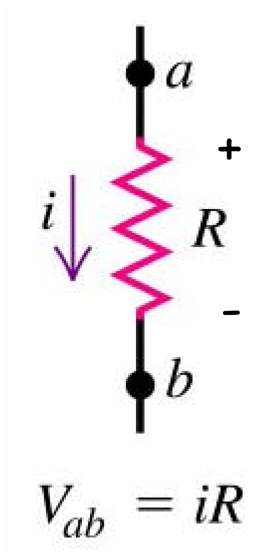
$$L = \frac{N\Phi_B}{i}$$

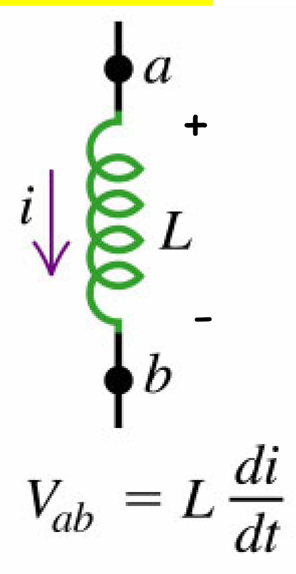
and we have the back emf,

$$\varepsilon = -L\frac{di}{dt}$$



Behavior of isolated coil in circuits





Resistor with current I has potential drop, V=iR from a to b

Coil with

a) constant current i has NO Voltage drop

b) di/dt>0, potential decreases from a to b, V=Ldi/dt

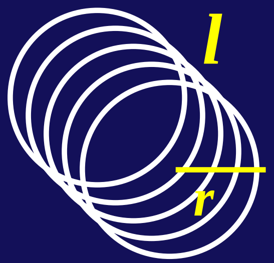
c) di/dt<0, potential increases from a to b, V=-L|di/dt|

Remember, emf in coil opposes current change.

Self inductance of long solenoid

- Long Solenoid:

  N turns total, radius *r*, Length *l*



$$r << l \Rightarrow B = \mu_0 \frac{N}{l} I$$

For a single turn, $A = \pi r^2 \Rightarrow \phi = BA = \mu_0 \frac{N}{l} I \pi r^2$

The flux through a turn is given by:

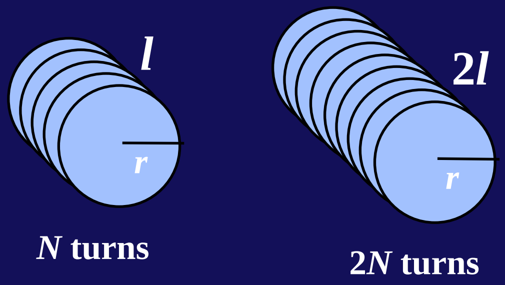
$$\Phi_B = \mu_0 \frac{N}{l} I \pi r^2$$

Inductance of solenoid can then be calculated as:

$$\boxed{L \equiv \frac{N\Phi_B}{I} = \mu_0 \frac{N^2}{l} \pi r^2 = \mu_0 \left(\frac{N}{l}\right)^2 l \pi r^2}$$

Lecture 17, Act 1

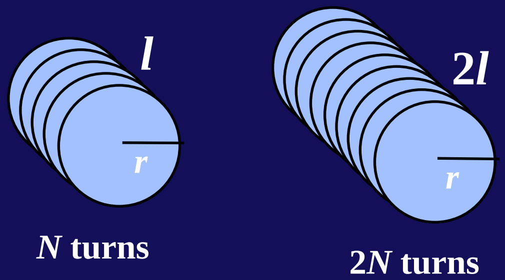
- Consider the two inductors shown:
  - Inductor 1 has length l, N total turns and has inductance L_1.
  - Inductor 2 has length $2l$, $2N$ total turns and has inductance L_2.
  - What is the relation between L_1 and L_2?



(a) $L_2 < L_1$ **(b)** $L_2 = L_1$ **(c)** $L_2 > L_1$

Lecture 17, Act 1

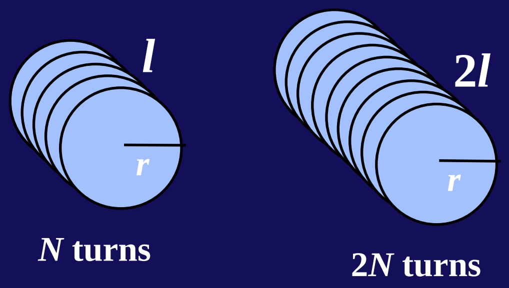
- Consider the two inductors shown:
  - Inductor 1 has length *l*, *N* total turns and has inductance L_1.
  - Inductor 2 has length 2*l*, 2*N* total turns and has inductance L_2.
  - What is the relation between L_1 and L_2? **(a)** $L_2 < L_1$ **(b)** $L_2 = L_1$ $\boxed{\textbf{(c)}\ L_2 > L_1}$



- To determine the self-inductance L, we need to determine the flux Φ_B which passes through the coils when a current I flows: $L \equiv N\Phi_B / I$.
- To calculate the flux, we first need to calculate the magnetic field B produced by the current: $B = \mu_0(N/l)I$
  - i.e., the B field is proportional to the number of turns per unit length.
  - Therefore, $B_1 = B_2$. But does that mean $L_1 = L_2$?

Lecture 17, Act 1

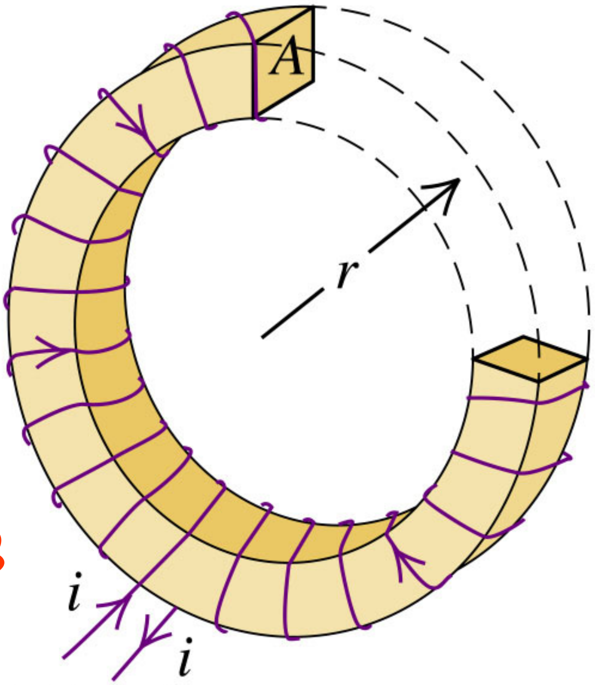
- To calculate L, we need to calculate the flux.
  - Since $B_1 = B_2$, the flux through any given turn is the same in each inductor
  - There are twice as many turns in inductor 2; therefore the net flux through inductor 2 is twice the flux through inductor 1! Therefore, $L_2 = 2L_1$.



Inductors in series *add* (like resistors): $L_{\text{eff}} = L_1 + L_2$

And inductors in parallel add like resistors in parallel: $\dfrac{1}{L_{\text{eff}}} = \dfrac{1}{L_1} + \dfrac{1}{L_2}$

Self Inductance of toroidal solenoid



The magnetic field in a toroid was

$$B = \frac{\mu_0 N i}{2\pi r}$$

and the net mag. flux is

$$\Phi_B = BA = \frac{\mu_0 N i}{2\pi r} A$$

Approximates B as value at center of coil.

Hence the self inductance is,

$$L = \frac{N\Phi_B}{i} = \frac{N\mu_0 N i}{i 2\pi r} A = \frac{N^2 \mu_0}{2\pi r} A$$

Example: N = 200, A = 5 cm^2, r = 0.1 m

$$L = \frac{200^2 4\pi \times 10^{-7}}{2\pi(0.1m)} (5 \times 10^{-4}\, m^2) = 4x10^{-5} H = 40\mu H$$