Faraday's Law

\[ \Phi_B \equiv \int \mathbf{B} \cdot d\mathbf{A} \]

\[ \varepsilon = -\frac{d\Phi_B}{dt} \ocomma \]
Recall the definition of magnetic flux is \[ \Phi_B = \int \vec{B} \cdot d\vec{A} \]

Faraday’s Law is the induced EMF in a closed loop equal the negative of the time derivative of magnetic flux change in the loop,

\[ \varepsilon = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A} = -\frac{d\Phi_B}{dt} \]

Constant B field, no induced EMF in loop

Changing B field, causes induced EMF in loop
Getting the sign EMF in Faraday’s Law of Induction

Define the loop and an area vector, \( \mathbf{A} \), whose magnitude is the area and whose direction is normal to the surface.

The choice of vector \( \mathbf{A} \) direction defines the direction of EMF with a right hand rule. Your thumb in \( \mathbf{A} \) direction and then your fingers point to positive EMF direction.
Positive flux ($\Phi_B > 0$)
Flux becoming more positive ($\frac{d\Phi_B}{dt} > 0$)
Induced emf is negative ($\mathcal{E} < 0$)

(a)

Positive flux ($\Phi_B > 0$)
Flux becoming less positive ($\frac{d\Phi_B}{dt} < 0$)
Induced emf is positive ($\mathcal{E} > 0$)

(b)
Lenz’s Law – easier way!

The direction of any magnetic induction effect is such as to oppose the cause of the effect.

⇒ Convenient method to determine $I$ direction

Example if an external magnetic field on a loop is increasing, the induced current creates a field opposite that reduces the net field.

Example if an external magnetic field on a loop is decreasing, the induced current creates a field parallel to the that tends to increase the net field.
Incredible shrinking loop: a circular loop of wire with a magnetic flux is shrinking with time. In which direction is the induced current?

(a) There is none.  (b) CW.  (c) CCW
Determine the direction of current in a - b when

1.) Switch is opened after having been closed for a long time.
   (a) No current.       (b) left to right.      (c) right to left.

2.) The resistance R is decreased while switch remains closed.
   (a) No current.       (b) left to right.      (c) right to left.
The area is $LS(t)$ and $dS/dt$ equals velocity of slider

\[ \mathcal{E} = -\frac{d}{dt} \Phi_B = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A} = -\frac{d}{dt} \int B \, dA \]

\[ = -\frac{d}{dt} BA = -\frac{d}{dt} BLS = -BL \frac{d}{dt} S = -BLv \]
we found
\[ \varepsilon = BLv \]
we now ask what is power dissipated, \( P_d \)
and power applied, \( P_a \)
assume total resistance is \( R \), then \( I = \varepsilon / R \) and
\[ P_d = I^2 R = B^2 L^2 v^2 / R \]

The power applied, \( P_a = Fv \), is the rate of work to the bar to the right against the force, \( F \), which is pointed to the left. \( F = ILB = (\varepsilon / R)LB = B^2 L^2 v / R \). Thus \( P_a = Fv = B^2 L^2 v^2 / R = P_d \) and we find the power put into the moving the bar equals the heat dissipated in the circuit.

\[ \Rightarrow \text{Power needed to move bar} = \text{Power dissipated in circuit} \]
Slide Wire Generator; use Lenz’s Law to get I direction

Lenz’s Law says direction creates field that opposes change in magnetic flux.

If we pull bar to right, the net magnetic flux in rectangle increases into screen, hence the I direction must induce opposite B field which is out of screen and is correct in drawing.

Suppose Lenz’s law were reversed, then I would be reversed and F would go right and the bar would be accelerated to the right, w/o need of external positive work and heat would be dissipated at the same time. This violates Conservation of Energy, so Lenz’s law is correct.
In Faraday’s Law, we can induce EMF in the loop when the magnetic flux, $\Phi_B$, changes as a function of time. There are two cases when $\Phi_B$ is changing,

1) Change the magnetic field (non-constant over time)
2) Change or move the loop in a constant magnetic field

The slide wire generator is an example of #2 and the induction of EMF via moving parts of the loop is called, *motional EMF.*
Wire loop area $A$ rotates with respect to constant magnetic field.

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = BA \cos \phi$$

If the angular frequency is $\omega$, then

$$\Phi_B = BA \cos(\omega t)$$

$$\frac{d}{dt} \Phi_B = -BA \omega \sin(\omega t)$$

and EMF in the loop is

$$\varepsilon = -\frac{d}{dt} \Phi_B = BA \omega \sin(\omega t)$$
Slide Wire Generator; revisited again

Suppose we move a conducting bar in a constant $B$ field, then a force $\vec{F}_B = q \vec{v} \times \vec{B}$ moves + charge up and - charge down. The charge distribution produces an electric field, $\vec{E}$, force, $\vec{F}_E$, and EMF, $\epsilon$, between $a$ & $b$. This continues until equilibrium is reached: $\vec{F}_E = -\vec{F}_B$.

$$\vec{E} = \frac{\vec{F}_E}{q} = -\frac{\vec{F}_B}{q} = -\frac{q\vec{v} \times \vec{B}}{q} = -\vec{v} \times \vec{B}$$  
$$\epsilon = \int_{a}^{b} \vec{E} \cdot d\vec{l} = vBl$$

In effect the bar motional EMF is an equivalent to a battery EMF.
If the rod is on the U shaped conductor, the charges don't build up at the ends but move around the U shaped portion. They produce an electric field in the U shaped circuit. The wire acts as a source of EMF – just like a battery. Called *motional electromotive force.*

\[
\varepsilon = \int_{a}^{b} \vec{E} \cdot d\vec{l} = vBL
\]
Direct Current Homopolar Generator invented by Faraday

Rotate a metal disk in a constant perpendicular magnetic field. The charges in the disk when moving receive a radial force. The causes current to flow from center to point b.

\[ \varepsilon = \int_0^R \omega Br \, dr = \frac{1}{2} \omega BR^2 \]

Faraday's original apparatus had the disk fixed with a meter and then he spun a magnet under the disk.

Emf induced across this segment is 
\[ d\varepsilon = vB \, dr = \omega Br \, dr \]