Kirchhoff’s Rules

Kirchhoff’s rules are statements used to solve for currents and voltages in complicated circuits. The rules are

Rule I. Sum of currents into any junction is zero.  \[ \sum I_i = 0 \]

Why? Since charge is conserved.

Rule II. Sum of potential differences in any loop is zero. (This includes emfs)  \[ \sum V_i = 0 \]

Why? Since potential (energy) is conserved
Kirchhoff’s Rules

(1) Set up current directions. The current is the same along single path and at a junction the sum of 2 currents entering a junction equals the current exiting the junction.

(2) Getting potential differences requires setting up travel path through a loop, either clockwise or counter clockwise. Positive for current flow – to + across a battery and negative for flow + to -. For a resistor, negative voltage drop if travel & I in same direction and pos. voltage increase if travel & I opposite.
Gustav Robert Kirchhoff (1824-1887)

Born in Prussia, Germany. Studied with Neumann and in 1841 published his famous Kirchhoff’s laws. He extended Ohm’s electrical theories. Later he studied spectra from various elements. He worked with Robert Bunsen and studied radiation spectrum from the sun. He also worked on black body radiation, which was very important in the development of Quantum Theory. After he was disabled in crutches and a wheelchair he turned from experimental physics to theoretical physics. He became Chair of Mathematical physics in Berlin. He was known as a masterful teacher with clarity and rigor in his thinking and teaching.
Loop Demo

\[ KVL: \quad \sum_{\text{loop}} V_n = 0 \quad \Rightarrow \quad -IR_1 - IR_2 - \varepsilon_2 - IR_3 - IR_4 + \varepsilon_1 = 0 \]

\[ \Rightarrow \quad I = \frac{\varepsilon_1 - \varepsilon_2}{R_1 + R_2 + R_3 + R_4} \]
Consider the circuit shown.

- The switch is initially open and the current flowing through the bottom resistor is $I_0$.
- After the switch is closed, the current flowing through the bottom resistor is $I_1$.
- What is the relation between $I_0$ and $I_1$?

(a) $I_1 < I_0$  
(b) $I_1 = I_0$  
(c) $I_1 > I_0$
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• Write a loop law for original loop:

$$12V + 12V - I_0R - I_0R = 0$$

$$I_0 = \frac{12V}{R}$$

• Write a loop law for the new loop:

$$12V - I_1R = 0$$

$$I_1 = \frac{12V}{R}$$
Two identical light bulbs are represented by the resistors $R_2$ and $R_3 \ (R_2 = R_3)$. The switch $S$ is initially open.

2) If switch $S$ is closed, what happens to the brightness of the bulb $R_2$?
   
   a) It increases            b) It decreases            c) It doesn’t change

3) What happens to the current $I$, after the switch is closed?
   
   a) $I_{\text{after}} = 1/2 \ I_{\text{before}}$
   
   b) $I_{\text{after}} = I_{\text{before}}$
   
   c) $I_{\text{after}} = 2 \ I_{\text{before}}$
How to use Kirchhoff’s Laws

A two loop example:

• Analyze the circuit and identify all circuit nodes and use KCL.
  \[ I_1 = I_2 + I_3 \]  

• Identify all independent loops and use KVL.
  \[ \varepsilon_1 - I_1R_1 - I_2R_2 = 0 \]  
  \[ \varepsilon_1 - I_1R_1 - \varepsilon_2 - I_3R_3 = 0 \]  
  \[ I_2R_2 - \varepsilon_2 - I_3R_3 = 0 \]
How to use Kirchoff’s Laws

• Solve the equations for \( I_1, I_2, \) and \( I_3 \):

First find \( I_2 \) and \( I_3 \) in terms of \( I_1 \):

\[
I_2 = \frac{(\varepsilon_1 - I_1 R_1)}{R_2} \quad \text{From eqn. (2)}
\]

\[
I_3 = \frac{(\varepsilon_1 - \varepsilon_2 - I_1 R_1)}{R_3} \quad \text{From eqn. (3)}
\]

Now solve for \( I_1 \) using eqn. (1):

\[
I_1 = \frac{\varepsilon_1}{R_2} + \frac{\varepsilon_1 - \varepsilon_2}{R_3} - I_1 \left( \frac{R_1}{R_2} + \frac{R_1}{R_3} \right)
\]

\[
\Rightarrow \quad I_1 = \frac{\varepsilon_1}{R_2} + \frac{\varepsilon_1 - \varepsilon_2}{R_3} \quad \frac{R_2}{1 + \frac{R_1}{R_2} + \frac{R_1}{R_3}}
\]
Let’s plug in some numbers

\[ \varepsilon_1 = 24 \text{ V} \quad \varepsilon_2 = 12 \text{ V} \quad R_1 = 5 \Omega \quad R_2 = 3 \Omega \quad R_3 = 4 \Omega \]

Then

\[ I_1 = 2.809 \text{ A} \quad \text{and} \quad I_2 = 3.319 \text{ A} \]

\[ I_3 = -0.511 \text{ A} \]

The sign means that the direction of \( I_3 \) is opposite to what’s shown in the circuit.
A) What are currents at junctions b, c, d and f?

\[ c : I_3 - 4A - 3A = 0; \quad I_3 = 7A \]
\[ d : 4A + 3A - I_{25} = 0; \quad I_{25} = 7A \]
\[ I_{25} = I_3 \]
\[ b : I_1 - I_{20} - I_3 = 0; \quad I_1 = \frac{199V}{20Ω} + 7A = 16.95A \]
\[ f : I_{20} + I_3 - I_1 = 0 \]

Note: d gives same as c
f gives same as b

B) What are voltages around loops, a-b-f-g, a-b-c-d-e-f-g, and b-c-d-e-f

\[ V_{25} = I_{25} (25 \Omega) = 175V \]
\[ V_6 = V_8 = 24V \]
\[ a - b - f - g : -199V + \varepsilon = 0; \quad \varepsilon = 199V \]
\[ a - b - c - d - f - g : -24 - 175 + \varepsilon = 0; \quad \varepsilon = 199V \]
\[ b - c - d - e - f : \]

exercise for student
Meters

Conventional needle meters are all based on the galvanometer (named after Luigi Galvani) which measures current.

Current in the red wire creates a magnetic field causes a torque inside an external magnetic field. This will be explained in Y&F 27.7. This torque causes the needle to rotate the needle proportionally to the current when connected to a restoring spring.

This galvanometer, can be configured in a circuit to measure amps, volts or ohms. A typical galvanometer measures 10milli-amps, at full scale deflection.
A) For an ammeter, what circuit measures 100 milliamp full scale if the intrinsic meter is a 10 milliamp meter and internal $R_c=10\Omega$?

B) For a voltmeter, what circuit measures 10 volts full scale?

C) For an ohmmeter, what circuit measures $R$ at half scale?, Assume $\mathcal{E}=1.5V$
A) For an ammeter, what circuit measures 100 milliamp full scale if the intrinsic meter is a 10 milliamp meter and internal $R_c=10\Omega$?

\[ I - I_{sh} = 10\ mA \]
\[ I = 100\ mA \]
\[ I_{sh} = 100 - 10 = 90\ mA \]

\[ (I - I_{sh})R_C = R_{sh}I_{sh} \]

\[ R_{sh} = \frac{(I - I_{sh})}{I_{sh}} R_C = \frac{10}{9}\ \Omega \]

*Meter: 10 mA full scale full scale when 100 mA Same potential difference*