

The shape of EOM-pulse-height distributions

Christoph Ley and Christopher H. Wiebusch

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Abstract

The *Amplitude-function* was designed as an easy function to describe the complex shape of pulse-width distribution as they are produced by EOM's for DUMAND II. This function is introduced and fits to calibration data sets are shown. The results of the EOM-calibrations are given. This parameterisation is not as good as a simple "straight-forward" parameterisation for only Mean and RMS, but provides details of the complex shape. This parameterisation may prove useful especially for simulation purposes.

1 Introduction

The European Optical Module (EOM [1]) is based on the large area phototube Philips XP2600 [2]. This so called „Smart PMT“ provides an excellent pulse-height resolution. Thus the read-out electronic was designed based on this characteristics. It is a charge integrating circuit producing a single ECL-pulse. It's leading edge is strongly correlated the start of the PMT-pulse and its width gives the integrated charge.

In the following *pulse-height* will be used for the number of photo-electrons (PE-space). This is **not** understood as with the actual *pulse-amplitude* (measured in Volt) of the PMT-signals. The word *pulse-energy* will be used in the following synonym with pulse-height. The *integrated charge* of a PMT-pulse, as it is measured by the read-out electronics (DMQT) is closely related to the pulse-height. For the EOM the measured charge is equivalent to the actual *pulse-widths* of the signals transmitted to the String Controller. Therefore the unit *ns* is appropriate for both, pulse-charge and pulse-width. In the low-PE-region the pulse-charge is also equivalent to the pulse-energy, but not for higher PE.

Figure 1 shows pulse-width distributions for EOM #26. For each distribution the PMT was illuminated with a fast green LED set to constant intensity. From the top-left (A) to the bottom-right (H) the intensity of the LED rises. The data was taken during calibrations at Kiel in July 1993.

At low light intensities, one can easily distinguish the peaks of 1PE, 2PE and more PE signals in the total distribution (e.g. fig 1-C). These peaks are linearly spaced.

For higher intensities ((E) - (H)), the energy scale changes from linear to logarithmic¹. The contributions for each PE do overlap and produce a single peak, that does not linearly move right at higher light intensities.

2 The amplitude function

The function, introduced in the following, is based on simple physical and electronic assumptions. The intrinsic parameters of this function have to be constant at different

¹This effect is produced by the read-out electronic. The input amplifier has a limited range for the pulse-amplitude of the PMT-signal, and cuts out amplitudes above this range.

light intensities.

2.1 The linear region

The following items are assumption that lead to the amplitude function for the linear region (eq.10).

- The production probability P_n of n photoelectrons at the photocathode due to the incoming light is given by a Poisson statistic²

$$P_n = \frac{N\bar{m}^n e^{-\bar{m}}}{n!} . \quad \begin{array}{ll} N & : \text{Normalisation} \\ \bar{m} & : \text{mean number of PE} \end{array} \quad (1)$$

This probability is normalised to the total number of events in a histogram (N) or $N = 1$, in case of a probability density.

- The position x_n of the n -th PE-peak scales linear

$$x_i = c_0 + n \cdot c_1 . \quad (2)$$

c_0 is a pedestrial, c_1 the time to integrate each PE charge.
Thus the position of the 1-PE-peak x_1 is

$$x_1 = c_0 + c_1 \quad (3)$$

and the distance between two peaks

$$x_i - x_j = (i - j) \cdot c_1 \quad (4)$$

The values of the parameters c_1 and c_2 depend on the actual gain and settings of the PMT and electronics.

- The pulse-height distribution for 1-PE and thus the distribution of charge, $G_1(x)$, is Gaussian³. The width is given by the parameter σ_1

$$G_1(x) = \frac{1}{\sqrt{2\pi} \cdot \sigma_1} \cdot e^{-\frac{(x - x_1)^2}{2\sigma_1^2}} . \quad (5)$$

σ_1 is given by the energy-resolution of the PMT. The energy resolution (FWHM) may be calculated via

$$\frac{\Delta E_{(\text{FWHM})}}{E} = 2 \cdot \sqrt{2 \cdot \ln 2} \cdot \frac{\sigma_1}{c_1} \approx 2,355 \cdot \frac{\sigma_1}{c_1} . \quad (6)$$

²This is only a rough approximation.

³Due to the high gain in the first stage of the PMT this is a good assumption.

- The distribution for n-PE signals is also Gaussian. Its width σ_n is

$$\sigma_n = \sqrt{n} \cdot \sigma_1 . \quad (7)$$

Thus the absolute width for n PE scales with $\sim \sqrt{n}$ and the relative width $\sim 1/\sqrt{n}$. The distribution $G_n(x)$ becomes

$$G_n(x) = \frac{1}{\sqrt{2\pi}\sigma_n} \cdot e^{-\frac{(x-x_n)^2}{2\sigma_n^2}} . \quad (8)$$

Everything put together:

$$G(x) = \sum_{n=1}^{\infty} P_n \cdot G_n(x) \quad (9)$$

gives

$$G(x) = \sum_{n=1}^{\infty} \frac{N\tilde{m}^n e^{-\tilde{m}}}{n!} \cdot \frac{1}{\sqrt{2\pi n} \cdot \sigma_1} \cdot \exp\left(-\frac{(x - nc_1 - c_2)^2}{2n\sigma_1^2}\right) . \quad (10)$$

The Amplitude-function derivatives maybe usefull for fitting purposes.

$$\begin{aligned} \frac{\partial G(x)}{\partial c_1} &= \sum_{n=1}^{\infty} \frac{N\tilde{m}^n e^{-\tilde{m}}}{n!} \cdot \frac{(x - nc_1 - c_0)}{\sqrt{2\pi n}\sigma_1^3} \cdot e^{-\frac{(x - nc_1 - c_0)^2}{2n\sigma_1^2}} \\ &= G(x) \cdot \frac{(x - nc_1 - c_0)}{\sigma_1^2} \end{aligned}$$

$$\begin{aligned} \frac{\partial G(x)}{\partial c_0} &= \sum_{n=1}^{\infty} \frac{N\tilde{m}^n e^{-\tilde{m}}}{n!} \cdot \frac{(x - nc_1 - c_0)}{\sqrt{2\pi n^3}\sigma_1^3} \cdot e^{-\frac{(x - nc_1 - c_0)^2}{2n\sigma_1^2}} \\ &= G(x) \cdot \frac{x - nc_1 - c_0}{n\sigma_1^2} \end{aligned}$$

$$\begin{aligned} \frac{\partial G(x)}{\partial \tilde{m}} &= \sum_{n=1}^{\infty} \frac{N\tilde{m}^n e^{-\tilde{m}}}{n!} \cdot \left(\frac{\sqrt{n}}{\tilde{m}\sqrt{2\pi}\sigma_1} - \frac{1}{\sqrt{2\pi n}\sigma_1} \right) \cdot e^{-\frac{(x - nc_1 - c_0)^2}{2n\sigma_1^2}} \\ &= G(x) \cdot \left(\frac{n}{\tilde{m}} - 1 \right) \end{aligned}$$

$$\begin{aligned} \frac{\partial G(x)}{\partial \sigma_1} &= \sum_{n=1}^{\infty} \frac{N\tilde{m}^n e^{-\tilde{m}}}{n!} \cdot \left(\frac{(x - nc_1 - c_0)^2}{\sqrt{2\pi n^3}\sigma_1^4} - \frac{1}{\sqrt{2\pi n}\sigma_1^2} \right) \cdot e^{-\frac{(x - nc_1 - c_0)^2}{2n\sigma_1^2}} \\ &= G(x) \cdot \left(\frac{(x - nc_1 - c_0)^2}{n\sigma_1^3} - \frac{1}{\sigma_1} \right) \end{aligned}$$

$$\begin{aligned}\frac{\partial G(x)}{\partial N} &= \sum_{n=1}^{\infty} \frac{\tilde{m}^n e^{-\tilde{m}}}{n!} \cdot \frac{1}{\sqrt{2\pi n\sigma_1}} \cdot e^{-\frac{(x - nc_1 - c_0)^2}{2n\sigma_1^2}} \\ &= G(x) \cdot \frac{1}{N}\end{aligned}$$

2.2 The logarithmic region

With the following assumption the description may be extended into the logarithmic region: If the amplitude of a PMT-pulse is higher than a certain threshold ($\sim 2V$), the pulse is cut off above this threshold by the electronic. Thus only a part of the pulse charge is integrated.

This introduces a new constant n_{log} which is the number of PE, when pulse-amplitudes of the PMT begin to reach this thresh value.

This PE-threshold constant n_{log} corresponds to a threshold constant x_{log} for the pulse width (integrated charge) given by

$$x_{log} = c_0 + n_{log} \cdot c_1. \quad (11)$$

The result of the loss of charge during the integration is a logarithmic charge integration of the n -th PE-peak. A first approximation⁴ of x_n is:

$$x_n = x_{log} \left(1 + \ln \frac{n}{n_{log}} \right). \quad (12)$$

The inverse equation

$$i(x) = n_{log} \cdot \exp \left(\frac{x}{x_{log}} - 1 \right) \quad (13)$$

may be interpreted as a PE number (in the PE-space) corresponding to a specific pulse-width x . The derivate

$$\frac{d}{dx} i(x) = \frac{1}{x_{log}} \cdot i(x) \quad (14)$$

will be used later.

The intrinsic probability function $\tilde{G}(i)$ for the PE-distribution of the PMT is not affected by the cut-off in the integration electronics.

$$\tilde{G}(i) = \sum_n P_n \tilde{G}_n = \sum_n P_n \cdot \frac{1}{\sqrt{2\pi} \cdot s_n} \cdot e^{-\frac{(i - n)^2}{2s_n^2}}. \quad (15)$$

⁴ Assuming an infinite rise-time, the integrated charge Q is the integral of an exponential decaying function. For amplitudes (I) higher than the threshold amplitude (I_t) Q consists out of the integral below this thresh. Thus $Q = I_t \cdot \tau + I_t \cdot \tau \cdot \ln \frac{I}{I_t}$, with the decay time τ . It is: $x_{log} = \tau \cdot I_t$ and $I \sim n$

The intrinsic energy resolution s_n is given by the relative width of the 1PE- peak (see eq.6)

$$s_n = \sqrt{n} \cdot \frac{\sigma_1}{c_1} \quad (16)$$

For the charge distribution $G(x)$ is

$$G(x)dx = \tilde{G}(i(x)) \cdot \frac{di(x)}{dx} \cdot dx = \tilde{G}(i)di . \quad (17)$$

Therefore

$$\begin{aligned} G(x) &= \sum_n P_n \frac{1}{\sqrt{2\pi} \cdot s_n} \cdot \frac{d}{dx} i(x) \cdot \tilde{G}_n(i(x)) \\ &= \sum_n P_n \frac{1}{\sqrt{2\pi} \cdot \sigma_n} \cdot \frac{n_{log} c_1}{x_{log}} \cdot \exp \left(\frac{x}{x_{log}} - 1 - \frac{c_1^2 \cdot (n_{log} \cdot e^{\left(\frac{x}{x_{log}} - 1\right)} - n)^2}{2\sigma_n^2} \right) . \end{aligned} \quad (18)$$

Note that eq.(18) has only one additional constant compared to eq.(10). Its characteristic width constant σ_i is the same as in the linear region.

3 Fit to the distributions

The following table lists the parameters⁵ which are calibrated in the following. After this is done, the shape of pulse-height distributions depend only on the external light intensity (or \bar{m} respectively).

Symbol	Meaning	typical value (with DMQT)
c_1	distance between two PE-peaks	120 ns
c_0	pedestrian of the pulse-widths-scale	40 ns
σ_1	standard deviation of the 1-PE-peak	35 ns
n_{log}	bin of logarithmic scaling (in PE-space)	3 PE

Figure 2 shows a fit of simple gaussian to those distributions of figure 1 which show a clear 1PE-Peak. Fitted parameters are the position of 1PE-peak x_1 (P1) and its width σ_1 (P2). The third constant is a normalisation factor (P3).

The results for these fits are averaged and the values for x_1 and σ_1 remain fixed for all later fits.

Figure 3 shows fits of the amplitude function to those distributions of figure 1 which show a clear low PE-peaks (linear region). The fitted parameters are the mean number of PE,

⁵The other parameters x_1 and x_{log} are calculated via eq.(3) and (11).

\bar{m} (P1), the distance between two PE-peaks⁶ c_1 (P3), and again a normalisation factor (P2).

The results for these fits are again averaged. Via eq.(3) c_0 is being calculated and the values for c_1 and c_0 are fixed to their mean for all later fits. Eq.(6) gives the energy resolution $\frac{\Delta E}{E}$ (FWHM).

Finally figure 4 shows fits of the amplitude function to those distributions of figure 1 which show a clear high PE-peak (logarithmic region). The fitted parameters are the beginning of the logarithmic threshold n_{log} (P1), the mean number of PE \bar{m} (P3) and a normalisation factor (P2).

The averaged n_{log} is used to calculate x_{log} (eq.(11)).

EOM No: #	x_1 [ns]	σ_1 [ns]	c_1 [ns]	c_0 [ns]	n_{log} [PE]	x_{log} [ns]	$\frac{\Delta E}{E}$ [%]
11	151.6	20.4	69	82.6	2.0	220.5	70
12	165.0	28.3	120.2	47.9	4.7	610.2	55
16	135.6	24.1	78.1	57.5	5.2	462.1	73
19	223.2	46.8	189.6	33.6	2.8	566.4	58
22	173.8	44.2	156.2	17.7	2.8	452.7	66
26	178.7	40.6	145.0	33.7	2.4	380.2	66
33	171.8	34.0	133.9	37.9	4.6	650.0	60
44	134.7	24.5	97.5	37.2	4.8	501.3	59
46	177.5	33.1	136.2	41.3	8.6	1219.5	57
53	186.3	36.3	149.6	36.7	2.6	463.0	57

Table 1: Calibration results

Table 1 shows the calibration results for all EOM's.

Figure 5 shows the pulse-height distributions of EOM #26 (figure 1). Superimposed is the calibrated amplitude function. The parameters are set to the average values. The mean number of PE \bar{m} is set to the fitted value.

Figure 6 shows the *amplitude function* with calibrated parameters for EOM #26. The mean number of PE \bar{m} scales from 0.5 (top left) to 12 (bottom right).

⁶see eq.(4)

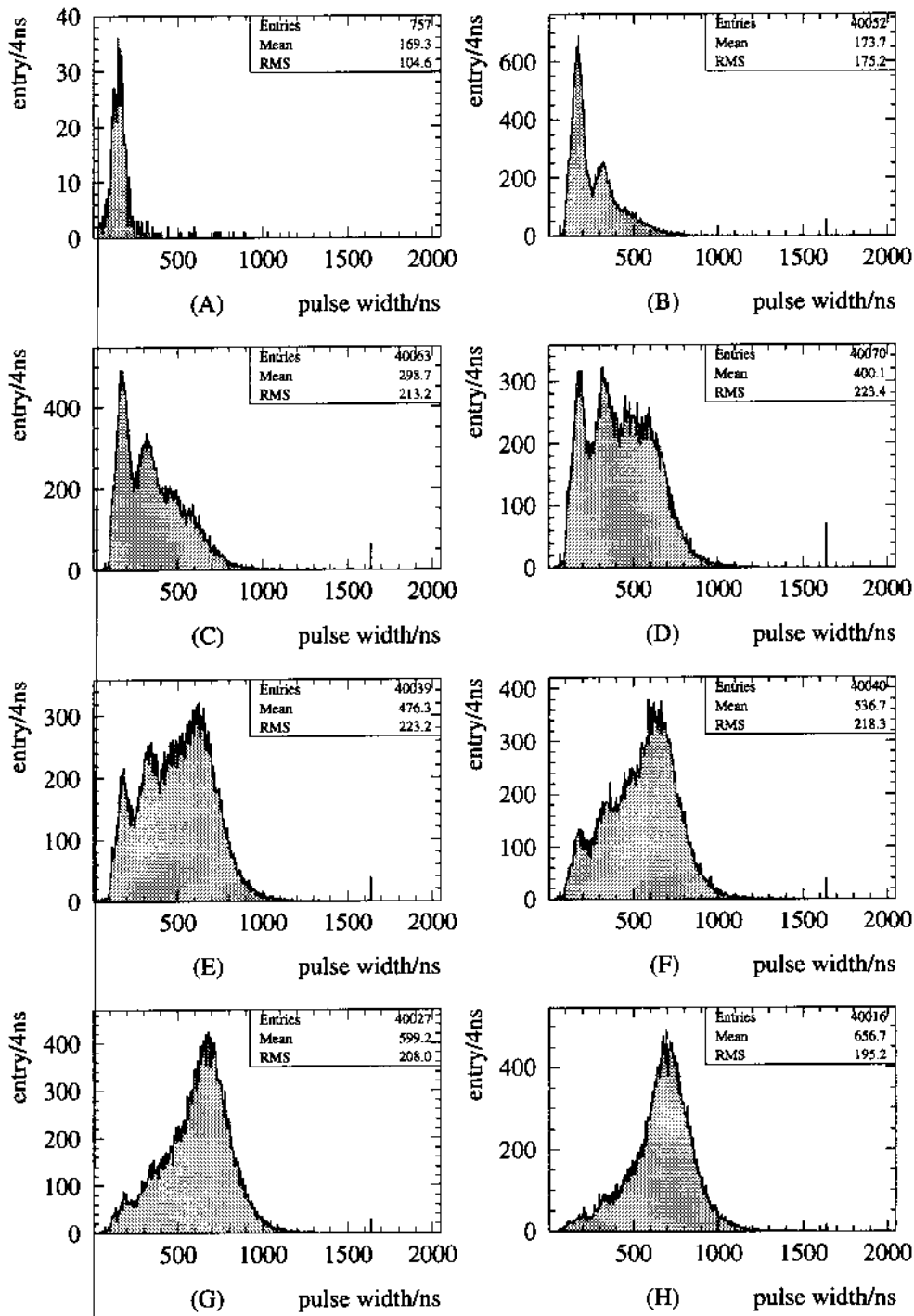


Figure 1: Pulse height distributions for EOM # 26 . (The intensity of the illumination increases from $A \rightarrow H$)

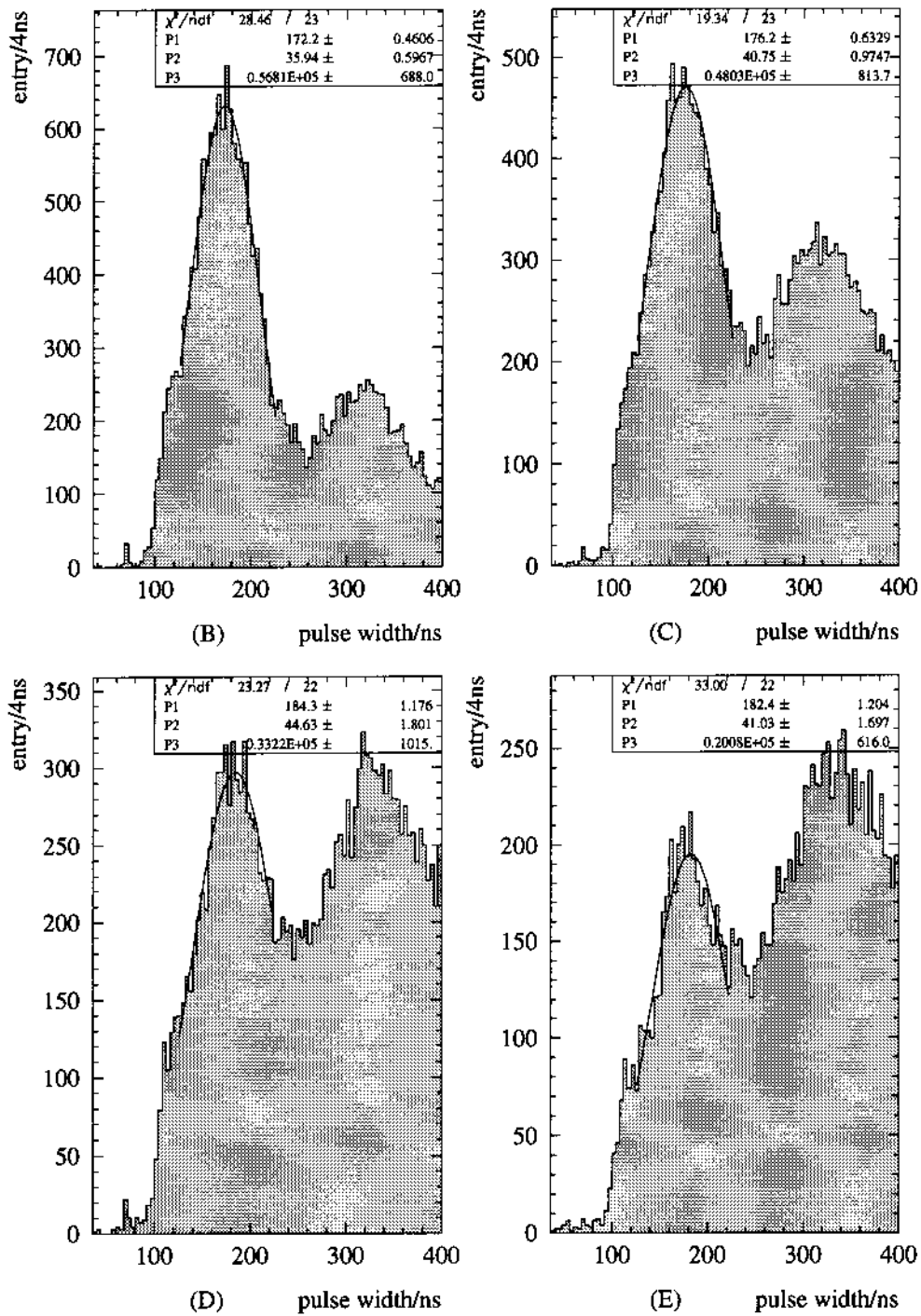


Figure 2: Fit to the 1-PE peak

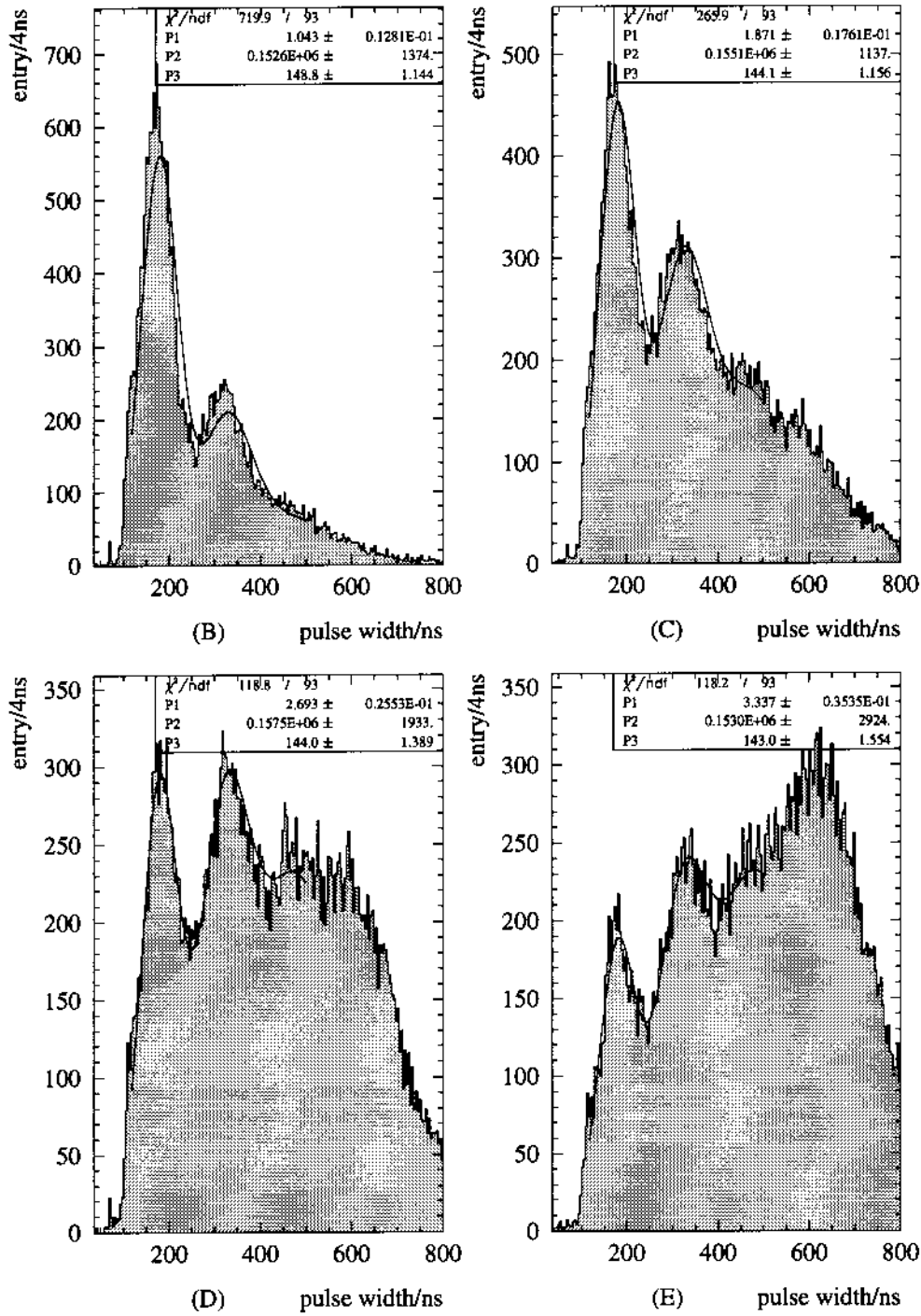


Figure 3: Fit to the linear region

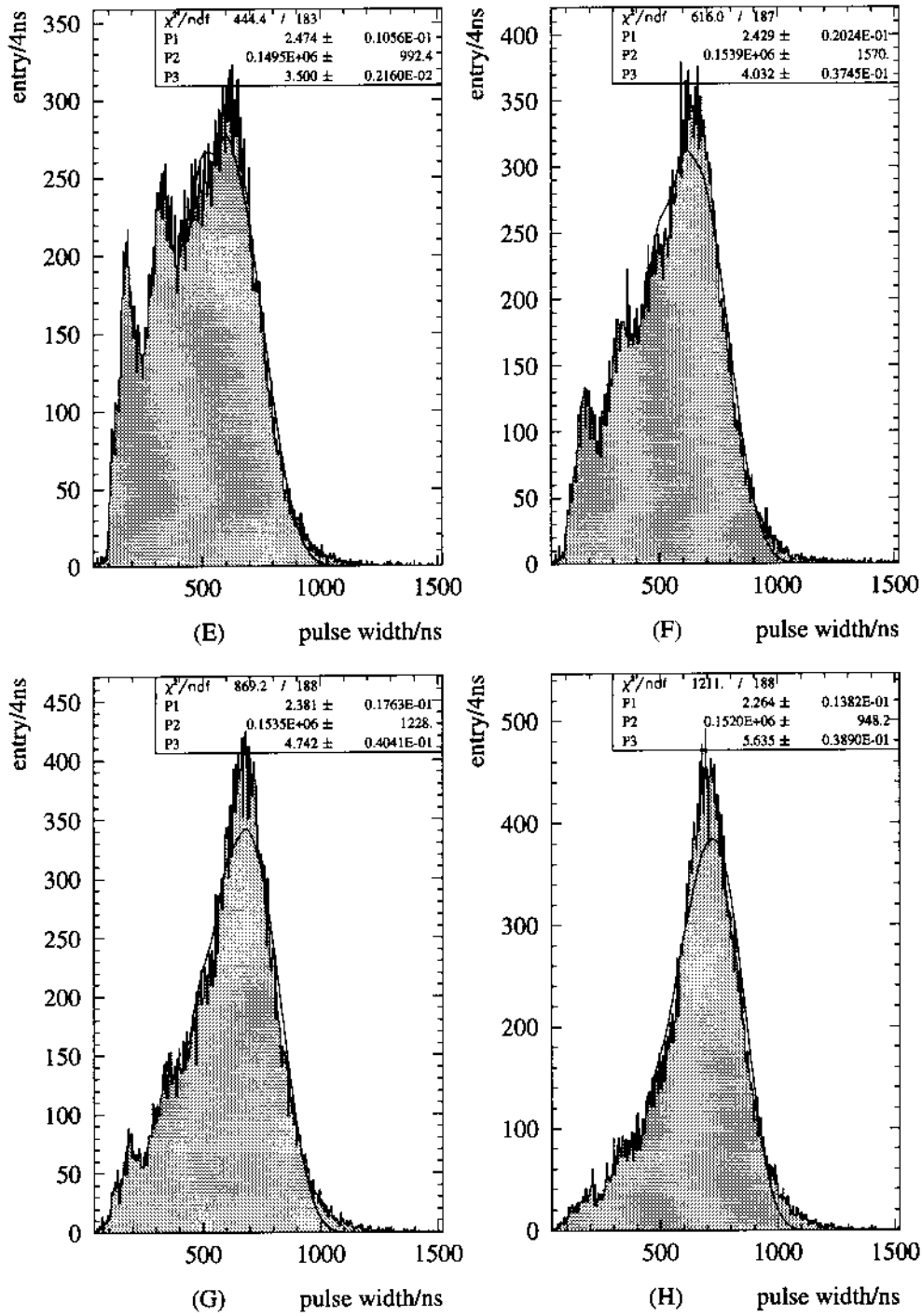


Figure 4: Fit to the logarithmic region

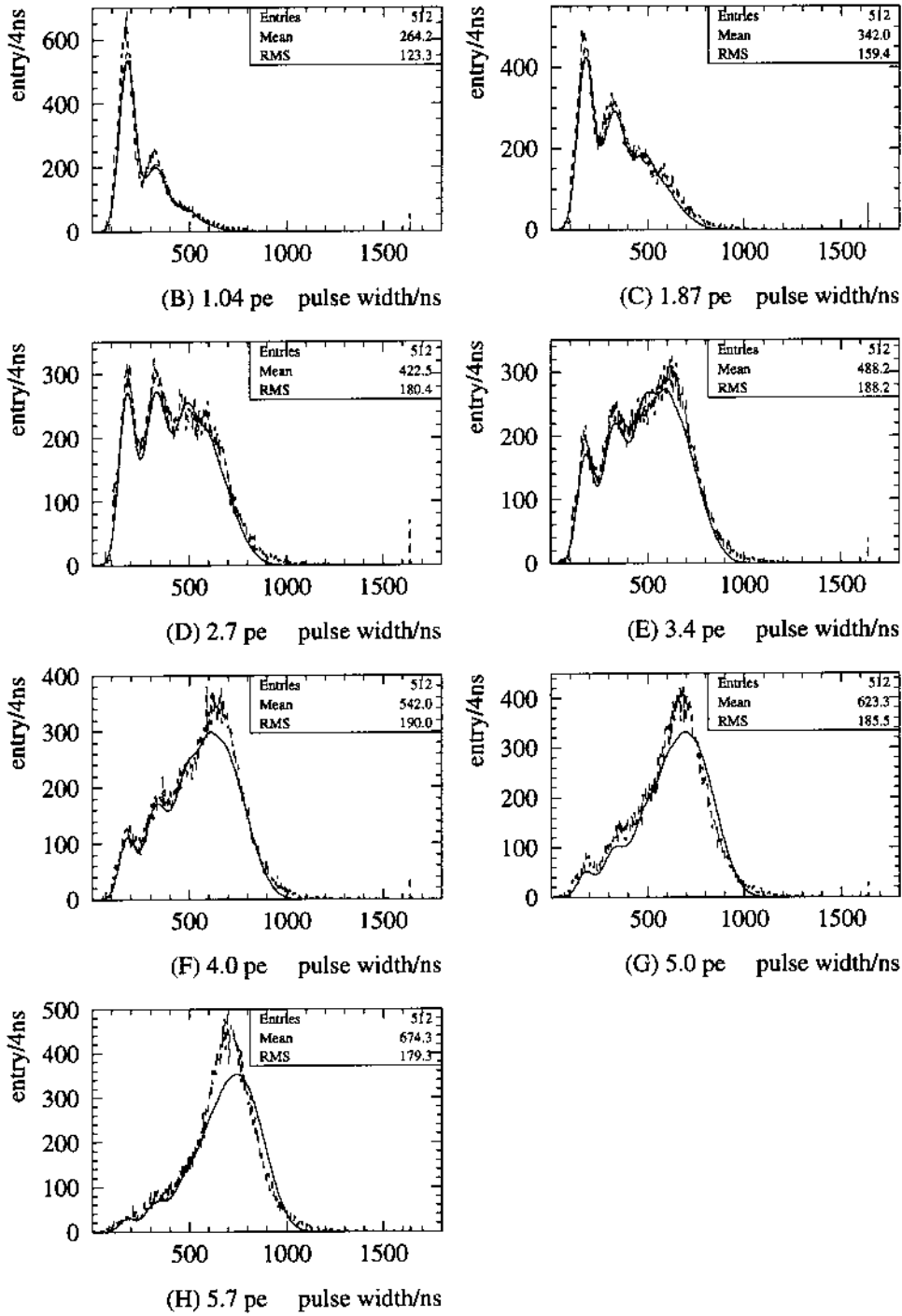


Figure 5: Amplitude function superimposed on the pulse-height distributions

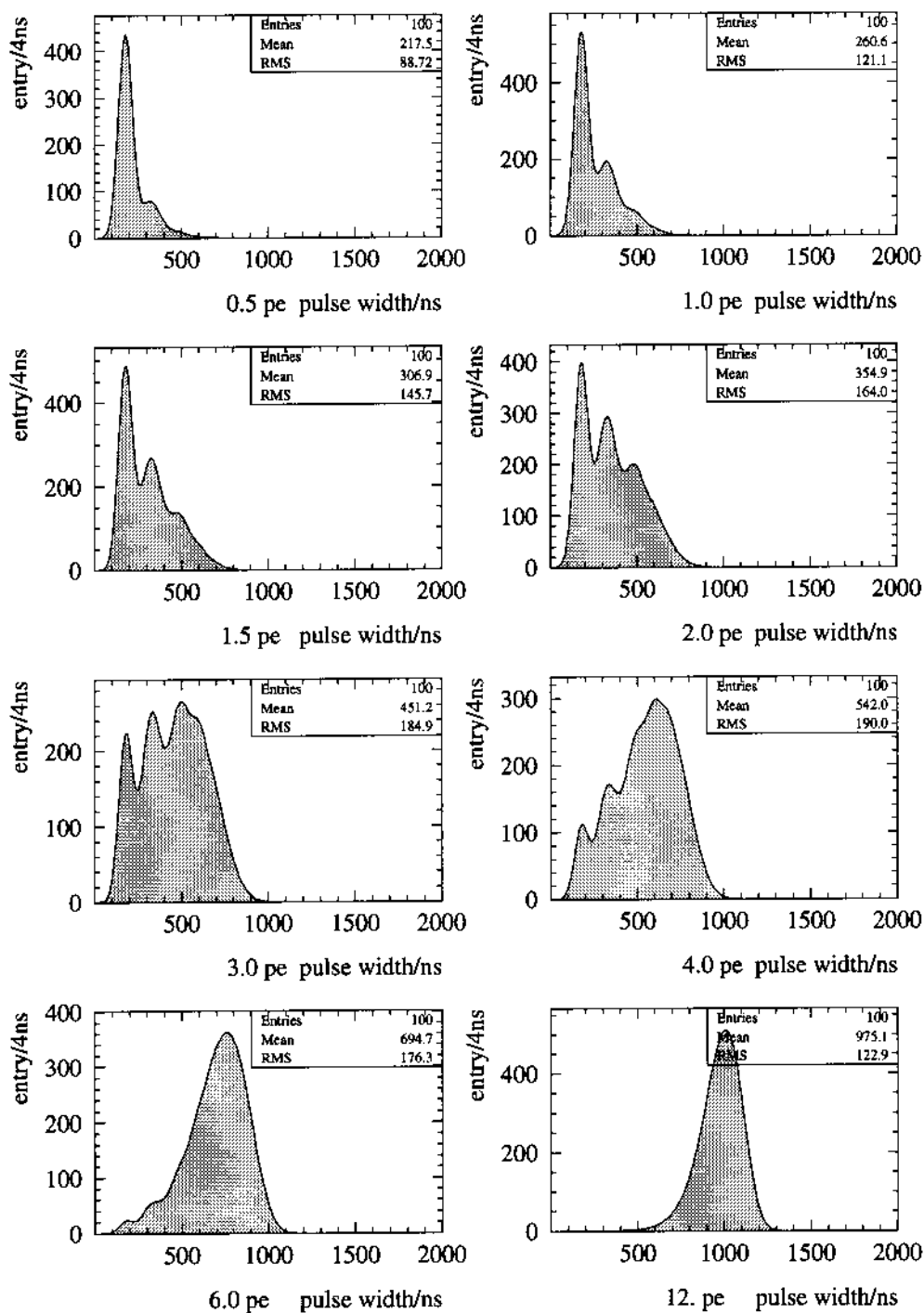


Figure 6: Calibrated amplitude function for different mean PE

4 Conclusions

The fits in the previous section show, that the *amplitude function*, which is based only on analytical assumption fulfills the following:

- This function is a good approximation of the, especially in the low PE-region, shape of pulse height distribution. The possible physical interpretation for each parameter proves, that the response function of the EOM to external light is well understood.
- The fitted calibration constants remain invariant under scaling of the light intensity. This means, that it is possible to have a complete description of the EOM, depending on intrinsic constants.
- This approximation is not perfect. Especially for actual data reconstruction in DUMAND II it may prove more reasonable to do a simple numerical parametrisation of the mean and RMS of the pulse-height versus PE. For other purposes like triggering [3, 4] the knowledge of this function in the low-PE region provides new opportunities. Also for the simulation of the detector the simulation of the actual shape instead of a simple mean-RMS simulation may be of interest.

A further improvement may be achieved, if a more accurate description of the actual PMT-pulseshape is used in the integration, that leads to eq.(12) (e.g.[4]—equation (4.5)).

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