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ν_e APPEARANCE FOR "$\nu_\mu - \nu_\tau$" OSCILLATIONS IN MATTER

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Abstract

New ranges of neutrino oscillation parameters can be probed by sending accelerator produced ν_μ to large neutrino detectors thousands of kilometers away. Since the neutrino propagation is through the Earth, the $\nu_e - \nu_\tau$ mixing can be enhanced by matter effects. Then a neutrino coupled dominantly to the ν_τ, with a mass squared of 10^{-3} eV^2 to 10^{-1} eV^2, may most easily be observed by a ν_e appearance experiment.

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Extending accelerator neutrino experiments to distances of thousands of kilometers is an often discussed method for probing new ranges of neutrino mass and mixing parameters. Such Extra Long Accelerator Neutrino (ELAN) experiments may become a reality in the near future [1,2]. One reason for this is the improved quality of neutrino beams that are possible using the FNAL Main Injector or the SSC injector. Another reason is that the evidence for massive neutrinos has never been stronger.

Solar neutrino observations provide compelling evidence for a neutrino mass squared difference in the range of 4×10^{-8} to 10^{-4} eV^2 (the Mikheyev-Smirnov-Wolfenstein solution, for reviews see [3,4]). Since there are three light neutrino flavors, there could also exist a more massive neutrino coupled dominantly to the ν_τ. In particular, such a neutrino could explain recent atmospheric neutrino observations [5].

ELAN experiments and atmospheric neutrino observations are sensitive to roughly the same range of neutrino masses. Which of these two type of experiments probes smaller neutrino mixings depends on the details of the experiment, however there can be a substantial difference. Atmospheric neutrino observations are, for the most part, disappearance experiments. There are two reasons for this. The first is because the atmospheric neutrino flux contains a mixture of ν_μ's and ν_e's. The second is because tau's decay too fast to be directly observed in large detectors, so their appearance is indistinguishable from neutral current events. Thus it is difficult to probe small values of the neutrino mixing using atmospheric neutrinos. But ELAN experiments can in principle do much better. The tau still decays too fast to be resolved (millimeter resolution in a megaton detector is too expensive), but accelerators can make a neutrino beam which is dominantly ν_μ or $\bar{\nu}_\mu$ with only a very small contamination of ν_e or $\bar{\nu}_e$. Thus a ν_e appearance experiment is possible. This would allow ELAN experiments to probe much smaller mixings than atmospheric neutrino experiments. Here we examine exactly what neutrino mass and mixing parameters an ELAN ν_e appearance experiment would probe.

A general discussion of three flavor effects for arbitrary neutrino masses in a matter background is quite involved. Herein we shall make the simplifying assumption that the vacuum oscillation wavelength associated with the lightest two neutrinos is larger than the radius of the Earth (as would be necessary for the MSW solution to the solar neutrino problem) and so it is a good approximation for terrestial experiments to take the vacuum masses as

$$m_1 = m_2 = 0, \quad m_3 > 0. \tag{1}$$

Although in this approximation there is only one massive neutrino, the dynamics are not equivalent to that of the two flavor approximation. Two relevant mixing parameters are necessary to specify the relation

$$|\nu_\alpha> = U_{\alpha i}|\nu_i> \tag{2}$$

between the flavor eigenstates, $\alpha = e, \mu, \tau$, and the mass eigenstates, $i = 1, 2, 3$. We choose the parametrization [6]:

$$U = \begin{vmatrix} 0 & \cos\phi & \sin\phi \\ -\cos\psi & -\sin\psi\sin\phi & \sin\psi\cos\phi \\ \sin\psi & -\cos\psi\sin\phi & \cos\psi\cos\phi \end{vmatrix} \tag{3}$$

where ϕ and ψ are mixing angles. With $m_1 = m_2$, one linear combination of these states may be chosen orthogonal to the ν_e, hence the zero element in Eq. (3). The usual two flavor approximation is attained in the limit $\phi = 0$.

In ELAN experiments, the neutrinos propagate a long distance through the Earth. The background of electrons induces a mass squared [7] for the electron-neutrino

$$\begin{aligned} A &= 2\sqrt{2}G_F(Y_e\rho/m_u)E \\ &= 0.76\times10^{-2}eV^2\left(\frac{Y_e\rho}{2.5\frac{gm}{cm^3}}\right)\left(\frac{E}{20GeV}\right) \end{aligned} \tag{4}$$

from the charged current interaction [3,4]. Here Y_e is the number of electrons per nucleon, ρ is the density, E is the neutrino energy, m_u is the nucleon mass; and for antineutrinos $A \to -A$. This mass squared scale corresponds to a general length scale for matter effects of

$$\lambda_m^0 = 6400km\left(\frac{2.5\frac{gm}{cm^3}}{Y_e\rho}\right) \tag{5}$$

For propagation lengths comparable to this or larger, the background matter plays an important role. Since this is less than the diameter of the Earth, matter effects can be relevant for ELAN experiments [8].

With the chosen three-flavor parametrization of Eq. (3), it follows [6] that ψ is independent of A while

$$\sin^2 2\phi_m = \frac{(m_3^2 \sin 2\phi)^2}{[(A - m_3^2 \cos 2\phi)^2 + (m_3^2 \sin 2\phi)^2]} \tag{6}$$

Similarly, the mass eigenstates in matter are

$$\begin{aligned} M_1^2 &= 0 \\ M_{3,2}^2 &= [\ (m_3^2 + A) \pm \sqrt{(A - m_3^2 \cos 2\phi)^2 + (m_3^2 \sin 2\phi)^2}\ ]/2 \end{aligned} \tag{7}$$

Naievely one expects the vacuum mixings to be small since those observed in the quark sector are. But when $A \approx m_3^2$ a resonance occurs and the effective $\nu_e - \nu_\tau$ mixing in matter, ϕ_m, becomes maximal. Then the two-flavor approximation of neglecting this mixing is not valid, unless ϕ is extremely small.

To illustrate the effects of the matter background on ELAN experiments, we give the relevant three-flavor oscillation probabilities for a constant matter density.

$$P(\nu_\mu \to \nu_e) = \frac{1}{2} \sin^2 \psi \sin^2 2\phi_m (1 - \cos[\beta_3 - \beta_2]) \tag{8}$$

$$\begin{aligned} P(\nu_\mu \to \nu_\mu) &= 1 - \frac{1}{2}\{\sin^2 2\psi [1 - \sin^2 \phi_m \cos[\beta_2 - \beta_1] - \cos^2 \phi_m \cos[\beta_3 - \beta_1]] \\ &\quad + \sin^4 \psi \sin^2 2\phi_m (1 - \cos[\beta_3 - \beta_2])\} \end{aligned} \tag{9}$$

$$\begin{aligned} P(\nu_\mu \to \nu_\tau) = \frac{1}{2} \sin^2 2\psi \{ & 1 - \sin^2 \phi_m \cos^2 \phi_m - \sin^2 \phi_m \cos[\beta_2 - \beta_1] \\ & - \cos^2 \phi_m \cos[\beta_3 - \beta_1] + \sin^2 \phi_m \cos^2 \phi_m \cos[\beta_3 - \beta_2]\} \end{aligned} \tag{10}$$

where the dynamical phase acquired by a neutrino mass eigenstate which propagates for a time t is

$$\beta_i \equiv \frac{M_i^2 t}{2E}. \tag{11}$$

The disappearance probability in Eq. (9) is rather insensitive to matter effects. However the situation is very different in Eq. (8). The probability for ν_e appearance *in vacuum* is suppressed by four powers of the off diagonal vacuum mixing matrix elements. But in matter the ϕ mixing parameter is enhanced and so the suppression by two powers of the off diagonal mixing parameters is removed for sufficiently long propagation lengths. ***Near the resonance, the probability of ν_e appearance is enhanced to be comparable to the probability of ν_τ appearance!***

For the purposes of this work, Eq. (8) was used for neutrino propagation in the mantle (Fig. (1)), and a two density model of the Earth was used to calculate an analytical probability for describing propagation through the core (Fig. (2)). This was checked against numerical integration over the full, inferred density distribution [9] and found to be a roughly accurate approximation. However to thoroughly account for the changing density values in the Earth requires the wave equation to be solved numerically. Fortunately, with the parametrization of Eq. (3) there is a general simplification. As is true for constant density it is not hard to show that also for arbitrary density distributions the expression for $P(\nu_\mu \rightarrow \nu_e)$ factorizes. It is always of the form of an overall factor of $\sin^2\psi$ times an expression which is independent of ψ and which in fact is just the same probability as calculated in the two flavor approximation. The $\nu_\mu - \nu_\tau$ mixing parameter only gives an overall modulation factor to ν_e appearance, it never enters into the dynamics.

The parameter regions accessible to ELAN experiments are estimated in Figs. (1) and (2). These figures show m_2^3 versus the $\nu_e - \nu_\tau$ mixing parameter $\sin^2 2\phi$ for two values of the $\nu_\mu - \nu_\tau$ mixing parameter, $\sin^2\psi = 0.3$ and 0.03. The ν_e appearance probability is chosen to be 0.01, comparable to the ν_e contamination estimated in [2]. The calculations are for propagations lengths of 6400 km and 11,400 km. The energy distribution of the neutrino event rate is taken from calculations for a short baseline experiment using the FNAL main injector [2] with 120 GeV protons. In an actual ELAN experiment, the beam energy and/or detector threshold could easily be varied by factors of 2 from these values. Approximately, this would shift the curves in Figs. (1) and (2) vertically by similar amounts.

In the figures, the large "nose" extending to small values of $\sin^2 2\phi$ is the result of the matter enhancement of the mixing. The enhancement is the most at the resonance where $A \approx m_3^2$ as given in Eq. (4). On the resonance the mass squared splitting is proportional to $\sin 2\phi$ so the contours extends to smaller values of the mixing for the longer propagation length, Eq. (11). Also the enhancement extends to much larger m_3^2 for the longer propagation length because then the neutrinos travel through the higher density matter of the core. In the parameter region below the resonance, $A >> m_3^2$ and the mixing is suppressed. For antineutrinos the mixing is never enhanced but is only suppressed by matter effects (assuming $\phi < \pi/4$), and so the corresponding contours for antineutrinos are given by the dotted lines.

No existing detector is ideal for an ELAN ν_e appearance experiment. However it may be feasible in planned neutrino detectors (such as DUMAND, Super Kamiokande, AMANDA, etc.) or by constructing a new detector. A choice must be made as to the "best" baseline length. To aid such a choice let us note that, because atmospheric neutrinos are mostly lower in energy than accelerator neutrinos, to completely cover the neutrino mass parameter region suggested by atmospheric neutrino experiments [5] requires an accelerator neutrino propagation length of at least a few thousand kilometers. At these distances matter effects are starting to become important and give advantages over pure vacuum oscillations. For a given value of $P(\nu_\mu \to \nu_e)$, the accessible mass versus mixing parameter region is *enlarged* by matter effects. In addition, the minimum measurable value of $P(\nu_\mu \to \nu_e)$ is decreased by matter effects. This is because the increased energy sensitivity (see Eqs. (4-7)), and the difference between ν's and $\bar{\nu}$'s, provide additional checks on the experimental results. Thus matter effects act to compenstate for the smaller event rates in ELAN experiments.

In summary, a ν_e appearance experiment with ELAN could probe very small mixing angles for neutrino masses down to 10^{-3} eV^2. In particular, for typical accelerator neutrino energies and core penetration, the mixing between the ν_e and a neutrino with a mass in the range 10^{-3} eV^2 to 10^{-1} eV^2 is enhanced by matter effects. If the massive neutrino couples dominantly to the ν_μ, the $\nu_\mu - \nu_e$ mixing matrix element is probed [8]. If the massive neutrino couples dominantly to the ν_τ (as may be the case if the solar neutrino problem is solved by the MSW effect), then the matter effects enhance the $\nu_\tau - \nu_e$ mixing. Then the probability of ν_e appearance can be comparable to the probability of ν_τ appearance in the ν_μ beam. Since it is much much easier to identify electrons than taus, this increases the observablility of oscillations.

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Figure Caption

Plot of vacuum mass squared versus $\nu_e - \nu_\tau$ mixing. The solid (dotted) contours are for ν ($\bar{\nu}$) oscillations with P($\nu_\mu \rightarrow \nu_e$)= 0.01 and $\sin^2\psi$ = 0.3 and 0.03. The dashed line is the constraint from the Gosgen reactor disappearance experiment [10].

1. An angle of 30 degrees below the horizon (the FNAL to DUMAND angle).

2. An angle 66 degrees below the horizon (the FNAL to AMANDA angle).

Figure 1

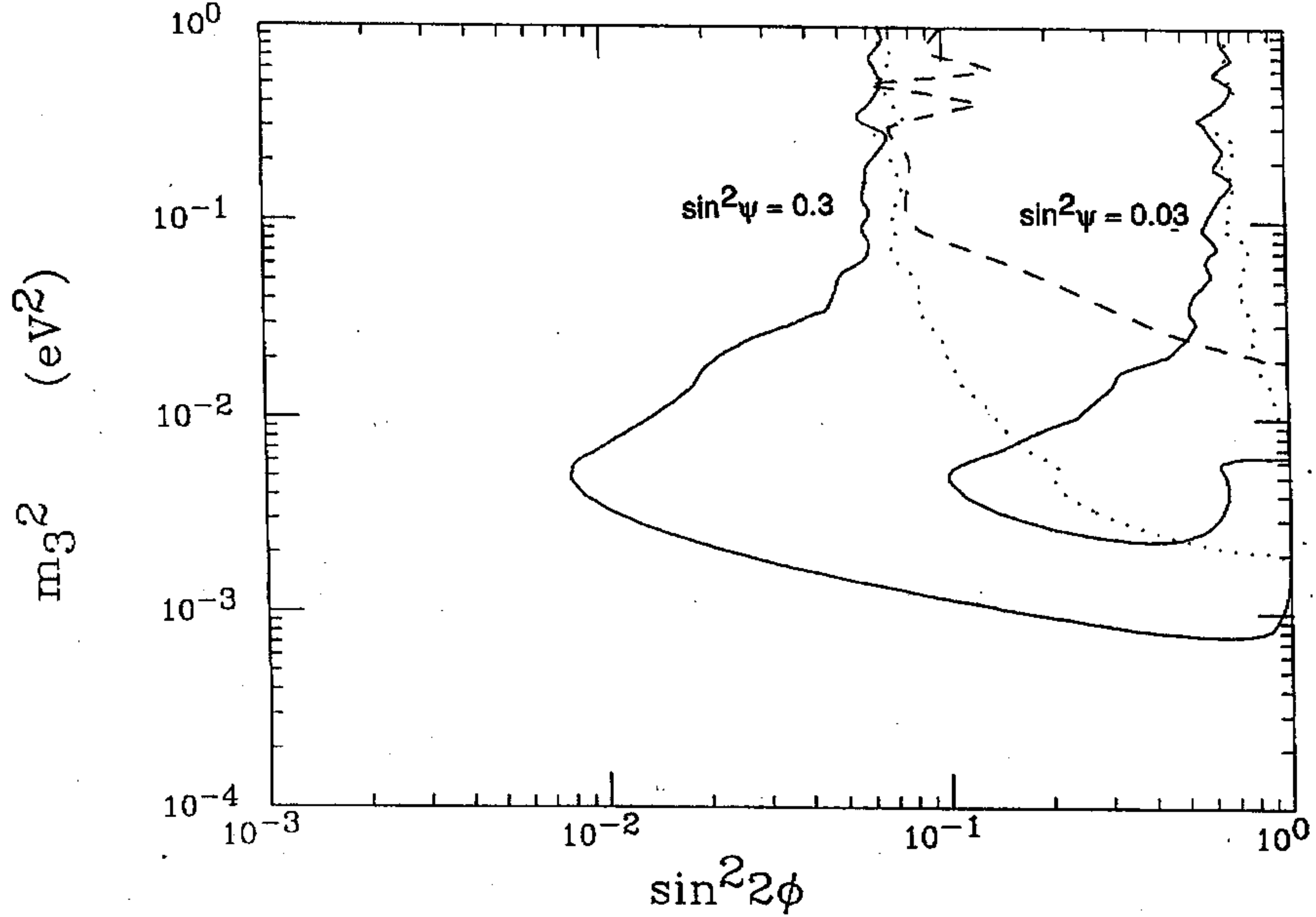

Figure 2