DUMAND Internal Report
DIR-5-86

DETERMINATION OF WATER ATTENUATION FROM SPS CALIBRATION RUNS

V.J. Stenger
December 1, 1986

There have been several informal notes circulated on how to use the SPS calibration runs to determine the water attenuation. This note describes how it is actually done in DECAL, the program developed to control the calibration run. The operation of DECAL will be described in a separate note.

The photoelectron charge collected by PMT_i from the light emitted by CM_j with step filter k is, in JGL's notation:

$$Q_{ijk} = \Phi_j \, \eta_i \, A_i \, F_i(\theta_{ij}) \, \exp(-\alpha \, r_{ij})/(4\pi r_{ij})^2 \, g_{jk} \, h_{ij} \tag{1}$$

where Φ_j is the total number of photons emitted by the CM ball (nominally 5×10^7), η_i is the quantum efficiency, A_i is the PMT area, $F_i(\theta)$ is the PMT angular response functions (nominally $0.55 + 0.45 \cos\theta$), r_{ij} is the distance from CM_j to PMT_i, g_{jk} is the step filter attenuation (1, 1/2, 1/4, 1/8) and h_{ij} is the angular filter on the laser ball (see Table I). We want to determine the <u>attenuation coefficient</u> $\alpha = 1/L$, where L is the <u>attenuation length</u> at the wavelength of the light emitted by the ball.

Dropping all but the PMT indices, I write for a given j and k:

$$Q_i = Q_{io}\exp(-\alpha \, r_i) \tag{2}$$

Q_{io} would be the charge for zero attenuation.

Since we measure the log of the charge anyway, I then form:

$$\chi^2 = \sum_i \frac{(\ln Q_i - \ln q_i)^2}{(\delta \ln q_i)^2} \tag{3}$$

where q_i is the measured charge in PMT_i.

Now let me take

$$\delta \ln q_i = \frac{\delta q_i}{q_i} = \frac{\sqrt{q_i}}{q_i} = \frac{1}{\sqrt{q_i}} \tag{4}$$

so that

$$\begin{aligned} \chi^2 &= \sum_i q_i(\ln Q_{io} - \alpha r_i - \ln q_i)^2 \\ &= \sum_i (a_i - \alpha b_i)^2 \end{aligned} \tag{5}$$

where

$$\begin{aligned} a_i &= \sqrt{q_i}\ (\ln Q_{io} - \ln q_i) \\ b_i &= \sqrt{q_i}\ r_i \end{aligned} \tag{6}$$

You might quibble that I should use $\sqrt{(Q_i + 1)}$, but this is just a weighting factor and should be good enough. PMT's with $q_i = 0$ are not used in the fit.

Differentiating (5) with respect to α and setting to zero one can solve for the value of α which minimizes χ^2:

$$\alpha = \frac{\sum_i a_i b_i}{\sum_i b_i^2} \tag{7}$$

Thus all PMTs are used to determine the best value of $L = 1/\alpha$.

The error in α from <u>one event</u> can be determined by

$$\delta\alpha = \left(\frac{\partial^2 \chi^2}{\partial \alpha^2}\right)^{-\frac{1}{2}} = \left(2\sum_i b_i^2\right)^{-\frac{1}{2}} = \left(2\sum_i q_i\ r_i^2\right)^{-\frac{1}{2}} \tag{8}$$

An upper limit on the error in α can be calculated just from geometry in the case where $q_i = 1$. This gives for the fractional errors, $\delta\alpha/\alpha = \delta L/L = 0.5$. S. Matsuno has shown (note of Nov. 21, 1987) that for the simple method of using the ratio of q's for two PMT's the fractional error in L is essentially equal to that for the ratio of charges, or about $\sqrt{2}$ for one CM pulse. For n pulses this is then reduced by $\sqrt{n}$. If n = 7 we get a 50% error, in agreement with the above.

In fact, we do better since $q_i \gg 1$. Using simulated values of q_i calculated from (1), but processed as real data, that is, converted to a simulated pulse width and then converted back as would be done with the real data, I get $\delta\alpha/\alpha = \delta L/L = 0.26$. The rounding errors associated with that operation, which uses 5 ns units, are included but there are no statistical effects, so this represents a lower limit on what can be determined from one event triggered by one CM. Assuming n independent events, we can then reduce this further by $\sqrt{n}$.

Table I. Values of the reciprocal of the attenuation factors h_{ij} defined in equation (1) which correspond to the angular filters installed on the CM ball, as proposed by M. Webster in his note of July 20, 1986.

PMT	1	2	3	4	5	6	7
CM 1	4	200	800	50	50	50	50
CM 2	1	1	1	1	25	150	25