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Track Fitting for the DUMAND II Octagon Array

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Abstract

Algorithms for fitting muon tracks in the DUMAND II Octagon Array are described. These algorithms have been developed and tested using the Hawaii Monte Carlo program DUMC. They include effective filtering of random optical module hits at singles rates at least as high as 100 KHz.

Introduction

The DUMAND II Octagon Array will be composed of 9 vertical strings arranged on the vertices of an octagon with 40 m sides plus a central string. Each string will contain 24 optical modules (OMs) spaced 10 m apart vertically. The photomultiplier tubes (PMTs) within each optical module will be a mixture of Hamamatsu and Philips tubes with similar characteristics. For the purposes of data analysis, the PMT's can be viewed as being interchangeable (although not necessarily identical).

For each trigger, the following data will be available on shore for each OM_i:

- position (x_i, y_i, z_i) in meters;
- hit time t_i in ns;
- charge q_i in photoelectrons (pe);
- second hit time t_i' for multiple hits;
- the Gaussian or Poisson error for each of these quantities, or the appropriate error distribution.

Since each OM will be normally be operated at a threshold below 1 pe equivalent, most recorded hits will usually be background. The task of the track fitter is to reconstruct the direction of the muon, or muons, in the presence of high background.

This report describes the algorithms developed using the Hawaii DUMAND Monte Carlo program DUMC to simulate data with realistic estimates of the errors and backgrounds. While the Monte Carlo has been extensively run to predict the array properties and optimize its design parameters, only representative results will be shown here. The exact capabilities of the array will depend on a number of adjustable parameters, such as the PMT thresholds, exact trigger algorithm, minimum number of strings and OMs used in the fit, and others. The purpose of this report is primarily to document the algorithms developed so far, so that they might serve as a starting point for the development of software for the analysis of the real experimental data and be compared with what has been done by others.

Track Reconstruction Procedure

Track reconstruction proceeds as follows:

1) A preliminary background filter (pre-filter) is applied.
2) A preliminary approximate fast track reconstruction (pre-fit) is performed that does not require any iterations in obtaining the χ^2 minimum.
3) This pre-fit may be repeated once or twice, with suspicious OM hits discarded. If none of the fits is satisfactory, the event is discarded.
4) Using the pre-fit to set the initial track parameters, a χ^2 minimum search is performed.
5) This search can be repeated with suspected points discarded.

Note that background hits are filtered out in stages, as our knowledge of the true track direction is improved. In principle, a track fitting procedure is the best one can do for removing background, since it uses all the information on the track. The only reason to use algorithms other than a direct χ^2 fit is computer speed. Although I have not made any computer speed measurements, I anticipate that the pre-filters and pre-fit can be performed in real time as the data comes to shore, as a monitor or event filter.

The details of the steps follow.

Background Filtering

Simple filters have been developed that reduce the background significantly:

1) All hits are compared with the trigger hit and tossed out if the hit time t_i in OM_i exceeds the time it would take the Cherenkov wave front to pass between that module and the trigger module. That is, if d_i is the distance between OM_i and the trigger module, and $\Delta t_i = t_i - t_o$, where t_o is the trigger hit time, hit i is removed if

$$\Delta t_i > \frac{n}{c} d_i \tag{1}$$

where n = 1.35 is the index of refraction of water at DUMAND depth.

2) Several iterations (typically three) are made on the spacetime pre-fit (see below). After each, the measured hit times t_i from each OM_i are compared with the expected from the fit $\tilde{t}_i$ and tossed out of the difference, or *pull*, $p_i = \tilde{t}_i - t_i$ exceeds one standard deviation from the mean pull.
3) As we will see below, the pre-fit uses the muon speed as a fit parameter. The event is discarded if the muon speed determined by the pre-fit is far from c (currently, off more than 0.1 m/ns).
4) Two iterations are performed on the χ^2 search, with the same hit toss-out procedure as 2) if it fails the first try.
5) Even if the event passes the χ^2 search fit, it is discarded if the number of strings used in the final fit is less than three and the number of OM hits is less than some minimum, currently 10. Note that this is applied after hits have been discarded in the filtering process. The cut on fit hits was imposed after it was found the the large reconstruction errors were associated with small numbers of hits. The effective area is not significantly compromised, remaining 20,000 m^2 averaged over angle.

Based on experience with the SPS, we estimate that the singles rates from K^{40} will be in the range 60–80 KHz, with occasional bursts from bioluminescence. I have made runs assuming a conservative single PMT noise rate of 100 KHz.

After the filters described above, 0.5 ± 0.7 background hits and 18 ± 7 track hits typically remain. In terms of the distribution of background hits, 67% of the events have zero background hits, 23% have one, 9% have two, and only 1% have three or more, when we demand at least 10 hits on at least three strings in the final fit.

Preliminary Track Reconstruction

A fast, efficient preliminary track reconstruction algorithm has been developed. This is used as the first guess *(pre-fit)* for the χ^2 search. It should also prove to be useful for fast analysis of data as they come ashore, for the purpose of monitoring the array. (Online track reconstruction onboard the ship during the SPS experiment proved to be invaluable, even though the actual results were not used in subsequent analysis. Seeing actual track fits on the computer monitor gave us a degree of confidence that everything was working as it should.)

Let the position vector of the muon at time t be written as in freshman physics:

$$\mathbf{r} = \mathbf{r}_o + \mathbf{v}\, t \tag{2}$$

Let the components of the vectors r, r_o and v be written as x^j, x^j_o and v^j, j = 1, 3. The trick is *not* to apply the constraint that the muon's speed is c, so that the x^j and v^j form a set of six symmetrical parameters.

Next define

$$\chi^2 = \sum_{j=1}^{3} \sum_{i=1}^{N} q_i (x^j_i - x^j_o - v^j t)^2 \tag{3}$$

where q_i is the measured charge of OM_i and N is the total number of hits. Differentiating this with respect to each of the six parameters x^j_o and v^j and setting the results equal to zero give the following six uncoupled equations for the best fit parameters:

$$v^j = \frac{\langle x^j t\rangle - \langle x^j\rangle\langle t\rangle}{\langle t^2\rangle - \langle t\rangle^2} \tag{4}$$

$$x^j_o = \langle x^j\rangle + v^j\langle t\rangle \tag{5}$$

Note that (4) just expresses the fact the velocity is a measure of the correlation of position and time.

It turns out that the numerator and denominator in (4) are small and rounding errors in the computer foul up the calculation. However, writing out the sums explicitly and doing a page of algebra gives the following forms that can be used in practice:

$$x^j_o = \frac{1}{D} \sum^{N} \sum^{N}_{i \neq k} q_i q_k \,[\, x^j_i (t^2_k - t_i t_k) + x^j_k (t^2_i - t_i t_k)] \tag{6}$$

$$v^j = \frac{1}{D} \sum^{N} \sum^{N}_{i \neq k} q_i q_k (x^j_i - x^j_k)(t_i - t_k) \tag{7}$$

$$D = \sum^{N} \sum^{N}_{i \neq k} q_i q_k (t_i - t_k)^2 \tag{8}$$

Since explicit solutions are provided, the pre–fit is very fast. Also, unlike a χ^2 search, it does not require an initial guess. In fact, this pre–fit is used to provide the first guess for the final χ^2 search. However, because of the approximations involved, the median reconstruction error is 9°. This is adequate for the first guess, but the χ^2 search must still be used for the final fit to achieve the desired angular resolution of 1°.

Final Track Reconstruction

The current final track fit is a χ^2 search for the following five parameters defining the track:

- $\{x_o, y_o, z_o\} \rightarrow x_o^j$, $j=1,2,3$ location of track at $t = 0$
- $\{\cos\theta, \phi\} \rightarrow \alpha^j$, $j=1,2,3$ direction of track $\hat{\mu}$

where α^j are the direction cosines. The starting values of the parameters are taken from the pre–fit described above.

Referring to Fig. 1, we calculate, for each hit i, the distance d_i from OM_i to the point of closest approach to the track, and the distance L_i from $\{x_o, y_o, z_o\}$ to that point:

$$L_i = \sum_{j=1}^{3} \alpha^j (x_i^j - x_o^j) \tag{9}$$

$$d_i^2 = \sum_{j=1}^{3} (x_i^j - \alpha^j L_i - x_o^j)^2 \tag{10}$$

With these definitions, the predicted time of arrival of the light from the track at OM_i is

$$\tilde{t}_i = (L_i + d_i \tan\theta_c)/c \tag{11}$$

where θ_c is the Cherenkov angle.

The χ^2 then contains two summations:

$$\chi^2 = \sum_{i=1}^{N_h} \frac{1}{\delta t_i^2} (t_i - \tilde{t}_i)^2 + \sum_{i=1}^{N} \frac{1}{\delta q_i^2} (q_i - \tilde{q}_i)^2 \tag{12}$$

where q_i is the measured charge and $\tilde{q}_i$ is the predicted charge in OM_i, and the first summation is over N_h = the number of hit OMs while the second is over N = the total number of operating OMs. The predicted charge in each hit OM_i is taken as that for a minimum ionizing track at distance d_i. For the $N-N_h$ OMs that have no recorded hit, the measured charge $q_i = 0$.

In (12), δt_i and δq_i are the standard deviation measurement errors in the arrival time and OM charge. Note that these will in general depend on q_i and will be different for each tube. The errors are assumed to be Gaussian. This was OK for the Monte Carlo, where Gaussian errors were used in the simulated events, but may be be inadequate in the real undersea world of Jacques Cousteau. To handle the more realistic situation, it may be necessary to develop a maximum likelihood procedure with non-Gaussian errors.

Also note that the charge is determined from the PMT pulse width (at least for the Hamamatsu tube, and from something similar for the Philips tube), so its error involves not only the PMT statistics, but conversion errors as well. The OM-makers in Japan and Germany need to calibrate their modules so that appropriate error distributions in t and q are determined, as a function of q. These may be different for each individual tube, in which case a table look-up or parameterization is needed.

As mentioned above, if the pulls for one or more hits exceeds one standard deviation those hits are tossed out and another search is performed on the remaining hits. This is the final background filter that is applied.

Again only representative results will be mentioned here, since the quality of the fit depend on various optional parameters such as the minimum number of good hits in the final fit. For a minimum of 10 OMs in the final fit and a time resolution δt = 2.5 ns, the median pointing error is 1.2°, when the singles rate is 100 KHz.

Alternate χ^2 Fit

When I was in Moscow last June, something was pointed out to me about the fitting procedure. If one of the three spatial parameters (x_o, y_o, z_o) is replaced by a time parameter, then an analytical solution can be obtained for the best value of that variable. This reduces the parameter space that must be searched for χ^2 minimum from five to four dimensions, which is faster and presumably more accurate.

I have implemented this idea by making the following parameter transformation: replace the three parameters (x_o, y_o, z_o) with (x_a, y_a, t_a), where (x_a, y_a) are the coordinates of the muon at $z = 0$ and t_a is the time that the muon passes that point. Let P_i be the distance from this point to the point of closest approach. Then, using the other definitions in Fig. 1,

$$P_i = \vec{R}_i \cdot \hat{\mu} = \sum_{j=1}^{3} \alpha^j (x_i^j - x_a^j) \tag{13}$$

$$d_i^2 = R_i^2 - P_i^2 \tag{14}$$

$$\tilde{t}_i = t_a + (L_i + d_i \tan\theta_c)/c \tag{15}$$

where α^j are the direction cosines of the muon as before, and $(x_a^j) \equiv (x_a, y_a, 0)$. Using just the first term in (12), setting $\partial\chi^2/\partial t_a = 0$ and solving for t_a gives the solution for this parameter.

$$t_a = \frac{1}{N} \sum_{i=1}^{N} [\, t_i - (L_i + d_i \tan\theta_c)/c] \tag{16}$$

The remaining four parameters $(x_a,\ y_a,\ \cos\theta,\ \phi)$ are then found by a search for χ^2 minimum using the full form in (12).

Unfortunately, for reasons I do not understand, this alternate fit does not work as well, with about 20 percent fewer successful fits. I invite other Monte Carlo technicians to try their hand, but in the meantime I have retained the five parameter search as the final fit.

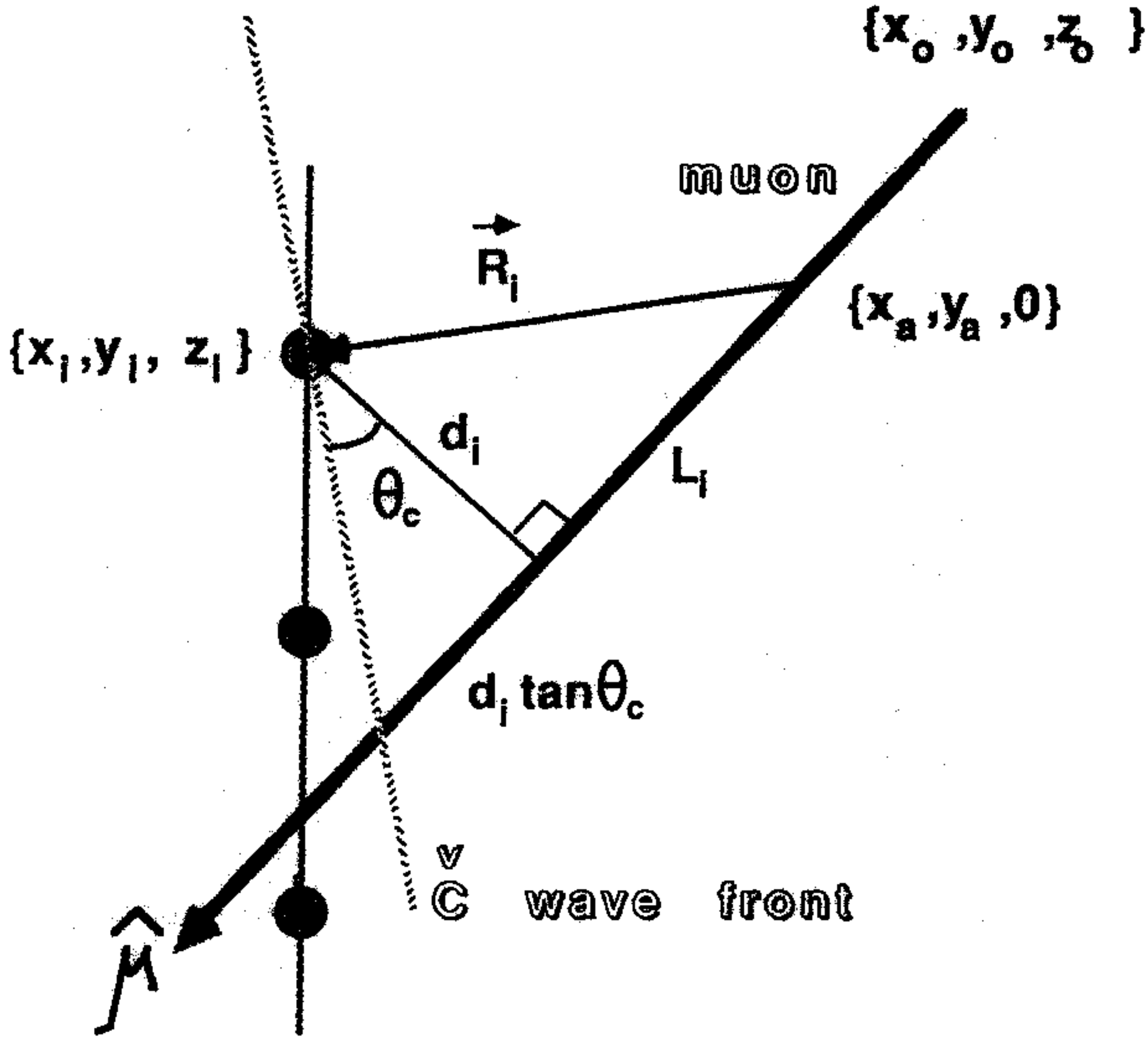

Fig. 1. Definition of terms used in text.