QUALIFYING EXAM

Part IA

February 22, 2019

8:30 - 11:30 AM

NAME ____

1. ____

2. ____

3. ____

4. ____

TOTAL ____

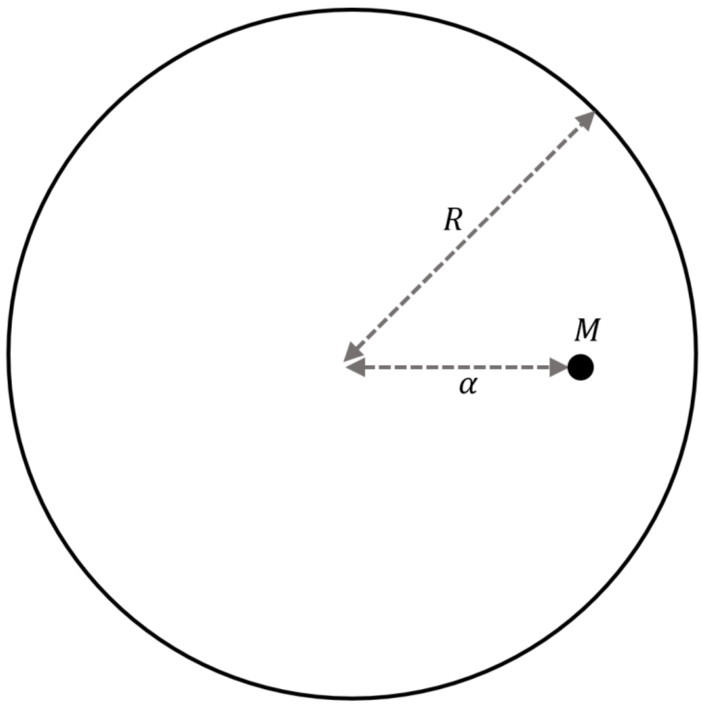
INSTRUCTIONS: CLOSED BOOK. A formula sheet is provided. WORK ALL PROBLEMS. Use back of pages if necessary. Extra pages are available. If you use them, be sure to write your name and make reference on each page to the problem being done.

PUT YOUR NAME ON ALL THE PAGES!

Physics Qualifying Exam 2019 Part IA Name ________________________

1. A 22nd century engineer constructs a huge massless balloon inside of which is a point mass of a super dense substance equal to the mass of the earth, offset from the center of the balloon by a distance α, as in the sketch below. A non-relativistic satellite with mass $m \ll M$ has an elliptical orbit tangent to the balloon at the point of closest approach.

R = radius of earth
M = mass of earth



(a) What is the semi-major axis of the elliptical orbit?

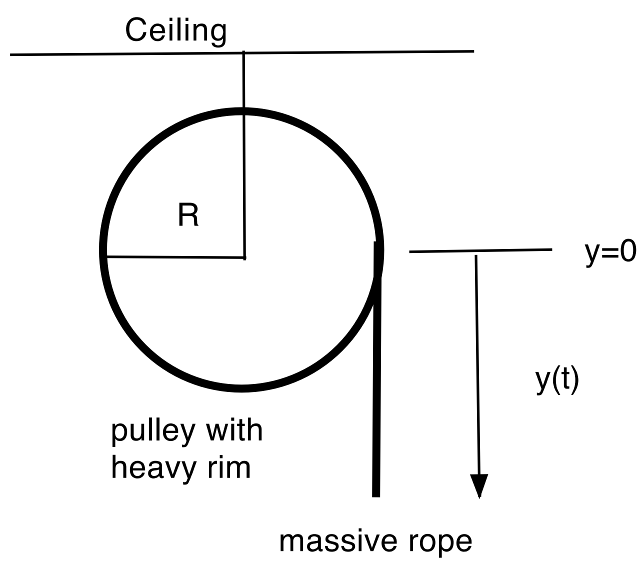
(b) What is the period of a satellite in orbit around this balloon for an orbit such that this period is as small as it can be (if the satellite is not to crash into the balloon).

Express your answer in terms of τ_0, the smallest period possible if the mass M was at the center of the balloon.

Physics Qualifying Exam 2/19 Part IA Name ________________________

2. A cylindrical pulley of mass M and radius R with a frictionless bearing is hung from the ceiling. The pulley has a very heavy rim so that it can be approximated as a hoop. A rope of length L, mass m, and negligible thickness is wrapped around the hoop many times. At $t = 0$, a portion of the rope hangs below the center of the hoop (see figure below, $y(t=0)=y_o$). The rope starts at rest and rolls without slipping over the hoop. The acceleration of gravity due to the Earth is g.

Note: The moment of inertia of a hoop is given by its total mass times its radius squared. The thickness of the rope is negligible so it does not affect the radius of the hoop but the mass of the rope is NOT negligible.



(a) Find the speed of the rope as a function of y (the amount of rope that hangs over at a given time) (Assume that the rope is still wrapped around the hoop many times such that there is as much of rope on the bottom half of the hoop as on the top half). Use a conservation law.

(b) Use Newton's Laws to find a differential equation for $y(t)$.

(c) Solve the differential equation from part (b) for $y(t)$.
(Note: If you calculate dy/dt from $y(t)$, it should agree with part (a).

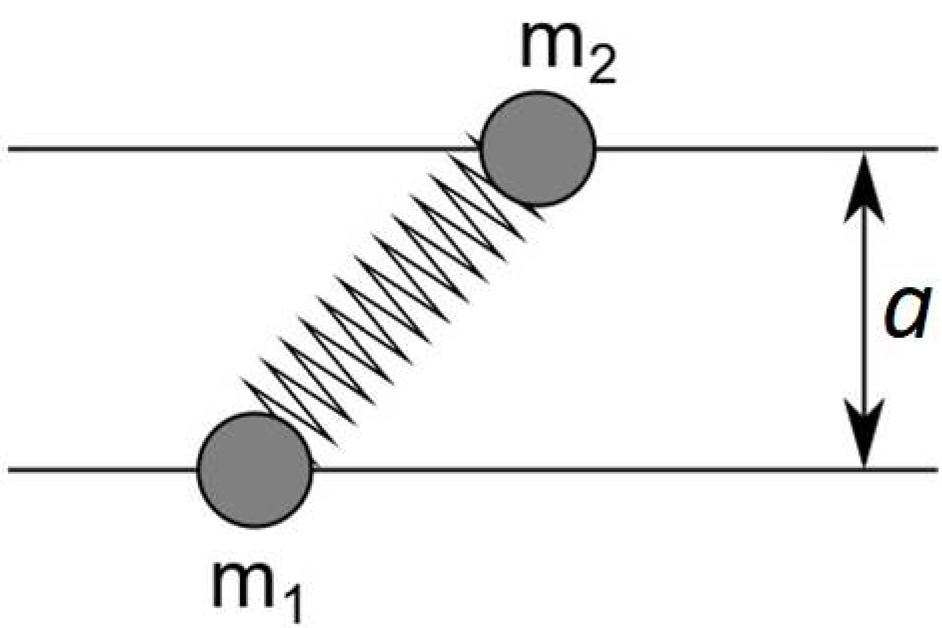
Physics Qualifying Exam 2019 Part IA Name ________________________

3. Two masses m_1 and m_2 are mounted on parallel straight rails. Assume that the masses can move freely on their corresponding rails, that they do not feel friction, and that they are not under the influence of gravity. The distance between the rails is a. The two masses are connected with an ideal spring, of spring constant k, and rest length ℓ.

(a) Formulate an expression for the potential energy and determine the rest position(s). Make a sketch of the potential energy as a function of the distance of the two masses for the cases of $a/\ell = 1$, $a/\ell > 1$, and $a/\ell < 1$. Are the rest position(s) stable? Assuming a constant ℓ and variable distance a, what is happening when the ratio a/ℓ changes from > 1 to < 1.

(b) For a fixed a, determine the Lagrangian of the system and formulate the equations of motion for both masses at the rest position(s).

(c) What are frequencies for small oscillations around equilibrium?



Physics Qualifying Exam 2019 Part IA Name ________________________

4. A kaon of mass 494 MeV/c^2 decays into a muon and a massless neutrino.

(a) Find the energies of the muon (mass =106 MeV/c^2) and the neutrino if the kaon decays at rest.

(b) What is the relativistic factor, $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$, for the muon?

(c) If the muon lifetime is 2.2 microseconds, how far does the muon travel in the laboratory frame before it decays?

(d) In the muon frame, how long does the muon live?

QUALIFYING EXAM

Part IB

February 22, 2019

1:30 - 4:30 p.m.

NAME ____

1. ____

2. ____

3. ____

4. ____

TOTAL ____

INSTRUCTIONS: CLOSED BOOK. A formula sheet is provided. WORK ALL PROBLEMS. Use back of pages if necessary. Extra pages are available. If you use them, be sure to write your name and make reference on each page to the problem being done.

PUT YOUR NAME ON ALL THE PAGES!

Physics Qualifying Exam 2019 Part IB Name ________________________

1. A piece of wire with resistivity ρ, cross-sectional radius s, and length $4L$ is formed into a square. An applied force moves the square at constant velocity v into a region of uniform magnetic field with magnitude B and oriented perpendicular out of the page, as shown in the figures.

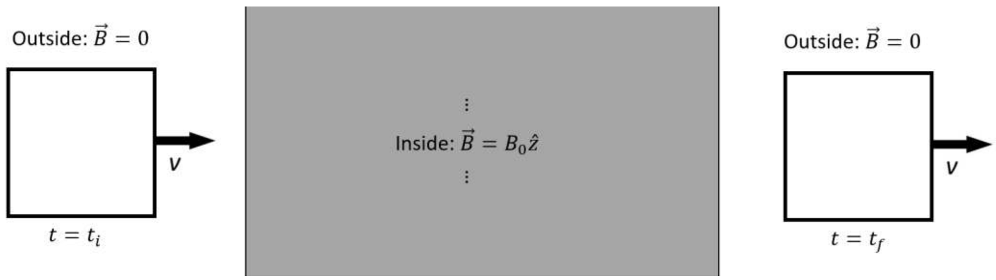


Figure 1

(a) What is the maximum current through the square loop as it moves from the position at t_i to the position at t_f (the path shown in Figure 1)?

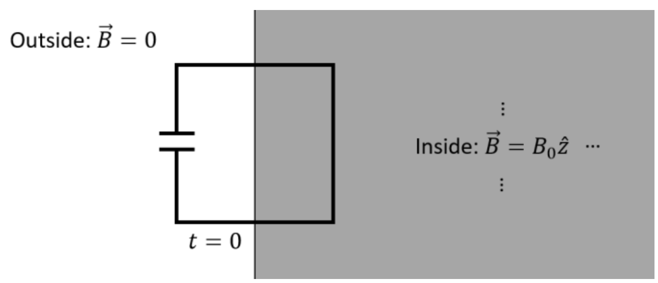


Figure 2

(b) The same loop is closed instead with a capacitor of capacitance C, as shown in Figure 2. The capacitor has initial charge Q_0, with the loop placed halfway in the field region, starting from rest. Determine the charge as a function of time, $Q(t)$. You may assume that the capacitor is of negligible size, and that the field region is long enough that the loop will not leave it on the right.

(c) Examine your solution to part (b) to see if it is has sensible behavior for all $t > 0$. If it does not, you may have missed a term in part (b). Add it in and re-solve for $Q(t)$. Describe the corresponding motion of the loop.

Physics Qualifying Exam 2019 Part IB Name ________________________

2. An optical plane wave is normally incident on a pair of slits. (The slit width is small compared to λ). The slits are separated by 100 μm and the fringes are observed on a screen 1 m away. If the plane wave consists of two monochromatic components with wavelengths of 425 nm and 595 nm, of arbitrary amplitude and phase, calculate the distance on the screen between the center axis and the first zero in the interference pattern in the following three cases.

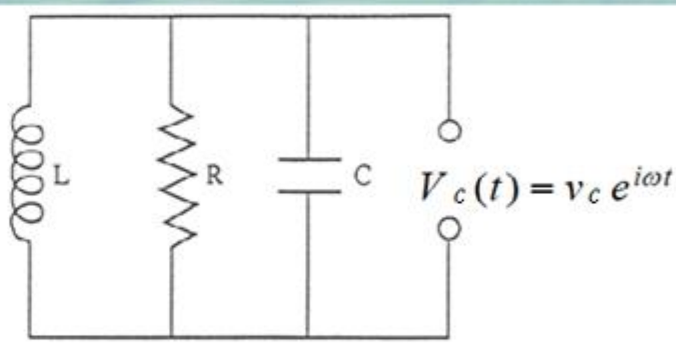
(a) for 425 nm light only,

(b) for 595 nm light only,

(c) and for both wavelengths together.

Physics Qualifying Exam 2019 Part IB Name ________________________

3. The photograph below is a Superconducting Radio Frequency structure used to accelerate electron bunches at the Thomas Jefferson National Laboratory near Norfolk, VA, to energies of billions of electron volts. The driving frequency is 1.497 GHz and can be modeled as an RLC circuit, as shown in the lower portion of the figure.



Ignore the complexity of the waveguide coupling to this structure in the following questions.

(a) What is an expression for the impedance of the structure, as a function of frequency?

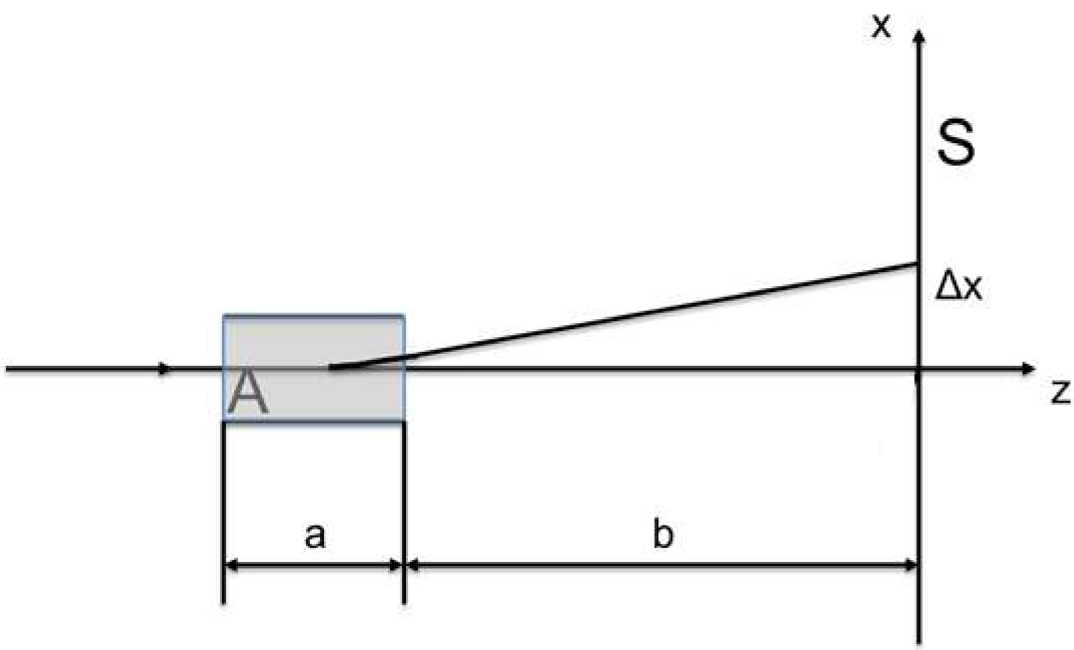
(b) What is an expression for the resonant frequency?

(c) The Q-factor of the cavity is 1×10^{10}, and can be expressed as $Q = \omega_0 RC$. To determine the capacitance of a cell, one first needs to determine the resistance. A power of 29 W is dissipated in the resistor and so R can be determined given that the accelerating field is 20 MV/m and the cavity length is 0.7 m. Calculate R and then C.

(d) All parameters for the cavity are now determined except for the inductance. What is this inductance L?

Physics Qualifying Exam 2019 Part IB Name ________________________

4. A beam of positively charged particles is moving along the z-axis through an area of space A in which there is an electric field of strength E and a magnetic field of strength B. Both fields are homogenous in area A. The directions of the E and B fields are perpendicular to one another, and perpendicular to the z-axis. If both fields are present, the beam impacts the screen at $x = 0$, the center of the screen. If the magnetic field is turned off, the beam's impact position on the screen is displaced by a distance Δx. Knowing the distances a and b, find the charge-to-mass ratio, q/m, for the particles in the beam.



QUALIFYING EXAM

Part IIA

February 25, 2019

8:30 - 11:30 AM

NAME ____

1. ____

2. ____

3. ____

4. ____

TOTAL ____

INSTRUCTIONS: CLOSED BOOK. A formula sheet is provided. WORK ALL PROBLEMS. Use back of pages if necessary. Extra pages are available. If you use them, be sure to write your name and make reference on each page to the problem being done.

PUT YOUR NAME ON ALL THE PAGES!

Physics Qualifying Exam 2019 Part IIA Name ________________________

1.

Consider a spin-1/2 particle with Hamiltonian

$$H = E_0 + \frac{\Delta E}{\hbar}\hat{S}_z.$$

At time $t = 0$, the particle is prepared in a state, which is an eigenstate of $\hat{S}_x$ with spin-projection $+\hbar/2$ along the x-axis.

a) Find the state at arbitrary $t > 0$ in terms of the eigenstates of $\hat{S}_z$, $|\uparrow\rangle$ and $|\downarrow\rangle$.

b) Suppose the spin projection of the particle along the x-axis is measured at time $T > 0$. What is the probability that one would measure $+\hbar/2$?

c) Suppose instead that the spin projection of the particle along the x-axis is measured N times, after equal time intervals $\Delta t = T/N$. What is the probability that the particle will be measured to have spin projection $+\hbar/2$ along the x-axis after every one of the N measurements?

d) Evaluate the above result in the limit where T is fixed, but $N \to \infty$. (Hint: You may use the fact that if $\lim_{m\to\infty} a_m = a$, then $\lim_{m\to\infty}(1 + a_m/m)^m = e^a$.)

Useful information: the Pauli spin matrices are

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Physics Qualifying Exam 2019 Part IIA Name ________________________

2. The wavefunction of a spinless particle moving in 3-D is given by
$\Psi\,(x, y, z) = N\, r \cos\theta \exp(-r/2a)$, where $r = \sqrt{x^2 + y^2 + z^2}$ and $\cos\theta = z/r$.
Here N and a are constants.

(a) What is the probability of finding the particle with $x > 0$, $y > 0$, $z > 0$ and $r > a$?

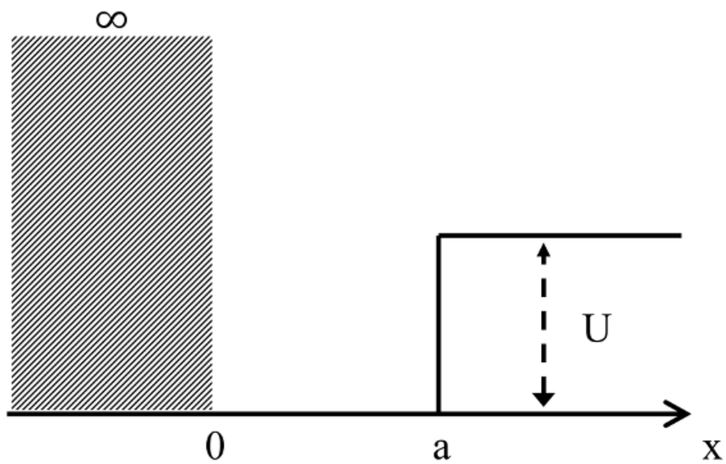
(b) An experimentalist makes a large number of measurements of p_x, the x-component of the momentum of the particle. What will be the average of the measurements? Will every measurement of p_x yield the same value or will there be a spread of values? Explain clearly.

(c) An experimentalist makes many measurements of J_Z, the z-component of the total angular momentum of the particle. What will be the average of the measurements? Will every measurement of J_Z yield the same value, or will there be a spread? Explain clearly.

(d) An experimentalist makes many measurements of J_X, the x-component of the total angular momentum of the particle. What will be the average of the measurements? Will every measurement of J_x yield the same value, or will there be a spread? Explain clearly.

Physics Qualifying Exam 2019 Part IIA Name ________________________

3. Consider the following one-dimensional semi-infinite square well potential:



$$V(x) = \begin{cases} \infty & x < 0 \\ 0 & 0 \le x \le a \\ U & x > a \end{cases}$$

(a) For which energy ranges can you expect bound states?

(b) Obtain general solutions to the Schrödinger equation for bound states inside the well ($0 \le x \le a$) and in the region beyond ($x > a$).

(c) What are the four appropriate boundary conditions?

(d) Enforce these boundary conditions to find a transcendental equation that determines the allowed energies of the system.

(e) Show how the transcendental equation could be solved graphically: Make a sketch and show the location of solutions. You do not need to find numerical values.

(f) Identify graphically the location of the solutions in the limiting case $U \to \infty$. Use this to solve for the exact energy eigenvalues (for $U \to \infty$) and comment on the result. Is it familiar?

Physics Qualifying Exam 2019 Part IIA Name ________________________

4. Go beyond the dipole approximation to study "forbidden" transitions. Consider a particle of mass m moving in a l-d harmonic oscillator (H.O.) potential of oscillator frequency ω. Its unperturbed Hamiltonian is

$$\hat{H}^{(0)} = \frac{\hat{p}}{2m} + \frac{m}{2}\omega^2\hat{x}^2$$

In the presence of a forcing field the system is perturbed by

$$\hat{H}^{(1)} = C\hat{x}\cos\left(k\hat{x} - \omega_0 t\right)$$

where C is a constant with units of force.

(a) First, assume $kx \ll 1$. Show that to first order in kx, the forcing field $C\cos(kx - \omega_o t)$ may be expanded as $C[\cos(\omega_o t) + kx\sin(\omega_o t)] + O((kx)^2)$.

(b) Now argue that to first order in C, the term $C\cos(\omega_o t)$ induces regular dipole transitions between H.O. eigenstates $|n\rangle \leftrightarrow |n+1\rangle$. Likewise, argue that the term $Ckx\,\sin(\omega_o t)$ induces forbidden transitions of the type $|n\rangle \leftrightarrow |n+2\rangle$.

(c) Use Fermi's Golden Rule to find the transition rate for both the regular dipole, and the forbidden transitions.

QUALIFYING EXAM

Part IIB

February 25, 2019

1:30 - 4:30 PM

NAME ____

1. ____

2. ____

3. ____

4. ____

TOTAL ____

INSTRUCTIONS: CLOSED BOOK. A formula sheet is provided. WORK ALL PROBLEMS. Use back of pages if necessary. Extra pages are available. If you use them, be sure to write your name and make reference on each page to the problem being done.

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PUT YOUR NAME ON ALL THE PAGES!

Physics Qualifying Exam 2019 Part IIB Name ________________________

1. A gasoline truck engine takes in 10,000 J of heat and delivers 2,000 J of mechanical work per cycle. The heat is obtained by burning gasoline with heat of combustion $L_c = 5.5 \times 10^4$ J/g.

   (a) What is the thermal efficiency of this engine?

   (b) If this is an ideal heat engine with the low temperature reservoir at room temperature (at 25°C) , find the operating temperature of the engine.

   (c) How much heat is dissipated in each cycle?

   (d) If the engine goes through 25 cycles per second, what is its power output in watts?

   (e) What mass of gasoline is burned per second?

Physics Qualifying Exam 2019 Part IIB Name ________________________

2. The energy of a nonmagnetic solid insulator is due the to the vibrational motion of the atoms (molecules) around equilibrium positions. For small oscillations, classical mechanics shows that the vibrational motion can be described in terms of generalized coordinates and generalized momenta such that the energy is the sum of independent normal mode harmonic oscillations. For an atomic solid (such as solid argon):

$$\mathcal{E}(q_1, \dots, q_s, p_1, \dots, p_s) = \sum_{\ell=1}^{s} \left(\frac{p_\ell^2}{2m} + \frac{m\omega_\ell^2}{2} q_\ell^2 \right),$$

where m is the mass of the atom, and ω_ℓ is the angular frequency of the ℓth vibrational normal mode. Since the number of particles, N, is very large for a solid, of the order 10^{25}, the number of vibrational normal modes $s = 3N - 6 \cong 3N$.

(a) Calculate the partition function of the solid.

(b) Calculate the entropy of the solid.

(c) Calculate the thermally averaged energy of the solid.

(d) Calculate the constant volume heat capacity of the solid.

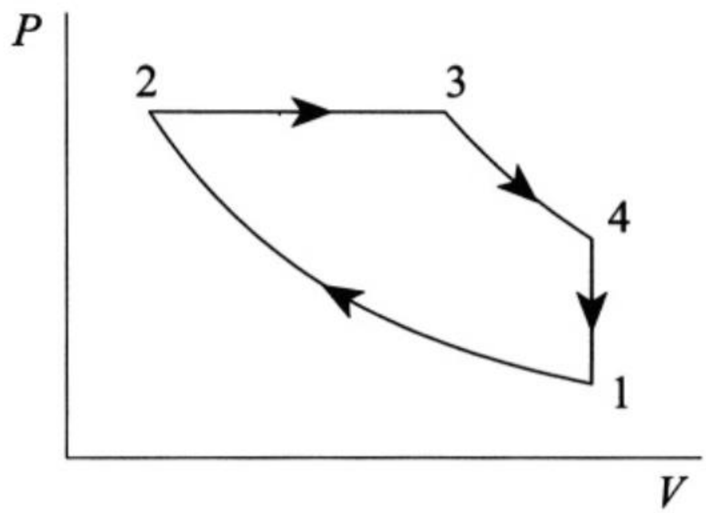
HINT: The following integral identity is useful:

$$\int_{-\infty}^{\infty} du\, e^{-(u-a)^2/2b^2} = b\sqrt{2\pi} \ ,$$

with constants a and b $(b > 0)$.

Physics Qualifying Exam 2019 Part IIB Name ________________________

3. A thermodynamic cycle consists of two adiabats (3 →4 and 1→2), an isobaric process (2→3), and a constant volume process (4→1) as seen below. Assume an ideal gas with constant C_v and C_p.



(a) Which temperature (T_1, T_2, T_3, T_4) is the highest? Which temperature is the lowest?

(b) Find the total work done by the cycle in terms of T_1, T_2, T_3, T_4 and C_p, C_v.

(c) Calculate the efficiency of the cycle in terms of the volumes V_1, V_2, V_3, V_4.

Physics Qualifying Exam 2019 Part IIB Name ________________________

4. Consider the Sun and the Earth, as idealized blackbodies, in otherwise empty flat space. The Sun is at a temperature T_S = 6,000 K, the Earth's surface temperature is uniform. The radius of the Earth is R_E = 6.4 x 10^6 m, the radius of the Sun is R_S = 7.0 x 10^8 m, and the Earth-Sun distance is *d* = 1.5 x 10^{11} m. The mass of Sun is M_S = 2.0 x 10^{30} kg.

(a) Find the steady state temperature of the Earth.

(b) Find the radiation force on the Earth.

(c) Repeat parts (a) and (b) for an interplanetary metallic "chondrule" in the form of a spherical, perfectly conducting blackbody with a radius *R* = 0.1 cm, moving in a circular orbit around the Sun at a radius equal to the Earth-Sun distance *d.*

(d) At what distance from the Sun would the metallic particle from part (c) melt (melting temperature T_m = 1,550 K)?

(e) For what size particle would the radiation force calculated in (c) be equal to the gravitational force from the Sun at a distance *d*?

Note: σ = 5.67 x 10^{-8} W/m^2/K^4