QUALIFYING EXAM

Part IA

February 23, 2018

8:30 - 11:30 AM

NAME ____

1. ____

2. ____

3. ____

4. ____

TOTAL ____

INSTRUCTIONS: CLOSED BOOK. A formula sheet is provided. WORK ALL PROBLEMS. Use back of pages if necessary. Extra pages are available. If you use them, be sure to write your name and make reference on each page to the problem being done.

PUT YOUR NAME ON ALL THE PAGES!

Physics Qualifying Exam 2018- Part IA
Problem 1

Name:____________________

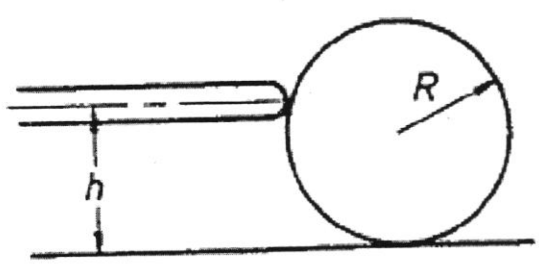
A ball of radius R and mass M (initially at rest on a frictionless surface) is struck with a horizontal impulse at height h, as shown below. Note that the moment of inertia of a sphere is $(2/5)\ MR^2$.

a) In the initial impact, is the angular momentum of the ball conserved? Is its linear momentum conserved? (Explain your reasoning.)

b) What is the velocity of the center of mass of the ball after impact, in terms h, R, and the angular velocity after impact, ω?

c) Find the value of h for which the ball will roll without slipping.

d) If the ball is instead a spherical shell with the same mass ($I = (2/3)MR^2$), what is the value of h for which the ball will roll without slipping?

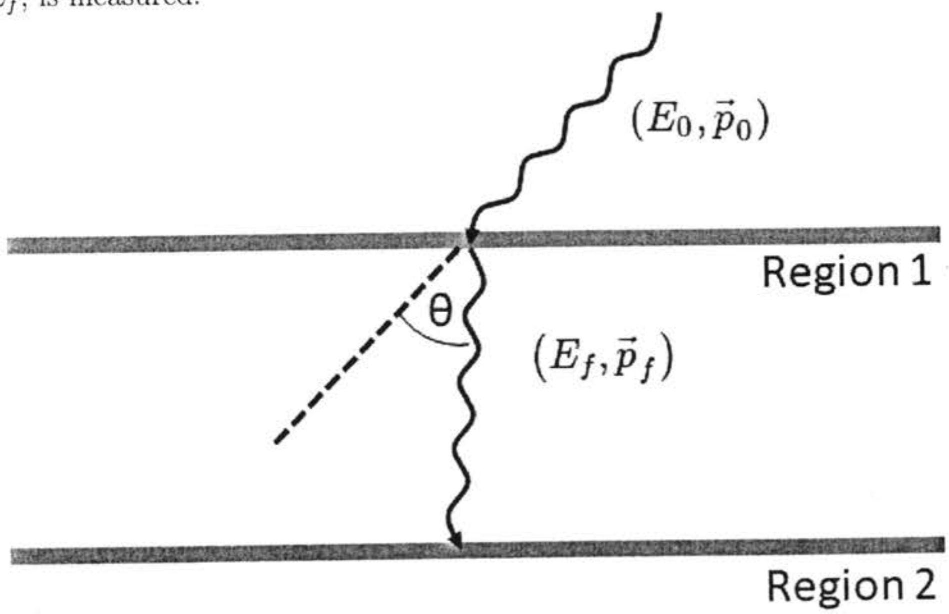


Physics Qualifying Exam 2018 - Part IA

Problem 2

Name:________________

A Compton imaging detector utilizes two interaction regions (for example, two parallel planes). In the first, an incoming gamma ray photon of four-momentum $(E_0, \vec{p}_0)$ elastically scatters off of an electron at rest, where E_0 is the unknown energy of the incoming gamma and $\vec{p}_0$ is its three-momentum. The scatter deposits energy E_e into the electron, and the scattered photon is then absorbed in a second interaction volume, where its full remaining energy, E_f, is measured.



(1) Begin with the simple case of the photon scattering in region 1 off of a free electron at rest (i.e., with four-momentum $(m_e, 0)$). Derive an expression for $\cos\theta$, where θ is the relative angle between the incoming and outgoing photon. Express $\cos\theta$ in terms of m_e and the incoming and outgoing gamma energies.

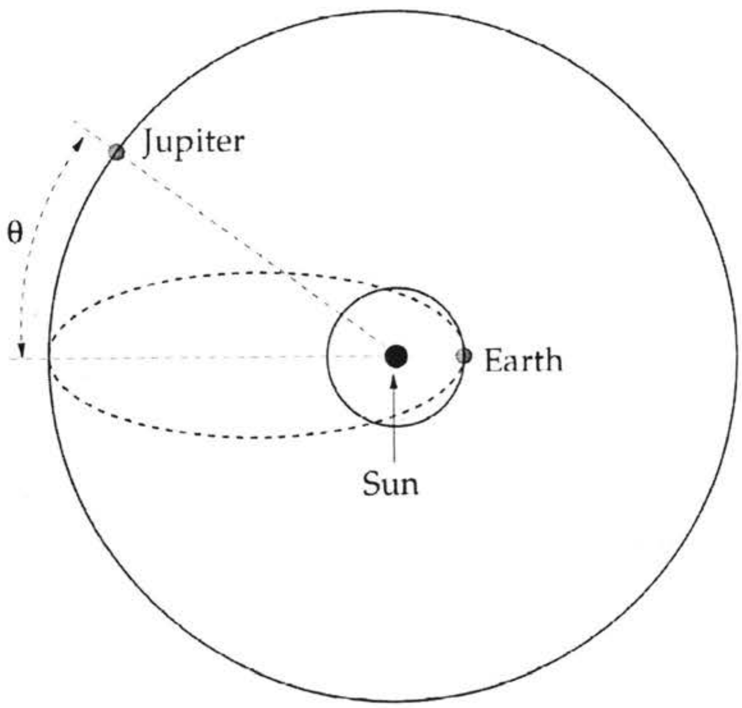
(2) Now assume that the scattered gamma is fully absorbed in the second detection region. Rewrite the expression from 1) in terms of only the electron mass, m_e, and the measured energies: E_e and E_f.

(3) Sketch the result of part (1). What three-dimensional geometric figure describes the constraint that can be inferred on the direction of the incoming gamma, and how is this figure oriented with respect to the two points of interaction?

(4) Now assume that in the first interaction region that the gamma interaction is not with an electron at rest, but with an electron with non-relativistic momentum $\vec{p}_e$. Show that this introduces an error term into the previous constraint proportional to the projection of $\vec{p}_e$ along the momentum-transfer vector of the gamma.

Physics Qualifying Exam 2018- Part IA
Problem 3

Name:____________________

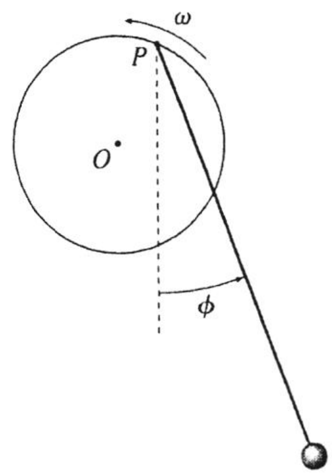


The most efficient way to launch a spacecraft to Jupiter is by boosting it into an elliptical orbit around the Sun, with the orbit's perihelion at Earth, and the aphelion at a point intersecting the orbit of Jupiter, as shown above, in a configuration known as a *Hohmann Transfer Orbit*. Determine the angle θ (at the time of launch) such that the spacecraft and Jupiter will arrive at the point where the two orbits intersect at the same time. Assume circular orbits (in the counter-clockwise direction) of radii 1.0 AU and 5.2 AU, for the Earth and Jupiter respectively.

Physics Qualifying Exam 2018- Part IA
Problem 4

Name:____________________

The figure below shows a simple pendulum (mass m, length l) whose support P is attached to the edge of the wheel (center O, radius R) that is forced to rotate at a fixed angular velocity ω. At $t = 0$, the point P is level with O on the right. Write down the Lagrangian and find the equation of motion for the angle φ. [Hint: be careful writing down the kinetic energy T. A safe way to get the velocity right is to write down the position of the bob at time t, then differentiate.] Check that your answer makes sense in the special case that $\omega = 0$.



QUALIFYING EXAM

Part IB

February 23, 2018

1:30 - 4:30 PM

NAME ______________________________

1. ______________________________

2. ______________________________

3. ______________________________

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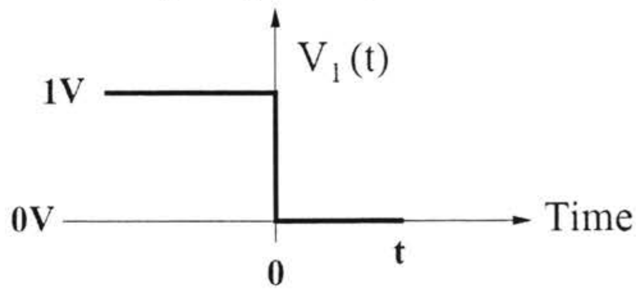
TOTAL ______________________________

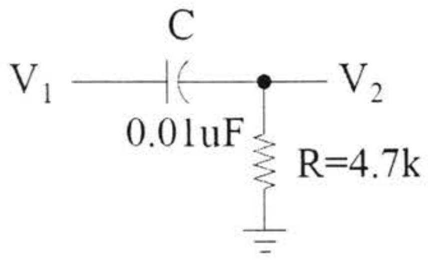
INSTRUCTIONS: CLOSED BOOK. A formula sheet is provided. WORK ALL PROBLEMS. Use back of pages if necessary. Extra pages are available. If you use them, be sure to write your name and make reference on each page to the problem being done.

PUT YOUR NAME ON ALL THE PAGES!

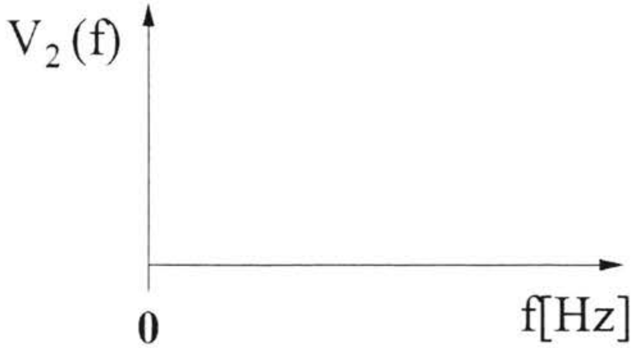
Name:____________________

A) For the figure below, if at time $t=0$ the input voltage V_1 steps down from 1 V to 0 V, calculate and plot the output voltage at V_2, indicating the characteristic time constant (τ) and the polarity of the signal.

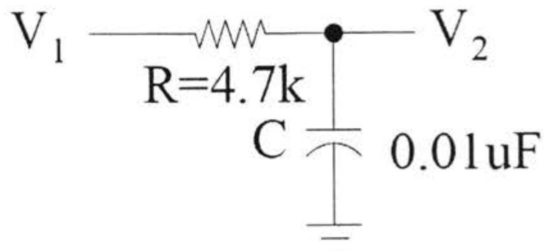


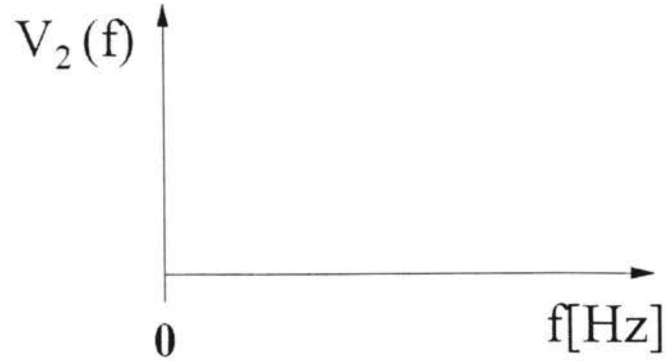


B) The signal now applied to V_1 is now a 1 V sine wave. Determine $V_2(f)$, the voltage out as a function of frequency and sketch the result, indicating the asymptotic behavior as $f \rightarrow 0$ and $f \rightarrow \infty$. Note also the frequency at which the output signal is $1/\sqrt{2}$ of the input signal. What type of filter is this?

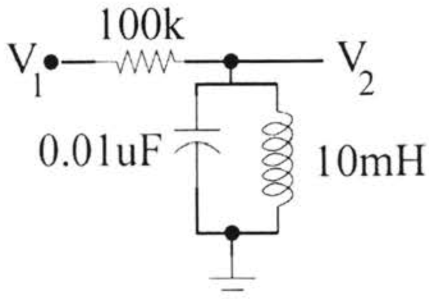


C) Using the same sine signal source as in part B, interchange *R* and *C* and repeat the calculations and plot. What type of filter is this?





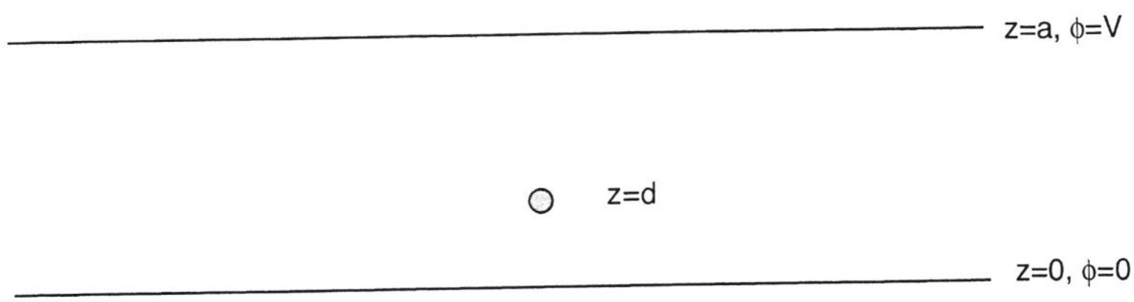
D) Calculate the resonance frequency of this circuit. What type of filter is this?



Physics Qualifying Exam 2018- Part IB
Problem 2

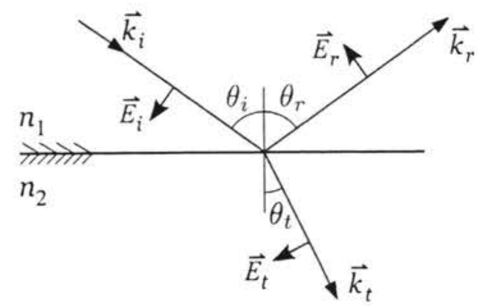
Name:____________________

Two infinite conducting plates are aligned with the *x-y* plane, one located at $z=0$ and one at $z=a$. The plate at $z=0$ is grounded while the plate at $z=a$ is held at an electrostatic potential V. An infinitely long line charge (charge per unit length= λ) is located between the two plates and is aligned along the *y*-direction at ($x=0$, $z=d$). Find the electrostatic potential $\phi(x,y,z)$ for every point in the region between the two plates. Use MKS units.



Name:____________________

An optical plane wave with wavevector $\vec{k}_i$ is incident on an interface between two dielectrics with refractive indices n_1 and n_2. The wavevectors $\vec{k}_r$ and $\vec{k}_t$ are the wavevectors of the reflected and transmitted plane waves, respectively, all lying in the plane of the figure. The electric fields $\vec{E}_{i,t,r}$ associated with these plane waves at a given point on the interface have the directions shown and lie in the plane of the figure.



a) How is θ_r related to θ_i? How is θ_t related to θ_i?

b) Write the boundary condition for the parallel component of the total **E**-field on either side of the interface (parallel to the interface). Write the boundary condition for the perpendicular component of the total **D**-field (electric displacement) on either side of the interface (perpendicular to the interface). Express both of these equations in terms of the magnitudes $E_{i,r,t}$, the angles $\theta_{i,t}$, and the indices $n_{1,2}$ as needed.

c) Rewrite the equations in part (b) in terms of the reflection and transmission coefficients $r_p \equiv E_r/E_i$ and $t_p \equiv E_t/E_i$. Simplify your equations by dividing out any common factors.

d) Solve the equations in part (c) for r_p and t_p in terms of $n_{1,2}$ and $\theta_{i,t}$.

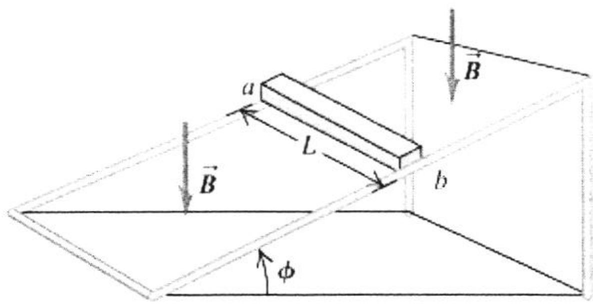
e) Show that the reflection coefficient r_p vanishes for angles $\theta_{i,t}$ satisfying

$$\frac{\tan\theta_t}{\tan\theta_i} = \frac{n_1^2}{n_2^2} \quad .$$

Physics Qualifying Exam 2018- Part IB
Problem 4

Name:____________________

A conducting bar with resistance R and length L is gliding down conducting rails (as seen in the figure). The whole apparatus is exposed to a uniform magnetic field of strength **B** in negative z direction. The resistance of the rails is negligible and the motion on the rail should be treated as frictionless. The angle between the rails and the horizontal is ϕ.



a) Calculate the terminal velocity of the bar.

b) Calculate the induced current at terminal velocity.

c) Calculate the energy dissipation at terminal velocity. Compare to the rate of the work being done on the bar by gravity.

QUALIFYING EXAM

Part IIA

February 26, 2018

8:30 - 11:30 AM

NAME ______________________________

1. ______________________________

2. ______________________________

3. ______________________________

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TOTAL ______________________________

INSTRUCTIONS: CLOSED BOOK. A formula sheet is provided. WORK ALL PROBLEMS. Use back of pages if necessary. Extra pages are available. If you use them, be sure to write your name and make reference on each page to the problem being done.

PUT YOUR NAME ON ALL THE PAGES!

Physics Qualifying Exam 2018 - Part IIA
Problem 1

Name:____________________

Be sure to explain your reasoning.

a) Consider an initial state consisting of two identical spin-1/2 fermions (assume spin is the only internal quantum number). If this initial state has an orbital angular momentum ℓ=0, then what is the total angular momentum j?

b) Suppose that these two particles annihilate, producing a final state consisting of two different *distinguishable* spin-1/2 fermions. (Again, assume spin is the only internal quantum number.) What are the possible ℓ, s, j quantum numbers for the two-particle final state?

c) Suppose the two outgoing particles are observed by two detectors placed far apart on the + $\hat{z}$- and - $\hat{z}$-axes. Alice measures the spin of the particle seen at the detector on the + $\hat{z}$-axis, while Bob measures the spin of the particle seen at the detector on the - $\hat{z}$-axis. The detectors are far enough away that neither Bob nor Alice can communicate their measurement to the other before the other completes his/her own measurement. Suppose Alice measures a spin projection +1/2 along the $\hat{z}$-axis. What is the probability that Bob will also measure a spin projection +1/2 along that axis, and what is the probability that Bob will instead measure -1/2?

Physics Qualifying Exam 2018 - Part IIA Name:____________________

Problem 2

State whether the following are true or false, and give a brief justification for your answer. Answers with no justification or an incorrect justification will receive no credit.

a) It is impossible to simultaneously specify the z-component of the spin of a particle and the z-component of its total angular momentum. TRUE or FALSE?

b) The Hamiltonian $H = \frac{\vec{p}^2}{2m} + \alpha(x^2 + y^2 + z^2)S_z + \beta(x^2 + y^2 + z^2)^2$ for a spin-1/2 particle conserves parity. TRUE or FALSE?

c) Since L_x and L_y (the x- and y-components of the orbital angular momentum) do not commute, it is impossible to find even a single state for which both L_x and L_y have a definite value. TRUE or FALSE?

d) If electrons had been spin-zero bosons instead of spin-1/2 fermions, then the ionization energy of a neon atom in its natural state wold have been larger than it is in the real world. TRUE or FALSE?

e) The wavefunction of a single spin-0 particle is given by $\psi(r, \theta, \phi) = Ae^{-r^4/a^4} \cos^2\theta \sin\phi$, where A and a are constants. A measurement of the z-component of the angular momentum would *always* yield $L_z = \pm\hbar$. TRUE or FALSE?

f) Let $O = (\vec{r} \cdot \vec{p} + \vec{p} \cdot \vec{r})e^{-\frac{\vec{r}\cdot\vec{p} + \vec{p}\cdot\vec{r}}{\hbar}}$, where $\vec{r}$ and $\vec{p}$ are the position and momentum operators, respectively. Then the matrix element

$$\langle n', \ell' = \ell + 2, m_{\ell'}, m_{s'} | O | n, \ell, m_\ell, m_s \rangle = 0,$$

for *all* values of ℓ, m_ℓ, m_s, $m_{\ell'}$, $m_{s'}$, n, and n'. TRUE or FALSE?

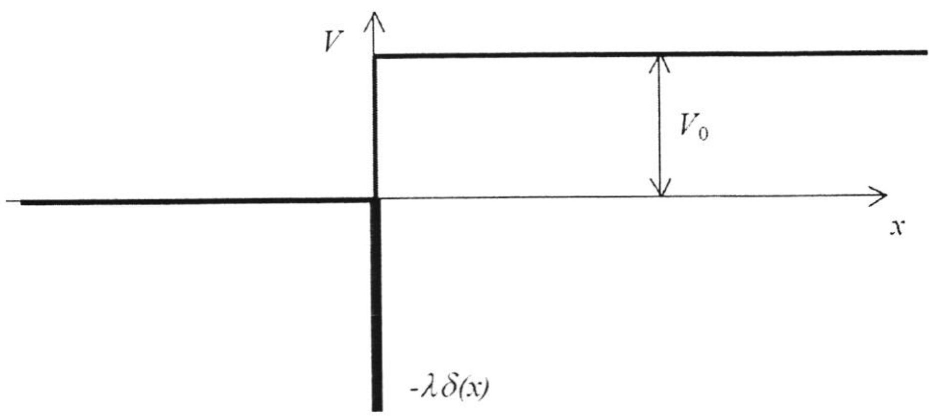
Physics Qualifying Exam 2018 - Part IIA
Problem 3

Name:____________________

An attractive delta-function well located at the origin is superimposed with a potential step at the origin:

$$V(x) = -\lambda\delta(x) + V_0\Theta(x),$$

where $\Theta(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$ is the step function, and λ, V_0 are positive quantities.



a) Find the form of the wave function in the regions $x > 0$ and $x < 0$, for a bound, energy eigenstate solution. Is the condition for a bound state $E < 0$ or $E < V_0$, and why?

b) Write down the boundary conditions at $x = 0$.

c) Sketch the wave function.

d) Without calculation, but using knowledge of the wave function, decide if for the bound state we should find $\langle x\rangle < 0$, $\langle x\rangle = 0$, or $\langle x\rangle > 0$?

e) Find an equation that determines the energy of the bound state in terms of fundamental constants and λ, V_0, m.

Physics Qualifying Exam 2018 - Part IIA
Problem 4

Name:________________________

Consider Yukawa scattering off the potential $V(r) = g^2 \frac{\exp(-\mu r)}{r}$, where g and μ are constants ($\mu \geq 0$). In the first Born approximation, the scattering amplitude of a particle with mass m and a momentum change $\mathbf{q}$ is given by

$$f(\mathbf{q}) = \frac{2g^2 m}{\hbar^2} \frac{1}{q^2 + \mu^2}.$$

a) Express the momentum change q in terms of the scattering angle θ and the energy E of the incident particle and schematically sketch the dependence of the cross-section $d\sigma/d\Omega = |f(\theta)|^2$ as a function of θ for $\mu = \sqrt{2Em}/\hbar$ and $\mu = 0$.

b) Find the dependence on energy E of the total cross-section

$$\sigma = 2\pi \int_0^\pi \sin\theta |f(\theta)|^2 d\theta.$$

c) What is the value of σ in the limit of Coulomb scattering (i.e. $\mu = 0$)? Explain.

QUALIFYING EXAM

Part IIB

February 26, 2018

1:30 - 4:30 PM

NAME ____

1. ____

2. ____

3. ____

4. ____

TOTAL ____

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PUT YOUR NAME ON ALL THE PAGES!

Physics Qualifying Exam 2018 - Part IIB
Name:________________________
Problem 1

Background: Large molecules (such as proteins) can have smaller molecules attached (bonded). Consider the case in which a large molecule can have only one small molecule bonded to it. The small molecule can orient itself over all solid angle 4π, however, only forms a strong bond for a restricted orientation of angle, θ, relative to the bonding site of the large molecule, where θ is the polar angle relative to the bonding axis (bonding direction). The solid angle for strong bonding is $\Omega = 2\pi(1-\cos\theta)$. The fraction of the number of "favorable" orientational states for strong bonding is $\Omega/4\pi$.

Consider a model in which the orientational states are discretized. The orientational state of the small molecule is specified by a discrete generalized coordinate, $q = 1, 2, 3, ..., n$, with total number of orientational states, n. The state $q = 1$ is the strongly bonded state, and the remaining states, $q \neq 1$, are un-bonded. Thus, the fraction of strongly bonded states is $1/n$. Thus, $n = \text{int}(4\pi/\Omega)$, where "int$(x)$" is the integer value of x. For example, for a hydrogen bond, $\theta \approx 5°$, $n \approx 526$.

Problem: The energy of the small molecule in orientational state q is $\epsilon(q) = -\epsilon_B\delta(q, 1)$, where $q = 1, 2, 3, ..., n$, where $\delta(a, b) = 1$ for $a = b$, $\delta(a, b) = 0$ for $a \neq b$, and where $\epsilon_B > 0$ is the binding energy.

a) Calculate the partition function of the small molecule in thermal equilibrium as a function of the absolute temperature, T.

b) Calculate the thermally averaged energy, $U = \langle\epsilon\rangle$ of the small molecule as a function of the absolute temperature, T. Make a qualitative sketch of U vs. T.

c) Calculate the entropy, S, of the small molecule as a function of the absolute temperature, T. Make a qualitative sketch of S vs. T.

d) Calculate the fraction of the number of small molecules in the strongly bonded state as a function of the absolute temperature, T.

e) Find the value of the absolute temperature, T, in Kelvin units, in which 60% of the small molecules are strongly bonded in thermal equilibrium. Use, $n = 500$, $\epsilon_B = 0.0434$ eV.

Physics Qualifying Exam 2018 - Part IIB
Problem 2

Name:____________________

1 mole of ideal gas with adiabatic index $\gamma \equiv C_P/C_V$, undergoes a process in which pressure changes with temperature as:

$$P = aT^{\alpha} \;,$$

where a and α are given constants.

a) Find the work done by the gas, if the temperature changes by ΔT during the process.
b) Calculate δQ for this process. For which values of α will δQ be negative?

Physics Qualifying Exam 2018 - Part IIB Name:____________________
Problem 3

A planet of radius R orbits its star at a distance D, with $R \ll D$. The star has a temperature T_S and a radius R_S.

(A) Treat the star as a perfect blackbody radiator. Find the effective temperature of the planet, the temperature where the planet is in blackbody equilibrium, assuming that it absorbs a fraction $(1 - \alpha)$ of the energy incident on it from the star (α is known as the albedo of an object), and emits radiation as a perfect blackbody. Use your result to calculate the effective temperature of the Earth, assuming that $T_S = 5778$ K, $R_S = 6.96 \times 10^8$ m and $\alpha = 0.3$.

(B) Now we will add an extremely simple model of the greenhouse effect. Assume a thin spherical shell of atmosphere around the Earth that is transparent to incoming solar radiation, but has emissivity ϵ with respect to the radiation coming from the Earth's surface and its own blackbody radiation. (The emissivity is the fraction of radiation emitted and absorbed relative to a perfect blackbody.)

(i) Why is it reasonable to treat the blackbody radiation from the sun and that from the Earth differently in this way?

(ii) Assume that the atmosphere is also in radiative equilibrium. (Remember that we treat the atmosphere as transparent to solar radiation, so none of the power emitted by the sun is absorbed by the atmosphere.) Find a relation between the temperature of the atmosphere, T_A, and the temperature of the planet's surface, T_P.

(iii) Use your answer from the previous part to find a new condition for radiative equilibrium of the Earth, since the Earth now sees the down-going contribution of radiation from the atmosphere.

(iv) Determine the temperature of the Earth's surface in terms of R, D, T_S, R_S, α, and ϵ.

(v) What value of ϵ gives a reasonable temperature for the surface of the Earth?

Physics Qualifying Exam 2018 - Part IIB Name:____________________

Problem 4

Let's consider a gas of N free (*i.e.* non interacting) electrons in a volume V:

(1) Show that the density of states with energy between ϵ and $\epsilon+d\epsilon$ is $g(\epsilon) = \frac{4\pi V}{h^3}(2M)^{3/2}\sqrt{\epsilon}$, where M is the electon mass and h the Planck constant.

(2) Find an expression for the chemical potential μ_0 at temperature $T = 0$ K (also known as Fermi energy), as a function of N and V.

(3) Derive the heat capacity at constant volume for low temperatures (you can approximate $\mu \approx \mu_0$), as a function of μ_0, N, and T.
You will need the identity:

$$\int_0^{+\infty} \frac{h(\epsilon)}{e^{\frac{\epsilon-\mu}{kT}}+1} d\epsilon = \left[1 + \frac{(\pi kT)^2}{6}\frac{\partial^2}{\partial\mu^2} + \cdots\right]\int_0^{\mu} h(\epsilon)d\epsilon,$$

where $h(\epsilon)$ is a generic function of the energy ϵ and μ is the chemical potential.

Cartesian Coordinates

1. $\nabla f = \dfrac{\partial f}{\partial x}\boldsymbol{i} + \dfrac{\partial f}{\partial y}\boldsymbol{j} + \dfrac{\partial f}{\partial z}\boldsymbol{k}$

2. $\nabla \cdot \boldsymbol{A} = \dfrac{\partial A_x}{\partial x} + \dfrac{\partial A_y}{\partial y} + \dfrac{\partial A_z}{\partial z}$

3. $\nabla \times \boldsymbol{A} = \left(\dfrac{\partial A_z}{\partial y} - \dfrac{\partial A_y}{\partial z}\right)\boldsymbol{i} + \left(\dfrac{\partial A_x}{\partial z} - \dfrac{\partial A_z}{\partial x}\right)\boldsymbol{j} + \left(\dfrac{\partial A_y}{\partial x} - \dfrac{\partial A_x}{\partial y}\right)\boldsymbol{k}$

4. $\nabla^2 f = \dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} + \dfrac{\partial^2 f}{\partial z^2}$

5. $\nabla^2 \boldsymbol{A} = \nabla^2 A_x\,\boldsymbol{i} + \nabla^2 A_y\,\boldsymbol{j} + \nabla^2 A_z\,\boldsymbol{k}$

Cylindrical Coordinates

1. $\nabla f = \dfrac{\partial f}{\partial \rho}\hat{\boldsymbol{\rho}} + \dfrac{1}{\rho}\dfrac{\partial f}{\partial \varphi}\hat{\boldsymbol{\varphi}} + \dfrac{\partial f}{\partial z}\hat{\boldsymbol{z}}$

2. $\nabla \cdot \boldsymbol{A} = \dfrac{1}{\rho}\dfrac{\partial}{\partial \rho}(\rho A_\rho) + \dfrac{1}{\rho}\dfrac{\partial A_\varphi}{\partial \varphi} + \dfrac{\partial A_z}{\partial z}$

3. $$\nabla \times \boldsymbol{A} = \left(\frac{1}{\rho}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right)\hat{\boldsymbol{\rho}} + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\hat{\boldsymbol{\varphi}} + \frac{1}{\rho}\left(\frac{\partial}{\partial \rho}(\rho A_\varphi) - \frac{\partial A_\rho}{\partial \varphi}\right)\hat{\boldsymbol{z}}$$

4. $\nabla^2 f = \dfrac{1}{\rho}\dfrac{\partial}{\partial \rho}\left(\rho\dfrac{\partial f}{\partial \rho}\right) + \dfrac{1}{\rho^2}\dfrac{\partial^2 f}{\partial \varphi^2} + \dfrac{\partial^2 f}{\partial z^2}$

Spherical Coordinates

1. $\nabla f = \dfrac{\partial f}{\partial r}\hat{\boldsymbol{r}} + \dfrac{1}{r}\dfrac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \dfrac{1}{r\sin\theta}\dfrac{\partial f}{\partial \varphi}\hat{\boldsymbol{\varphi}}$

2. $\nabla \cdot \boldsymbol{A} = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}(r^2 A_r) + \dfrac{1}{r\sin\theta}\dfrac{\partial}{\partial \theta}(A_\theta \sin\theta) + \dfrac{1}{r\sin\theta}\dfrac{\partial A_\varphi}{\partial \varphi}$

3. $$\nabla \times \boldsymbol{A} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\varphi \sin\theta) - \frac{\partial A_\theta}{\partial \varphi}\right]\hat{\boldsymbol{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \varphi} - \frac{\partial(rA_\varphi)}{\partial r}\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial(A_r)}{\partial \theta}\right]\hat{\boldsymbol{\varphi}}$$

4. $\nabla^2 f = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial f}{\partial r}\right) + \dfrac{1}{r^2\sin\theta}\dfrac{\partial}{\partial \theta}\left(\sin\theta\dfrac{\partial f}{\partial \theta}\right) + \dfrac{1}{r^2\sin^2\theta}\dfrac{\partial^2 f}{\partial \varphi^2}$

Selected integrals and sums

1. $\displaystyle\int_0^\infty dz\, e^{-az^2} = \frac{1}{2}\sqrt{\frac{\pi}{a}}$

2. $\displaystyle\int_{-\infty}^\infty dz\, e^{-az^2-2bz} = \sqrt{\frac{\pi}{a}}\, e^{b^2/a}$

3. $\displaystyle\int^x \frac{du}{\sin u} = \ln\left[\tan\left(\frac{x}{2}\right)\right]$

4. $\displaystyle\int^x \frac{du}{\cos u} = \ln\left[\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right]$

5. $\displaystyle\sum_{n=0}^\infty x^n = \frac{1}{1-x}\ ;\ |x| < 1$

Spherical harmonics $Y_{lm}(\theta,\phi)$

$Y_{00} = \dfrac{1}{\sqrt{4\pi}}$

$Y_{11} = -\sqrt{\dfrac{3}{8\pi}}\sin\theta\, e^{i\phi}$

$Y_{10} = \sqrt{\dfrac{3}{4\pi}}\cos\theta$

$Y_{22} = \dfrac{1}{4}\sqrt{\dfrac{15}{2\pi}}\sin^2\theta\, e^{i2\phi}$

$Y_{21} = -\sqrt{\dfrac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$

$Y_{20} = \sqrt{\dfrac{5}{4\pi}}\left(\dfrac{3}{2}\cos^2\theta - \dfrac{1}{2}\right)$

$Y_{33} = -\dfrac{1}{4}\sqrt{\dfrac{35}{4\pi}}\sin^3\theta\, e^{i3\phi}$

$Y_{32} = \dfrac{1}{4}\sqrt{\dfrac{105}{4\pi}}\sin^2\theta\cos\theta\, e^{i2\phi}$

$Y_{31} = -\dfrac{1}{4}\sqrt{\dfrac{21}{4\pi}}\sin\theta(5\cos^2\theta - 1)e^{i\phi}$

$Y_{30} = \sqrt{\dfrac{7}{4\pi}}\left(\dfrac{5}{2}\cos^3\theta - \dfrac{3}{2}\cos\theta\right)$

Constants

$e^2/\hbar c|_{\text{cgs gaussian}} = e^2/(4\pi\epsilon_0\hbar c)|_{\text{SI}} = 1/137$

$\hbar c = 197$ MeV$\cdot$ fm

$h = 6.626\times 10^{-34}$ J$\cdot$s $= 4.136\times 10^{-15}$ eV$\cdot$ s (Planck)

$c = 299{,}792{,}458$ m/s

$e = 1.602\times 10^{-19}$ C $= 4.803\times 10^{-10}$ esu

$R = 8.31$ J/(K$\cdot$ mol)

$k = 1.38\times 10^{-23}$ J/K $= 8.62\times 10^{-5}$ eV/K (Boltzmann)

$N_A = 6.022\times 10^{23}$ /mol (Avogadro)

$G = 6.674\times 10^{-11}$ N $\cdot$ m^2/kg^2

$\mu_0 = 4\pi\times 10^{-7}$ (N/A^2 or H/m or T$\cdot$ m/A)

$\epsilon_0 = 1/(\mu_0 c^2) = 8.854\times 10^{-12}$ F/m

$1/(4\pi\epsilon_0) = 8.99\times 10^9$ N $\cdot$ m^2/C^2

$g = 9.81$ m/s^2

$R_{\text{earth}} = 6.37\times 10^6$ m (radius of earth)

$M_{\text{earth}} = 5.97\times 10^{24}$ kg (mass of earth)

$M_{\text{sun}} = 1.98\times 10^{30}$ kg (mass of sun)

1 A. U. $= 1.5\times 10^{11}$ m (distance from sun to earth)

$m_e = 9.11\times 10^{-31}$ kg $= 0.511$ MeV/c^2 (electron mass)

$m_p = 1.673\times 10^{-27}$ kg $= 938.3$ MeV/c^2 (proton mass)

$m_e = 1.675\times 10^{-27}$ kg $= 939.6$ MeV/c^2 (neutron mass)

$m_\pi^\pm = 139.6$ MeV/c^2 (charged pion mass)

$m_\pi^0 = 135$ MeV/c^2 (neutral pion mass)

$m_K^\pm = 494$ MeV/c^2 (K-meson mass)

$m_\mu = 106$ MeV/c^2 (muon mass)

Conversion factors

$1\,\text{m} = 10^{10}$ Å $= 10^{15}$ fm

$1\,\text{T} = 1\,\text{Wb/m}^2 = 10^4$ G

$1\,\text{eV} = 1.60\times 10^{-19}$ J

$1\,\text{year} = 3.16\times 10^7$ s

$T/\text{K} = T/{}^\circ\text{C} + 273$

$1\,\text{cal} = 4.186$ J