QUALIFYING EXAM

Part IA

November 21, 2008

8:30 - 11:30 AM

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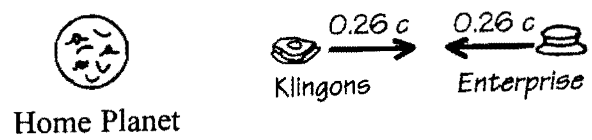
INSTRUCTIONS: CLOSED BOOK. Integral tables are permitted. WORK ALL PROBLEMS. Use back of pages if necessary. Extra pages are available. If you use them, be sure to make reference on each page to the problem being done.

PUT YOUR NAME ON ALL THE PAGES!

Physics Qualifying Exam 11/08 Part IA Name ______________________

Part I. A Klingon battleship has a proper length of 780 m and travels at speeds of 0.26 c with respect to its "home planet." The Klingons prepare to battle the Enterprise, which is moving at the same speed in the opposite direction with respect to the "home planet."



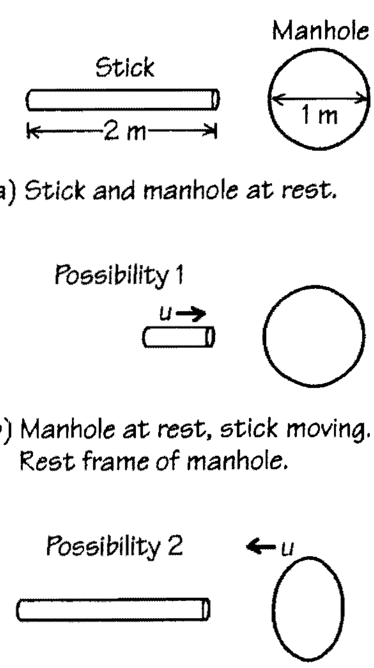
(a) Calculate the speed of the Enterprise in the Klingon frame.

(b) What is the length of the Klingon ship as measured by Captain Kirk on the Enterprise?

Part II. A perfectly smooth horizontal road has an open manhole 1 m in diameter. A stick of proper length 2 m is aligned with the road as shown in the figure below. The stick slides along the road with a constant speed u so that its length in the rest frame of the open manhole is 0.5 m. Therefore, a worker in the open manhole says that the stick will slide in. However, a second observer in the rest frame of the stick sees the manhole approaching with the same speed u and concludes that the length of the hole along the direction of motion is only 25 cm and hence the stick will not fall in.

How can these two points of view be reconciled?

HINT: Is a rigid body well defined in special relativity?

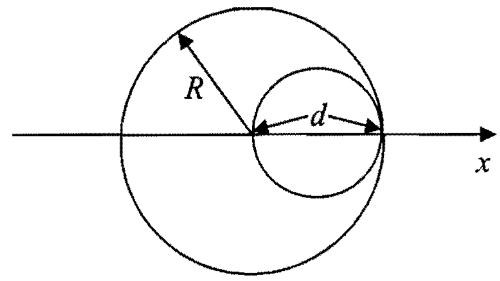


(a) Stick and manhole at rest.

(b) Manhole at rest, stick moving.
Rest frame of manhole.

(c) Stick at rest, manhole moving.
Rest frame of stick.

2. A solid sphere of radius R has a hole of diameter d ($d = R$) cut out from it as illustrated in the figure.



(a) What is the moment of inertia of the "hollowed out sphere" about the x direction shown in the figure?

(b) This object is taken into outer space, far from any objects so that any forces on it can be neglected.

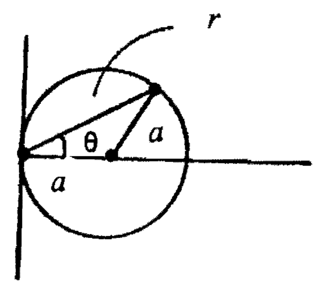
(i) Is there any direction such that if this object is set rotating about an axis in that direction it will continue to rotate about that axis? If yes, find ALL such directions. If no, EXPLAIN clearly why there are none.

(ii) About what direction of rotation will the angular momentum of the object be the smallest, assuming the angular speed of rotation is the same?

(iii) If this object is set rotating, are there any fixed points in the object that do not change their location as the object rotates. If yes, find the location of ALL these fixed points. If your answer is no, explain clearly why there can be no point that remains fixed.

3. A particle of mass m moves under the action of a central force. Its orbit is a circle of radius a which passes through the center of force.

   (a) Find the equations of motion.



   (b) Find the radial dependency of the force (i.e., $F(r) \sim \frac{1}{r^n}$).

HINT: The equation of the orbit is $r = 2a \cos\theta$.

4. Suppose that a galaxy of total (visible) mass M, consists of stars with uniform density in a central spherical nucleus of radius R, and stars in the disk of the galaxy beyond R (whose total mass we can ignore) but whose speeds, v, we can measure as a function of radius r, from the galactic center.

(a) Find the speed of the stars for $r \leq R$ and $r > R$.

(b) If the galaxy is submerged in invisible "dark matter", which has gravitational interactions only, dominates the mass of the galaxy, and has a density $\rho(r) \propto \frac{1}{r^2}$, find the radial dependence of the orbital speed of the stars in the disk.

(c) Do the results of (b) depend on the mass of the particles that make up the dark matter? How could one distinguish experimentally in the laboratory between heavy or light dark matter particles?

QUALIFYING EXAM

Part IB

November 21, 2008

1:30 - 4:30 PM

NAME ______________________________

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INSTRUCTIONS: CLOSED BOOK. Integral tables are permitted. WORK ALL PROBLEMS. Use back of pages if necessary. Extra pages are available. If you use them, be sure to make reference on each page to the problem being done.

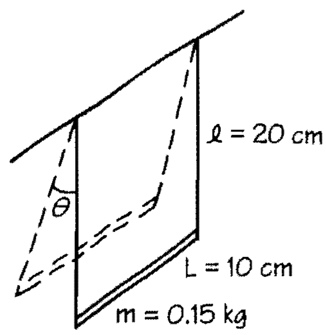
PUT YOUR NAME ON ALL THE PAGES!

1. A pendulum consists of a metal bar suspended from two thin metallic wires attached to a fixed conducting bar. The resistance of this closed circuit is $0.03\,\Omega$. The pendulum is placed in a vertical magnetic field of magnitude $0.12\,T$.

The pendulum is displaced by a small angle from its equilibrium position and allowed to oscillate.

(a) What is the induced voltage across the suspended metal bar as a function of time?

(b) What is the ratio of the average power dissipated to the mechanical energy of the oscillator?

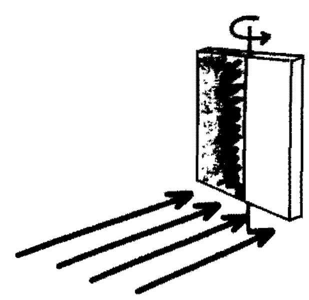


2. The short side of a thin, stiff rectangle $3.0cm \times 1.0cm$ is attached to a fixed vertical axis. Half of each side is painted black and is fully absorbent; the other half is a shiny, reflecting metal. The back side of the black part is shiny; the back side of the shiny part is black. There is no friction at the axis. The apparatus is bathed in a well-collimated (non-spreading) beam of light whose Poynting vector has magnitude $0.5kg/s^3$ and travels perpendicular to the vertical axis.

   (a) Calculate the initial force that the light beam exerts on the rectangle.

   (b) Calculate the initial torque on the rectangle's surface.

   (c) What is the average value of the torque due to the light over a full rotation about the axis?



3. A spherical volume of radius, a, is filled with volume charge density (in units of Coul/m^3),

$$\rho(r) = \frac{5g}{8\pi a^{5/2}} \frac{1}{\sqrt{r}}$$

where r is the radial distance, and g is a constant.

(a) Find the total charge on the sphere.

(b) For $r > a$, find the electric field and the electric potential.

(c) For $r < a$, find the electric field and the electric potential.

(d) Find the electric potential at the center of the sphere.

(e) Plot the electric field and the electric potential for all values of r.

4. According to the Liénard formula, the instantaneous power radiated by a point particle of charge q, mass m, velocity $\vec{\upsilon}$, and acceleration $\vec{a}$, is given by

$$P_{rad} = \left[\frac{1}{4\pi\varepsilon_0}\right] \frac{2}{3} \frac{q^2}{c^3} \gamma^6 \left[|\vec{a}|^2 - \left|\frac{\vec{\upsilon}}{c} \times \vec{a}\right|^2\right], \text{ where } \gamma = \frac{1}{\sqrt{1-\left(\frac{\upsilon}{c}\right)^2}}$$

(a) For an electron moving in a circular orbit in a plane perpendicular to a constant, uniform magnetic field $\vec{B}_0$, show that the Larmor radius is $R = \dfrac{\gamma m \upsilon}{q B_0}$.

(b) Find the power radiated by the electron in (a).

(c) Find P_{rad} in the high-energy limit where $\upsilon \approx c$ in terms of $q,\ m,\ c,\ \mathrm{B}_0$ and γ.

(d) Let the kinetic energy of the electron be 999 times its rest mass energy. Find an approximate value of the instantaneous power radiated by the electron if $\mathrm{B}_0 = 10\mathrm{T}$.

QUALIFYING EXAM

Part IIA

November 24, 2008

8:30 - 11:30 AM

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INSTRUCTIONS: CLOSED BOOK. Integral tables are permitted. WORK ALL PROBLEMS. Use back of pages if necessary. Extra pages are available. If you use them, be sure to make reference on each page to the problem being done.

PUT YOUR NAME ON ALL THE PAGES!

1. A beam of spin-½ particles is scattered off a spinless target. The initial and final momenta are given by $\boldsymbol{p}_i$ and $\boldsymbol{p}_f$, respectively. The scattering amplitude can be written as

$$f = a + b\boldsymbol{\sigma} \cdot \hat{\boldsymbol{n}}$$

where the spin of the beam particle is $\boldsymbol{S} = \frac{\hbar}{2}\boldsymbol{\sigma}$, σ_λ (λ = 1, 2, 3) are the Pauli matrices, and a, b are functions of energy E and scattering angle θ.

(a) Show that invariance under parity and time reversal implies that the vector $\hat{\boldsymbol{n}}$ can only be:

$$\hat{\boldsymbol{n}} = \hat{\boldsymbol{p}}_i \times \hat{\boldsymbol{p}}_f, \quad (\hat{\boldsymbol{p}}_i = \boldsymbol{p}_i / |\boldsymbol{p}_i|, \ \hat{\boldsymbol{p}}_f = \boldsymbol{p}_f / |\boldsymbol{p}_f|)$$

(b) Calculate the unpolarized cross-section (i.e., initial beam unpolarized, final polarization not measured).

(c) Show that the polarization, $\boldsymbol{P}$, of the final beam is given by $\boldsymbol{P} = \alpha\,\hat{\boldsymbol{n}}$ where $\alpha = \dfrac{2Re(a^* b)}{|a|^2 + |b|^2}$.

(d) Indicate how one might measure this polarization.

2. Consider the quantum Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \lambda m^2 \omega^3 x^4$$

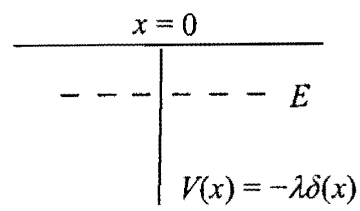
where the last term may be thought of as a small perturbation.

(a) Find the eigenenergies to first order in λ.

(b) Find the eigenstates to first order in λ, in terms of the unperturbed eigenstates of the harmonic oscillator.

HINT: $x = \left(a + a^+\right)\sqrt{\frac{\hbar}{2m\omega}}$, $a^+|n\rangle = \sqrt{n+1}|n+1\rangle$, $a|n\rangle = \sqrt{n}|n-1\rangle$

3. Demonstrate that the potential $V(x) = -\lambda\delta(x)$, $\lambda > 0$, has at least one bound state.



(a) From the Schrödinger equation show that $\left.\frac{d\psi}{dx}\right|_{x=+\varepsilon} - \left.\frac{d\psi}{dx}\right|_{x=-\varepsilon} = \frac{-2m\lambda}{\hbar^2}\psi(0)$, in the limit where $\varepsilon \to 0^+$.

(b) For $x \neq 0$ show that $\psi(x) = e^{-\kappa|x|}$ is a solution, and find κ.

(c) From (a) and $|E| = \frac{\hbar^2\kappa^2}{2m}$ find the bound state energy.

4. A particle of mass m is confined to a cube in which each side has length L. (In other words, the potential energy is infinite outside the cube and zero inside.)

(a) Find the normalized energy eigenfunctions.

(b) Find the energy eigenvalues.

(c) Determine the degeneracy of the lowest 3 energy levels.

(d) Find the density of states (the number of states per unit energy) in the limit of large quantum numbers.

QUALIFYING EXAM

Part IIB

November 24, 2008

1:30 - 4:30 PM

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INSTRUCTIONS: CLOSED BOOK. Integral tables are permitted. WORK ALL PROBLEMS. Use back of pages if necessary. Extra pages are available. If you use them, be sure to make reference on each page to the problem being done.

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1. An important contribution (20%) to radiogenic heating in the earth comes from radioactive decay of ^{40}K (potassium 40).

   (a) Estimate the heat production per unit mass of ^{40}K in units of W/kg using the half-life $t_{1/2} = 1.3\times10^{9}$ y and assuming the average energy deposited per decay is 0.65 MeV.

   (b) The equation governing the thermal evolution of the Earth is

   $$MC\frac{\partial T}{\partial t} = M\eta - 4\pi R^2 q,$$

   where M is the mass of the Earth (6.0×10^{24} kg), R is the radius of the Earth (6.4×10^{6} m), C is the specific heat capacity of the Earth [8.0×10^{2} J/(kg K)], q is the heat flow per unit area, and η is the radiogenic heat production rate per unit mass (W/kg). Typical values: the geothermal gradient (dT/dr = 2.0×10^{-2} K/m) and thermal conductivity [$k = 4.0$ W/(m•K)]. The abundance of ^{40}K in the earth is 26ppb (26 x 10^{-9}).

   Use your value of the heat production of ^{40}K to determine the rate of heating or cooling of the Earth in units of K/My. Is this heating or cooling?

2. A Carnot heat engine operates between a finite heat source (initial temperature is 300 K and heat capacity is 100J/K) and an infinite heat sink (temperature is 25 K). The engine eventually stops.

   (a) What is the final temperature of the heat source?

   (b) What is the total amount of heat extracted from the heat source?

   (c) What is the total work output?

3. The energy density of radiation in a blackbody cavity at a temperature T is given by the Planck formula,

$$\rho(\lambda)\ d\lambda = \frac{8\pi hc}{\lambda^5}\ d\lambda\ \frac{1}{\exp(hc/\lambda kT)-1}$$

where h is Planck's constant, c, the speed of light and k is Boltzman's constant.

(a) What is the average energy of a photon in this cavity?

(b) What would the answer in (a) have been had you ignored the indistinguishability of photons with the same quantum numbers?

(c) How would your answer be different had the photons been identical particles that also obeyed the Pauli exclusion principle?

You may use the results from the table below if you wish.

k	$\Gamma(k+1)$	$I_k = \int_0^\infty \frac{x^k dx}{e^x - 1}$	$J_k = \int_0^\infty \frac{x^k dx}{e^x + 1}$
$\frac{1}{2}$	$\frac{1}{2}\sqrt{\pi}$	2.315	0.678
1	1	$\frac{\pi^2}{6} = 1.645$	$\frac{\pi^2}{12} = 0.8225$
$\frac{3}{2}$	$\frac{3}{4}\sqrt{\pi}$	1.783	1.152
2	2!	2.404	1.803
3	3!	$\frac{\pi^4}{15} = 6.494$	$\frac{7}{120}\pi^4 = 5.682$

4. Consider a molecular crystalline solid of N molecules that have internal degrees of freedom (such as various intra-molecular conformations). The internal, non-degenerate ground state energy of each molecule is taken as zero. Each molecule has a single, four-fold degenerate, internal excited state of energy $\varepsilon_i>0$. The molecular internal states of each molecule are statistically independent of the other molecules.

   (a) Calculate the equation for the partition function of the molecular internal states of the N-molecule solid at absolute temperature T.

   (b) Calculate the equation for the entropy of the internal molecular states of the N-molecule solid at absolute temperature T. Discuss and sketch the (i) low-temperature ($kT<<\varepsilon_i$), and (ii) high-temperature ($kT>>\varepsilon_i$) limits (k is Boltzmann's constant).

   (c) Calculate the equation of the thermally averaged number of molecules in the internally excited state, n_i, at absolute temperature, T, where n_i is defined from the total thermally averaged internal excitation energy, $E_i=n_iN\varepsilon_i>0$, of the N-molecule solid.

   (d) Calculate the value of the internal excitation energy, $\varepsilon_i>0$ (in eV), for the case in which n_i is one part in a million at room temperature (300K).