

QUALIFYING EXAM

Part IA

November 26, 2007

8:30 - 11:30 AM

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TOTAL _____

INSTRUCTIONS: CLOSED BOOK. Integral tables are permitted. WORK ALL PROBLEMS. Use back of pages if necessary. Extra pages are available. If you use them, be sure to make reference on each page to the problem being done.

PUT YOUR NAME ON ALL THE PAGES!

1. A spaceship starts at rest in an inertial frame fixed relative to the sun and accelerates at 10 m/s^2 until it reaches a velocity of $0.5c$.
 - (a) How long does this take according to a clock on the spaceship?
 - (b) What is the final four-momentum of this 1000kg spacecraft, if the direction of acceleration was along the z -axis of the inertial frame?
 - (c) What is the magnitude of the four-momentum vector at $t=0$ and when it reaches velocity $0.5c$?

2. Consider an elastic collision in one dimension. Mass m_1 collides with mass m_2 , initially at rest. The energy transfer ratio TE is defined as

$$TE = \frac{\frac{1}{2}m_2v_{2f}^2}{\frac{1}{2}m_1v_{1i}^2} = \frac{E_{2f}}{E_{1i}},$$

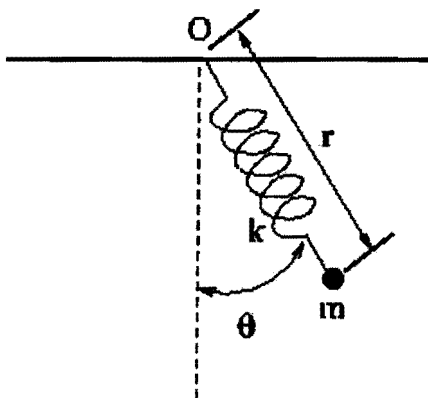
where v_{1i} is the initial velocity of m_1 and v_{2f} is the final velocity of m_2 .

- (a) Find an expression for TE in terms of $\mu = \frac{m_2}{m_1}$. For what value of μ is TE a maximum? What is the value of TE at maximum? Describe the collision for the case where TE is at maximum.

- (b) What is the ratio in terms of μ in center-of-mass coordinates $\frac{\frac{1}{2}m_2v_{2f-cm}^2}{\frac{1}{2}m_1v_{1i-cm}^2}$?

HINT: In the center-of-mass, how are the momenta of the two particles related?

3. A pendulum consists of a spring with a mass attached at one end, and the other end is attached to the ceiling (free to swing); see the figure below. The spring constant is k , the unstretched length of the spring is r_0 , the mass of the spring is assumed negligible, and the mass attached at the end can be considered as a point mass (m). The pendulum is released at an angle θ_0 with zero initial velocity, and the spring is initially not stretched nor compressed. The acceleration due to gravity is g . Assume no friction and no air-resistance.



For parts (a) to (e), DO NOT assume the small angle approximation.

- Write down the Lagrangian for the system in (r, θ) coordinates.
- Deduce the equations of motion (Euler-Lagrange equations) for $r(t)$ and $\theta(t)$.
- Is the angular momentum of the mass m about the point O a conserved quantity? If it is, what is its value?
- Is the total energy of the system a conserved quantity? If it is, what is its value?
- Is it possible for the pendulum to reach a larger angle than the initial angle at a later time? Explain.
- Comparing the period of this pendulum with that of a simple pendulum with fixed length r_0 , does this pendulum have a longer or shorter period? Explain.
- Assuming that the spring constant is very “large” but not infinite, obtain an approximate expression for the period of this pendulum assuming θ is small. Your expression need not be exact, but should reflect the effect of the spring.
- Determine a dimensionless parameter that can be used to quantify the statement that “the spring constant is very large” in part (g).

4. A particle of mass m and charge q (<0) moves under the influence of an infinite line charge of linear charge density λ (>0).
- (a) Obtain the equations of motion for the particle.
 - (b) Show that uniform circular motion in a plane is possible.
 - (c) Show that such circular motion is stable under small radial perturbations.
 - (d) Find the angular frequency of small radial oscillations about the stable orbit.
 - (e) Do small radial oscillations retrace themselves after a finite number of orbits?

QUALIFYING EXAM

Part IB

November 26, 2007

1:30 - 4:30 PM

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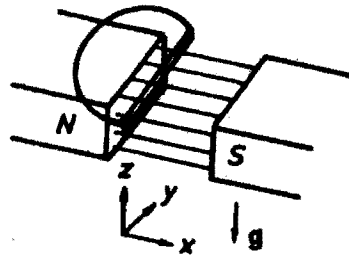
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1. A long straight wire oriented in the y direction lies in a uniform B field pointing in the x direction, as shown below. The mass per unit length of the wire and resistance per unit length of the wire are ρ and λ , respectively. The wire may be considered to extend to the edges of the field, where the ends are connected to one another by a massless perfect conductor that lies outside the field. Fringe effects and air resistance can be neglected.



- (a) Explain qualitatively why the wire will reach a terminal velocity as it falls in the earth's gravitational field.
- (b) If the wire is allowed to fall under the influence of gravity, calculate its terminal velocity (in terms of B , ρ , λ , g).

2. Consider the forces acting between two infinitely long line charges with charge/unit length equal to λ :

- (a) What is the vector force per unit length between these line charges if the two line charges are oriented parallel to each other at distance a ?
- (b) What is the net vector force acting between the two line charges if they are oriented at right angles to each other with distance of closest approach equal to a ?

Now consider the forces acting between two infinitely long thin wires each carrying a current I :

- (c) What is the vector force per unit length acting between the two wires if the two wires are oriented parallel to each other at distance a , and the currents are in the same direction?
- (d) What is the net vector force acting between the two wires if they are oriented at right angles to each other with distance of closest approach equal to a ?

3. A sphere of radius a has a radial polarization given by $\vec{P} = \alpha r^m \hat{r}$ (MKS units).
- (a) Find the bound-charge volume density and the bound-charge surface density.
 - (b) Find \vec{E} outside and inside the sphere.
 - (c) Find \vec{D} outside and inside the sphere.
 - (d) Verify that your results for \vec{E} satisfy the appropriate boundary conditions.
 - (e) Sketch $\vec{E}(r)$ as a function of r .

4. A coil of N closely packed turns is wound on a toroidal magnetic core of magnetic susceptibility χ_m . The torus has a square cross section of side a and a central radius of b (from the center of the torus to the midpoint of the square cross section).
- (a) Find the magnetic energy when there is a current I in the windings.
- (b) Find the self-inductance. (Assume the material is linear, isotropic, and homogeneous.)

QUALIFYING EXAM

Part IIA

November 27, 2007

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1. State true or false and give a clear reason for your answer. No reason or wrong reason means no credit.

- (a) Since the commutator $[P_x, L^2] \neq 0$, a free particle can never have a well-defined value of the squared angular momentum.

True/False

Reason:

- (b) For a quantum mechanical system, if the probability current density vanishes in a region, the probability density also vanishes in that same region.

True/False

Reason:

- (c) Eigenvalues and eigenfunctions of a Hermitean operator are always real.

True/False

Reason:

- (d) The third excited state of a spinless particle of mass m moving in a spherically symmetric potential $V(r) = \frac{1}{2}m\omega^2 r^2$ is not degenerate.

True/False

Reason:

- (e) The density matrix that describes the spin states of a spin- $\frac{1}{2}$ particle in the basis where S_x is diagonal is given by $\begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$. This system cannot be described by any wave function.

True/False

Reason:

- (f) Imagine a fictitious spin 1/2 particle X^- with the same mass and charge as the electron but distinguishable from an electron. Since the e^-e^- and e^-X^- experience the same Coulomb potential, the differential scattering cross-sections for e^-e^- and e^-X^- scattering must be identical.

True/False

Reason:

- (g) The scattering cross-section for a particle with momentum \vec{p} scattering off a potential never exceeds $\frac{20\pi\hbar^2}{p^2}$ in the second partial wave ($l=2$)

True/False

Reason:

- (h) The matrix element $\langle 2,1,n|\vec{r}\cdot\vec{p}|1,1,n\rangle$ always vanishes where the states are written in the angular momentum basis $|J, J_z, n\rangle$, where n denotes any other quantum numbers.

True/False

Reason:

- (i) In quantum mechanics it is impossible for two identical particles in a single system to have the same set of quantum numbers.

True/False

Reason:

- (j) The Born approximation for the scattering cross-section always works well as long as the energy in the scattering process is high enough.

True/False

Reason:

2. An electron has an energy $+E_0$ when its spin is aligned with the $+x$ direction and $-E_0$ when its spin is anti-aligned with that direction. The electron's spin at time $t=0$ is along the $+z$ axis.
- (a) Find an expression of spin state for some later time t .
 - (b) What is the probability of finding the spin along $-z$ at some later time t .

3. Consider a thin rigid rod rotating about an axis that passes through its center of mass and is perpendicular to the rod. The moment of inertia about this axis is I .
- (a) Write down the Hamiltonian for this rotational motion, find the energy eigenfunctions and show that the eigenvalues are given by

$$E_n = \frac{\hbar^2 n^2}{2I} \quad (n = 1, 2, \dots)$$

- (b) If this rod has an electric dipole moment \vec{d} and is placed in an electric field \vec{E} , (constant and uniform),
- (i) find the shift of the ground ($n=1$) state to first order
- (ii) find the shift of the ground ($n=1$) state to second order.

HINTS: (1) The Hamiltonian $H = T = KE$ in this case.

(2) The perturbation due to the dipole interacting with the electric field is

$$\delta H = \vec{d} \cdot \vec{E} = |\vec{d}| |\vec{E}| \cos \phi$$

4. (a) Show that the operator $D = 1 - \frac{ip\delta a}{\hbar}$, where p is the momentum operator for the x-direction, has the property that when acting on a position eigenket $|x'\rangle$ it takes $|x'\rangle$ into another eigenket of position with eigenvalue $x' + \delta a$.
- (b) If the operator D acts on a momentum eigenket $|p'\rangle$, does the new state remain a momentum eigenket?
- (c) Does the action of D on a free-particle state change its energy?

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Part IIB

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1. An ideal monatomic gas has a molar heat capacity at constant volume of C_V , at constant pressure of C_p , and $\gamma = C_p/C_V$. A sample of this gas initially fills volume V_0 at pressure P_0 and absolute temperature T_0 . The gas expands isobarically to twice its initial volume, then further expands adiabatically by another factor of two in volume.
 - (a) What is the total work done by the gas for this sequence of processes?
 - (b) What is the final temperature of the gas?

2. Consider a gas of photons inside a volume V at a temperature T .
- (a) Derive an expression for the number of modes within the frequency region $(\omega, \omega + d\omega)$
 - (b) Treat the photons as quantized harmonic oscillators and write down the partition function (Z) for this photon gas without evaluating it.
 - (c) Evaluate the Helmholtz free energy, $F(V, T) = -kT \ln Z$, and the internal energy, $U(V, T)$, of this photon gas. What are their volume and temperature dependence? You may leave any definite integrals alone without evaluating them.
 - (d) The universe is pervaded by the $3 K$ black body radiation. This is supposed to arise from the adiabatic expansion of a previously much hotter photon cloud produced in the big bang. Determine the temperature of the black body radiation when the universe expands to twice the present volume, assuming the expansion is isentropic.

3. Two identical silicon (*Si*) solids each consists of 10 moles of *Si* atoms in a rigid cubic shape. The solids are thermally isolated. One of the *Si* solid cubes is at absolute temperature $T = 400K$ and the other is at $T = 100K$. These solids are used as reversible heat sources to drive a reversible engine that delivers the maximum amount of work to a reversible non-thermodynamic (purely mechanical) system. Calculate the maximum work (in *Joules*) delivered. (Hint: the process stops when the heat sources reach the same final temperature.) The heat capacity of solid *Si* is:

$$C_V = nRAT^3$$

where n is the number of moles, $R = 8.31 \frac{J}{mole-K}$ is the molar gas constant, and $A = 8.71 \times 10^{-7} K^{-3}$.

4. An igloo, a hemispherical enclosure built of ice (thermal conductivity = $1.67 \text{ J/m-s-}^\circ\text{C}$), has an inner radius of 2.50 m. The thickness of the ice is 0.30 m.
- (a) Estimate the rate at which thermal energy must be generated by a heater to maintain the air inside the igloo at 0°C when the outside temperature is -40°C ? (Assume the air temperature inside the igloo is uniform and the only heat loss mechanism is via conduction through the hemispherical portion of the igloo.)
 - (b) If the heater in part (a) shuts off, what is the temperature inside the igloo as a function of time? Assume that the total heat capacity of everything inside the igloo is C (ignore the heat capacity of the ice and assume that the air temperature inside the igloo is uniform.) If you are unable to derive the exact result, partial credit will be given if you can describe the qualitative behavior of the time dependence of the temperature correctly.
 - (c) If the heater puts out thermal energy at a rate twice of that from part (a), qualitatively describe the new steady state of the igloo.