QUALIFYING EXAM

Part IA

March 17, 2006

8:30 - 11:30 AM

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INSTRUCTIONS: CLOSED BOOK. Integral tables are permitted. WORK ALL PROBLEMS. Use back of pages if necessary. Extra pages are available. If you use them, be sure to make reference on each page to the problem being done.

PUT YOUR NAME ON ALL THE PAGES!

Physics Qualifying Exam 3/06 Part IA Name______________________

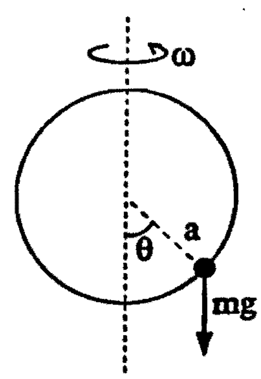
1. A bead of mass m slides without friction on a circular loop of radius a. The loop lies in the vertical plane and rotates about a vertical diameter with constant angular velocity ω.

   (a) What is the potential energy of the bead?

   (b) What is the Lagrangian of the bead?

   (c) For angular velocity ω greater than some critical angular velocity ω_c, the bead can undergo small oscillations about some stable equilibrium point θ_o. Find ω_c and θ_o.

   (d) Obtain the equations of motion for small oscillations about θ_o as a function of ω and find the period of oscillations.



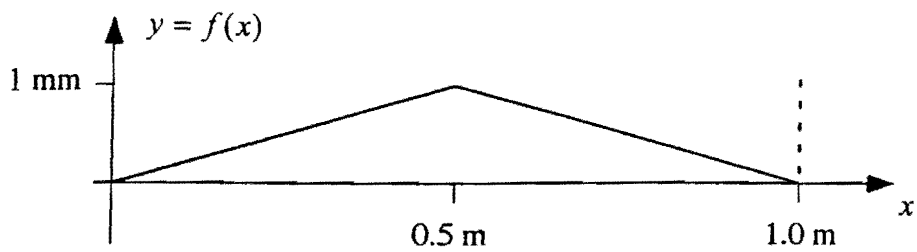
2. A wire of length $L = 1.0$ m and mass density $\mu = 0.2$ g/m, fixed at both ends under 100 N of tension, is plucked at the midpoint as shown in the figure and released at time $t = 0$.

(a) What is the frequency f of the subsequent oscillations? (Give a numerical value in Hz.)

(b) Calculate the Fourier coefficients of the Fourier series expansion for the initial transverse displacement $y = f(x)$.

(c) Modify the series expansion obtained in part (b) to give the full time-dependent solution for the transverse displacement $y(x,t)$ as a function of x and t.

(d) Sketch the transverse displacement of the wire at time $t = T/8$, where $T = 1/f$ is the period of oscillation.

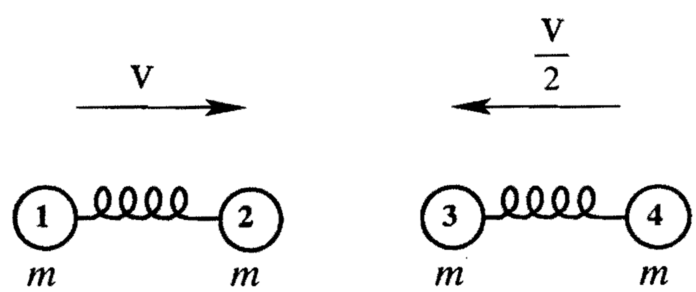


3. Consider a collision (in one dimension) of two bodies (see figure), each represented by two masses connected by springs with spring constant k. The collision is totally inelastic, i.e. the two masses, labeled 2 and 3 in the figure, become a single body of mass $2m$.

(a) Find the velocities of all masses immediately after the collision in the center-of-mass frame.

(b) Find the normal modes.

(c) Combine your answers to parts (a) and (b) to derive the trajectories after the collision: $x_1(t)$ for mass 1, $x_{23}(t)$ for the combined masses 2 and 3, and $x_4(t)$ for mass 4.



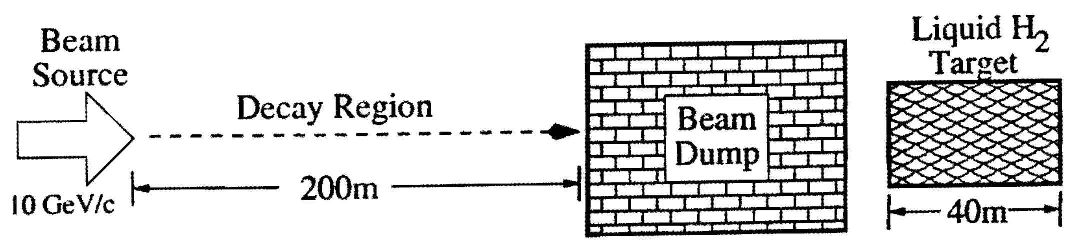
4. A momentum-selected beam of 10 GeV/c π^+ mesons passes through a 200m–long decay region and then is stopped in a heavily shielded "beam dump." Neutrinos produced by $\pi^+ \rightarrow \mu^+ \nu$ decays in the decay region pass through the beam dump and enter a 40m-long target filled with liquid hydrogen. $M_{\pi^+} = 140$ MeV/c^2, $M_{\mu^+} = 106$ MeV/c^2, $M_\nu \approx 0$.

(a) What fraction of the π^+ mesons decay in the decay region? (The rest-frame average lifetime of π^+ is $\tau_{\pi+} = 26$ nsec.)

(b) What is the maximum laboratory energy neutrino produced in the decay?

(c) If the neutrino-proton total cross section for all these neutrinos is $7\mathrm{x}10^{-38}$ cm^2, what fraction of the neutrinos entering the target interact?

{Density of Liquid H_2: 0.07 gm/cc}



QUALIFYING EXAM

Part IB

March 17, 2006

1:30 - 4:30 PM

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INSTRUCTIONS: CLOSED BOOK. Integral tables are permitted. WORK ALL PROBLEMS. Use back of pages if necessary. Extra pages are available. If you use them, be sure to make reference on each page to the problem being done.

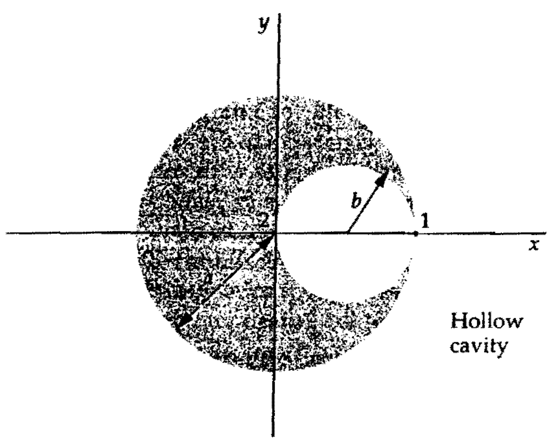
PUT YOUR NAME ON ALL THE PAGES!

Physics Qualifying Exam 3/06 Part IB Name________________________

1. A uniformly charged nonconducting sphere of radius a with center at the origin has volume charge density of ρ.

   (a) Show that at a point within the sphere a distance r from the center,
   $\vec{E} = \frac{\rho}{3\varepsilon_0} r \hat{r}$.

   (b) Material is removed from the sphere leaving a spherical cavity of radius $b = \frac{a}{2}$ with its center at $x = b$ on the x axis. Find the electric field in the cavity.



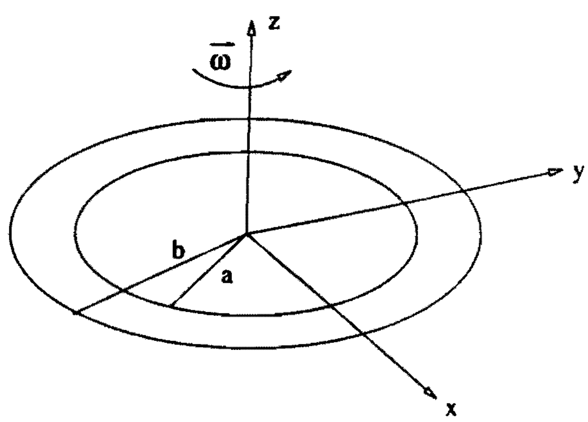
2. An infinitesimally thin annular disk of mass M lying in the $x-y$ plane is centered on the origin, with inside radius a and outside radius b. The disk spins with constant angular velocity $\vec{\omega} = \omega\,\hat{z}$ about the z-axis.

(a) Find the angular momentum $\vec{L}$ of the disk.

(b) If the disk has a charge Q spread uniformly over its surface, find its magnetic dipole moment $\vec{m}$.

(c) Find the gyromagnetic ratio: the ratio of the magnetic dipole moment to the angular momentum.

(d) A uniform magnetic field, $\vec{B}_0 = B_0\,\hat{x}$ is applied to the disk. Find the torque $\vec{\tau}$ on the disk.



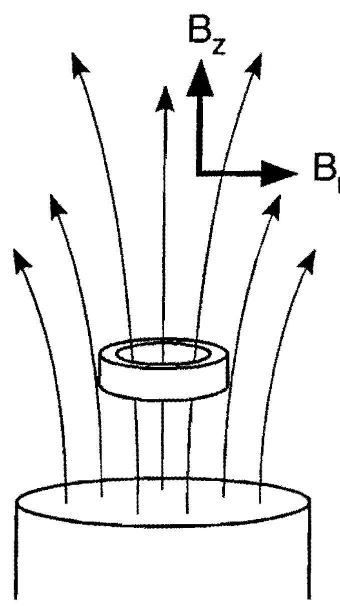
3. A thin, superconducting (*zero resistance*) ring of mass 50 mg, radius 0.5 cm, and self-inductance $L = 1.3 \times 10^{-8}$ H is held above a vertical, cylindrical magnetic rod, as shown in the figure. The axis of symmetry of the ring is the same as that of the rod. The cylindrically symmetric magnetic field around the ring can be described approximately in terms of the vertical and radial components of the magnetic field vector as

$$B_z = B_0(1-\alpha z)$$

and

$$B_r = B_0\beta r$$

where $B_0 = 0.01$ T, $\alpha = 2\ \mathrm{m}^{-1}$, $\beta = 32\ \mathrm{m}^{-1}$, and z and r are the vertical and radial position coordinates respectively. Initially, the ring has no current flowing in it. When released from rest at $z = 0$, it starts to move downwards under the influence of gravity with its axis still vertical.



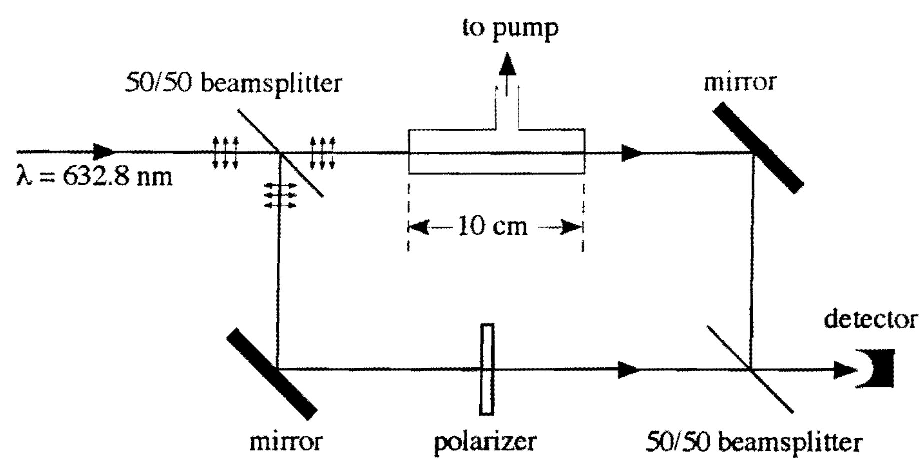
(a) Write an expression for the total magnetic flux through the ring. (Note that there are two contributions!)

(b) By appealing to Ohm's law, argue that the induced emf (electro-motive force) around the ring must be zero, and use this fact to determine a constant value for the total flux.

(c) Find the current in the ring as a function of position z.

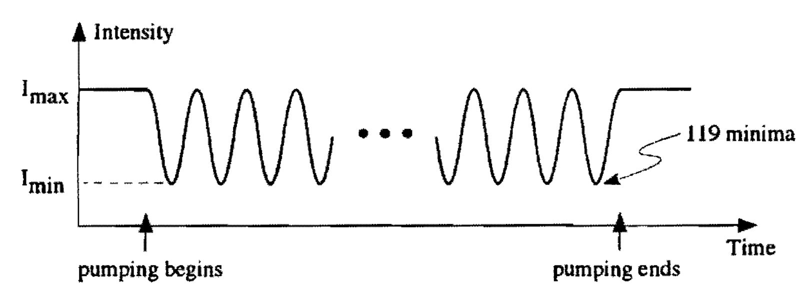
(d) Solve Newton's law of motion for the total force acting on the ring in the vertical direction to derive an expression for the current as a function of time. What is the maximum value of the current flowing in the ring?

4.



An interferometer is injected with a horizontally polarized, continuous-wave HeNe laser beam as shown in the figure above. One of the arms of the interferometer passes through a 10-cm gas cell that can be pressurized with a gas whose refractive index n varies with pressure P according to the relation $n = 1+AP$, where A is a constant. The other arm contains a linear polarizer. Initially, the gas cell is filled to an absolute pressure of 2.3 atm.

The gas cell is now slowly and completely evacuated, and during the evacuation 119 interference minima are observed in the intensity recorded by the detector, as illustrated below:



(a) Calculate the constant A in the expression for the refractive index.

(b) Write an expression for the fringe visibility $V_f = (I_{max}-I_{min})/(I_{max}+I_{min})$ as a function of the angle θ of the transmission axis of the polarizer with respect to the horizontal (ignore reflection losses from all optical surfaces.)

(c) If the polarizer is replaced with a polarization rotator (which simply rotates the plane of linear polarization by angle θ without attenuation), write an expression for the fringe visibility as a function of the rotation angle θ.

QUALIFYING EXAM

Part IIA

March 20, 2006

8:30 - 11:30 AM

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TOTAL ______________________________________

INSTRUCTIONS: CLOSED BOOK. Integral tables are permitted. WORK ALL PROBLEMS. Use back of pages if necessary. Extra pages are available. If you use them, be sure to make reference on each page to the problem being done.

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1. A particle of rest energy mc^2 is confined in a square well of width a.

(a) What are the energy eigenvalues and eigenfunctions?

(b) Find the first order relativistic correction to the kinetic energy and treat it as a perturbation to find the correction to the energy eigenvalues.

(c) For the ground state evaluate the energy and the correction given that:

$$mc^2 = 938 \text{ MeV}$$
and $$a = 2 \text{ fm}$$

[Note that $\sqrt{1+\varepsilon} = 1 + \frac{1}{2}\varepsilon - \frac{1}{8}\varepsilon^2 + \ldots$]

2. A particle in the ground state of an impenetrable one dimensional box with ends located at $x = \pm\frac{L}{2}$, but where the particle moves freely in the box.

(a) (i) Work out the ground state wave function.

(ii) Does this wave function have a definite parity?

(iii) Is it possible to find an excited state wave function of this system that does not have definite parity? If yes, give an example. If no, explain clearly why not.

(b) At time $t = 0$, the box is suddenly removed so that the particle is free to move everywhere. Find an equation for the most likely value of the particle momentum at a time just after $t = 0$. Solve for this value as completely as you can.

3. Consider a system of two spin ½ particles (labeled 1 and 2), with total spin $\vec{S} = \vec{S}_1 + \vec{S}_2$. The system is prepared to be in a state of total spin 0.

(a) Construct this state and verify explicitly that it has an eigenvalue 0 for both S_z and S^2.

(b) If no measurement is made on particle 2, what is the probability to find (i) S_{1z} to be $+\frac{\hbar}{2}$ or (ii) S_{1x} to be $+\frac{\hbar}{2}$?

(c) If S_{2z} is measured to be $+\frac{\hbar}{2}$, what is expected if immediately after the measurement of S_{2z} (i) S_{1z} is measured or if (ii) S_{1x} is measured?

(In parts (b) and (c) explain your answers.)

4. A system is described by a Hamiltonian $H_0 = E_A |A\rangle\langle A| + E_B |B\rangle\langle B|$ where $\langle A|A\rangle = \langle B|B\rangle = 1$ and $\langle A|B\rangle = \langle B|A\rangle = 0$.

(a) What are the eigenvalues and eigenkets of H_0?

(b) The system is perturbed by a term $H_1 = \Delta\,[|A\rangle\langle B| + |B\rangle\langle A|]$. Find the corrections to the energies up to second order in H_1.

(c) Find the approximate eigenkets of H.

(d) Is it possible to find exact eigenvalues for the full Hamiltonian $H = H_0 + H_1$? If so, find them and confirm the results of part (b).

(e) Give an example of a physical system to which the above might apply.

QUALIFYING EXAM

Part IIB

March 20, 2006

1:30 - 4:30 PM

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TOTAL ________________________________

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Physics Qualifying Exam 3/06 Part IIB Name_______________________

1. One kilogram of water is heated by an electrical resistor from 20° C to 99° C at constant atmospheric pressure. (Take the specific heat of water to be 1 cal/g.)

(a) What is the change in the internal energy of the water?

(b) What is the entropy change of the water?

(c) By what factor has the number of accessible quantum states of the water increased?

(d) What is the maximum achievable mechanical work by using this water as a heat reservoir to run an engine whose heat sink is at 20° C.

(e) 0.2 kg of ice at 0° C are now dropped into a beaker containing 1 kg of water initially at 99° C. (The heat of fusion of ice is 80 cal/g.) Find the final temperature of the mixture neglecting the heat capacity of the beaker.

2. Consider one mole of monatomic ideal gas.

(a) Calculate the entropy change of the gas if it undergoes an isothermal expansion from v_1 to v_2, where v_1 and v_2 are the initial and final molar volumes. Is it positive or negative?

(b) Calculate the entropy change of the gas if it undergoes an isobaric expansion from v_1 to v_2. Is it positive or negative?

(c) If the gas undergoes an isentropic expansion, from (v_1, T_1) to (v_2, T_2), where T_1 and T_2 are the initial and final temperatures. Calculate T_2 in terms of v_1, v_2, and T_1.

(d) If the gas undergoes an expansion process where the intermediate temperature is not constant but the initial temperature and the final temperature are the same, i.e., from (v_1, T) to (v_2, T). Is the entropy of this process greater than, or less than, or equal to that of the isothermal expansion in part a? (Assume the initial and final T of this process is the same as the isothermal temperature of part a).

3. A group of N non-interacting spin-$\frac{1}{2}$ fermions move in one dimension in a simple harmonic oscillator potential.

(a) Find the average occupation of the states just above and just below the Fermi energy.

(b) Determine the specific heat of the system near $T = 0$.

(c) Determine the specific heat of the system at high temperatures.

4. Consider a paramagnetic material where a certain fraction of the molecules have one unpaired electron which behaves to first order as if it were free. The material is immersed in a magnetic field $\vec{B} = B_0\,\hat{z}$ which causes some fraction of the electron spins to align parallel with $\vec{B}$.

(a) What is the energy difference of the spin-parallel state compared to the spin-antiparallel state?

(b) At temperature T, what fraction of the electron spins are in each of the two states?

(c) What frequency of electromagnetic radiation will be absorbed by the population of free electrons in the material at the spin-flip resonance?

(d) How much energy would have to be absorbed from an incident radiation field at the resonance to equalize the population of the two spin states? Estimate the absorbed energy for this case for $B_0 = 0.1$ Tesla, and material at a cryogenic temperature $T = 1°$ K with $N_0 = 10^{18}$ available electron spins.