QUALIFYING EXAM

Part IA

March 11, 2005

8:30 - 11:30 AM

NAME____

1. ____

2. ____

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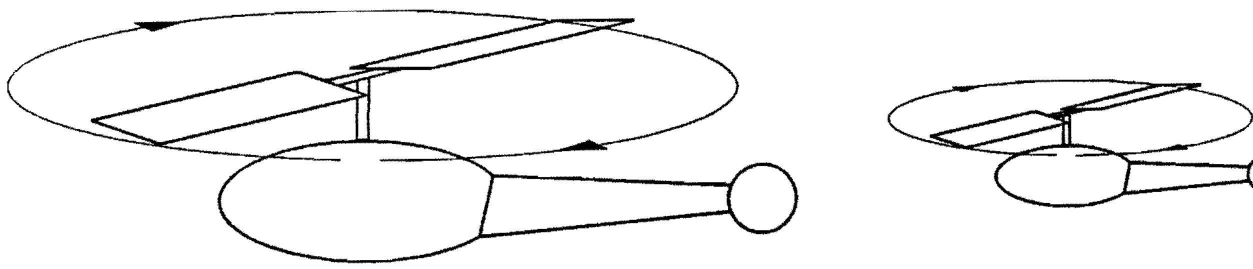
TOTAL ____________________________________

INSTRUCTIONS: CLOSED BOOK. Integral tables are permitted. WORK ALL PROBLEMS. Use back of pages if necessary. Extra pages are available. If you use them, be sure to make reference on each page to the problem being done.

PUT YOUR NAME ON ALL THE PAGES!

1. Consider two helicopters hovering in the air. The helicopters are structurally identical in every respect, except that the linear dimensions of the second one are half those of the first.



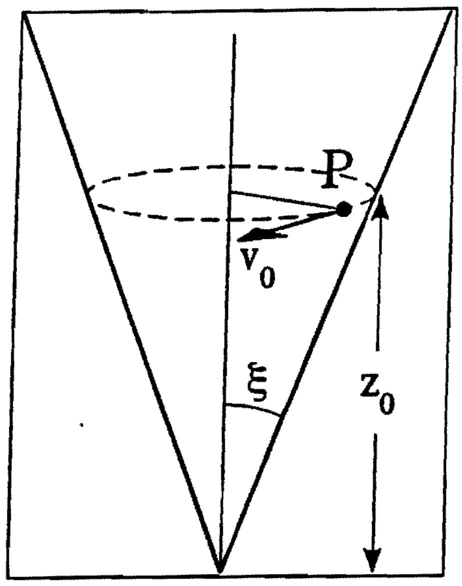
(a) Derive an expression for the lift force on the first helicopter (to within an overall constant of proportionality), as a function of the angular velocity of the rotor, the density of air, and any relevant geometric parameters of the rotor such as its outer radius, width, tilt angle, etc.

(b) What is the ratio of the angular velocities of the rotors for the two helicopters, given that both helicopters are hovering at a fixed elevation?

(c) If the first helicopter can hover when the power output of its engine is P, what power output is needed to enable the second helicopter to hover?

2. A wooden cube with edge length a and mass m floats in water with one of its faces parallel to the water surface. The density of the water is ρ. Give an expression for the vertical position of the cube's center of mass as a function of time if the cube is initially displaced downward by a small distance y_0 and released from rest.

3. A particle of mass m is constrained to slide frictionlessly under gravity on the inside surface of an inverted cone with semi-vertical angle ξ.



At time $t = 0$, this particle is shot with speed v_0 from a point P whose vertical height above the apex of the cone is z_0. Its initial velocity is horizontal.

(a) If z denotes the vertical height of this particle from the apex of the cone $[z(t=0) = z_0]$ obtain either z(t) or t(z) in the form of an integral $z(t) = \int dt\, f(t)$ or $t(z) = \int dz\, g(z)$.

You do not need to evaluate the integral.

(b) Analyze your answer in (a) and discuss whether or not z(t) can ever exceed z_0. If the answer is NO, explain why. If the answer is YES, discuss quantitatively when this is possible and when it is not.

HINT: Look at the conserved quantities for this system.

4. Part I

A Λ hyperon with laboratory momentum 10 GeV/c decays in flight to a π^+ and a proton.

Compute the maximum and minimum proton lab momentum.

$$M_\Lambda c^2 = 1115 \text{ MeV}$$

$$M_{\pi^+} c^2 = 139 \text{ MeV}$$

$$M_{\text{proton}} c^2 = 938 \text{ MeV}$$

Part II

A spaceship, starting from rest on Earth, accelerates towards a distant galaxy 2 billion light years away in such a way that its occupants experience a constant, comfortable acceleration of $g = 9.81 \text{ m/s}^2$ for the entire trip. How much time will elapse on the spaceship's clocks during this trip?

HINT: Apply the velocity addition formula to obtain a relation between an increment $d\tau$ of spaceship proper time and an increment dv of spaceship velocity relative to Earth, and use the relation between incremental proper time $d\tau$ and Earth-time dt to show that

$$\frac{dv}{dt} = \frac{g}{\gamma^3},$$

where $\gamma = \left(1 - v^2/c^2\right)^{-1/2}$ is the gamma factor for the spaceship relative to Earth.

QUALIFYING EXAM

Part IB

March 11, 2005

1:30 - 4:30 PM

NAME____

1. ____

2. ____

3. ____

4. ____

TOTAL ____

INSTRUCTIONS: CLOSED BOOK. Integral tables are permitted. WORK ALL PROBLEMS. Use back of pages if necessary. Extra pages are available. If you use them, be sure to make reference on each page to the problem being done.

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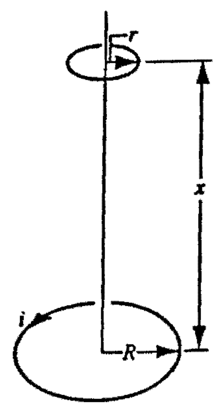
1. Consider two parallel loops as shown. The smaller loop (radius r) is a distance x above the larger loop (radius R) with x>>R.

The current i in the larger loop is constant. Suppose the distance x is increasing at a rate dx/dt=v, where v is a constant.

(a) What is the B field from the larger loop at the location of the smaller loop? (You can assume it is uniform over the smaller loop).

(b) What is the induced emf in the smaller loop?

(c) What is the direction of the induced current in the smaller loop?



2. Part I

(a) Calculate the energy density of the electric field at a distance r from an electron at rest (presumed to be a point-like particle), in terms of e and r.

(b) Assume now that the electron is not a point particle but a sphere of radius R over whose surface the electron charge is uniformly distributed. Determine the total energy associated with the electric field in vacuum as a function of R.

(c) If we associate the energy in (b) with the mass of electron (using $E_0 = m\,c^2$), we can determine a value for R, the classical radius of the electron. Calculate the value of R in meters.

$$m = 9.109\,(10^{-31})\text{ kg}$$

$$c = 299{,}792{,}458\text{ m/s}$$

Part II

In the Bohr planetary model of the hydrogen atom, the electron circulates around the nucleus in a circular path of radius 5.29×10^{-11} m at a frequency $f = 6.60 \times 10^{15}$ Hz.

(d) What is the B field at the center of the orbit?

(e) What is the associated magnetic energy density?

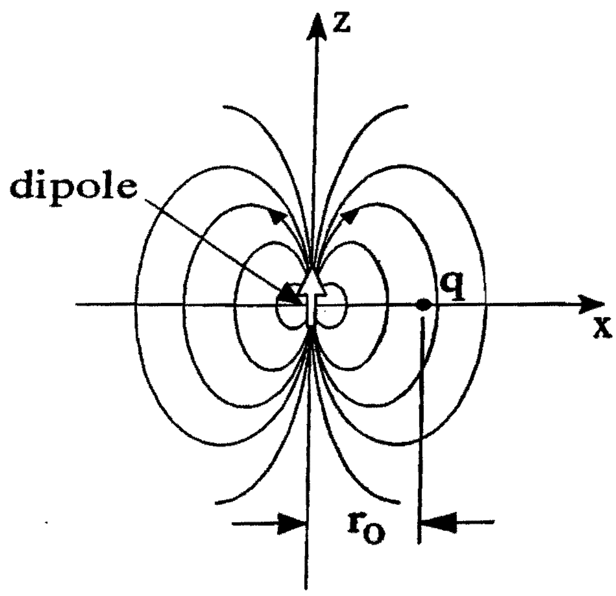
3. Light of 633 nm wavelength is used to uniformly illuminate three thin slits separated by 300 μm (slit-to-slit), and the resulting diffraction pattern is displayed on a screen 5 m away.

(a) Calculate the positions and relative intensities of the interference maxima at the screen, and sketch the diffraction pattern.

(b) If a thin glass plate were placed over the middle slit, by what ratio would the maximum intensity of the interference pattern change if the glass plate introduces a phase shift of 90°?

(c) If a second thin glass plate were placed over one of the remaining slits, what phase shift would restore the *general shape* of the original diffraction pattern?

4. An electric dipole of dipole moment $\vec{p}$ is placed at the origin and is directed along the z-axis. A point charge q is placed at a distance r_0 from the origin on the x-axis. If the point charge is released from rest, calculate the shape of the trajectory, $r = r(\theta)$, followed by the point charge. (r, θ are the usual spherical coordinates).



The electric field of a dipole is given by $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_o}\frac{1}{r^3}[3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}]$, where $\hat{r}$ is the unit vector in the radial direction.

QUALIFYING EXAM

Part IIA

March 14, 2005

8:30 - 11:30 AM

NAME____

1. ______________________________________

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TOTAL ___________________________________

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Physics Qualifying Exam 3/05 Part IIA Name______________________

1. Consider a one-dimensional quantum oscillator, described by the Hamiltonian:

$$H = \frac{p^2}{2m} + U(x, t) \text{ where } U(x, t) = \begin{cases} \dfrac{m\omega_1^2 x^2}{2}, & t < 0 \\ \dfrac{m\omega_2^2 x^2}{2}, & t > 0 \end{cases}$$

For $t \leq 0$, the oscillator is in the ground state $|0\rangle$.

(a) For $t > 0$, find the probability that the oscillator is in excited states $|1\rangle$, $|2\rangle$.

(b) For $t > 0$, find the probability that the oscillator is in excited state $|n\rangle$.

(c) If $T = \pi/(2\omega_2)$, what is the probability that at $t = T$, the oscillator is at the same state $|0\rangle$ as it was for $t < 0$?

HINT: Use creation and annihilation operators to solve this problem.

2. A bound state of the b quark and the b anti-quark is described by the following Hamiltonian:

$$H = \frac{\vec{p}_1^{\,2}}{2m_b} + \frac{\vec{p}_2^{\,2}}{2m_b} - \frac{\alpha_s}{|\vec{r}_1 - \vec{r}_2|} + \Lambda|\vec{r}_1 - \vec{r}_2|,$$

where $m_b = 4.9$ GeV and $\alpha_s = 0.5$.

(a) Treating $\Lambda|\vec{r}_1 - \vec{r}_2|$ as a perturbation, compute the energy levels of the nS states, for $n = 1, 2$ through first order in Λ. Why is treating $\Lambda|\vec{r}_1 - \vec{r}_2|$ as a perturbation guaranteed to fail for high enough values of the principal quantum number n?

(b) The two lightest $\Upsilon(1S)$, $\Upsilon(2S)$ resonances, the S-wave states of b and $\bar{b}$, have masses $m_1 = 9.46$ GeV, $m_2 = 10$ GeV. Determine Λ by comparing the result of your perturbative calculation for $n = 1$ with m_1 and check if you get a reasonable result for m_2. Is treating $\Lambda|\vec{r}_1 - \vec{r}_2|$ as a perturbation a good approximation for m_2?

3. Let $\vec{A}$ and $\vec{B}$ be any vector operators (under rotation) and $\vec{J}$ the angular momentum operator. Which of the matrix elements listed below are <u>necessarily</u> zero? Give a <u>short reason</u> for each of your answers.

(i) $<\alpha,\ j=1,\ m=0 \mid \vec{J}\cdot\vec{A} \mid \alpha, j=2, m=0>$

(ii) $<\alpha,\ j=1,\ m=0 \mid \vec{J}\cdot\vec{A} \mid \alpha', j=1, m=0>,\ \alpha\neq\alpha'$

(iii) $<\alpha,\ j=1,\ m=0 \mid \vec{J}\cdot\vec{J} \mid \alpha', j=1, m=0>,\ \alpha\neq\alpha'$

(iv) $<\alpha,\ j=1,\ m=0 \mid J_z^2 \mid \alpha, j=2, m=0>$

(v) $<\alpha,\ j=1,\ m=0 \mid J_z A_z \mid \alpha, j=2, m=0>$

(vi) $<\alpha,\ j=1,\ m=1 \mid J_z A_z \mid \alpha', j=2, m=1>,\ \alpha\neq\alpha'$

(vii) $<\alpha,\ j=3,\ m=2 \mid \vec{A}\cdot\vec{B} \mid \alpha', j=2, m=2>,\ \alpha\neq\alpha'$

(viii) $<\alpha,\ j=1,\ m=0 \mid A_z^2 \mid \alpha', j=1, m=0>,\ \alpha\neq\alpha'$

(ix) $<\alpha,\ j=7,\ m=0 \mid A_z B_z \mid \alpha, j=2, m=0>$

(x) $<\alpha,\ j=1,\ m=1 \mid J_z A_z \mid \alpha, j=2, m=0>$

Here, α, α' denote other quantum numbers such that states with $\alpha\neq\alpha'$ are orthogonal.

4. Ten non-interacting electrons are confined to a rectangular box with dimensions $1\text{Å} \times \sqrt{2}\,\text{Å} \times 2\,\text{Å}$. ($1\text{Å} = 10^{-8}$cm).

   (a) Compute the ground state energy for 10 electrons.

   (b) Identify the principal quantum numbers of the first degenerate single electron energy level and determine its energy.

QUALIFYING EXAM

Part IIB

March 14, 2005

1:30 - 4:30 PM

NAME______________________________________

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TOTAL ________________________________

INSTRUCTIONS: CLOSED BOOK. Integral tables are permitted. WORK ALL PROBLEMS. Use back of pages if necessary. Extra pages are available. If you use them, be sure to make reference on each page to the problem being done.

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1. Show that the energy, U, of n moles of an ideal gas at fixed temperature is independent of the volume. Use only the equation of state, pV = nRT, and thermodynamic identities to demonstrate that $\left(\frac{\partial U}{\partial V}\right)_T = 0.$

2. An engineer is designing a heat engine that operates between two **finite** heat baths. The initial temperatures of these two baths are $T_1 = 500\ K$ and $T_2 = 300\ K$. The engine extracts heat energy from the hot bath, performs work, and expels the remaining heat energy to the cold bath. The (temperature dependent) heat capacities of the baths are $C_1 = 10\ T^3\ [J/K^4]$ and $C_2 = 5T^3\ [J/K^4]$.

(a) Theoretically, what is the maximum amount of work that can be performed by this heat engine?

(b) What are the final temperatures of the two heat baths when all the work is done?

(c) How much heat is removed from the hot bath?

(d) What is the entropy change of the whole system, i.e. heat baths and engine?

3. N atoms are located at N lattice sites of a crystal in an external magnetic field $\vec{B}$. Each atom has an angular momentum $\vec{J}$ and its magnetic moment is given by $\vec{m} = \mu_B \vec{J}$ where μ_B is the Bohr magneton. Assume these magnetic moments do not interact with each other.

(a) Find the magnetization $\vec{M}$ of the system as a function of temperature T.

($\vec{M}$ = *average* magnetic moment per volume = $\dfrac{<\vec{m}>}{V}$),

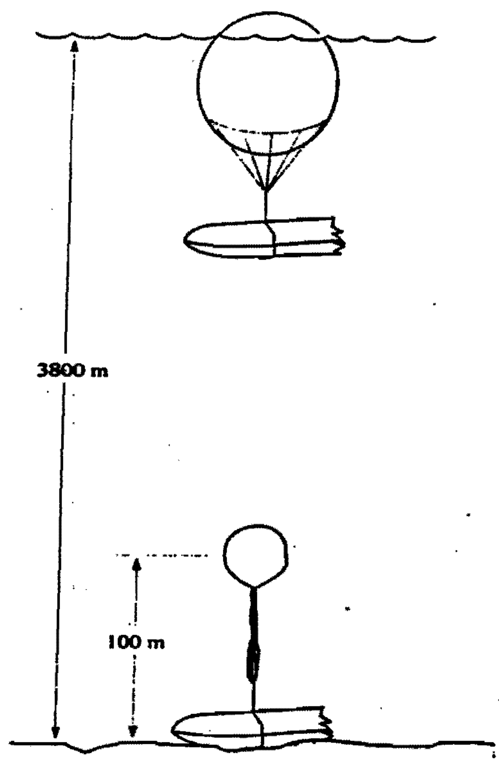
DO NOT leave your answer as a sum; evaluate the expression in closed form.

(b) What is $\vec{M}$ as $T \to 0$?

(c) What is $\vec{M}$ as $T \to \infty$?

(d) Find the first non-vanishing temperature-dependent term for $\vec{M}$ as $T \to \infty$.

4. Design a "lift bag" (a partially inflated bag) to raise each of the two sections of the wreck of the RMS Titanic. Use the following data and assumptions:



- Each section is at a depth of $D = 3800$ m with a mass of $M = 2.5 \times 10^7$kg.
- The water temperature in that region of the North Atlantic is nearly constant with depth. Assume $T = 0^\circ$ C for the whole depth, and the density of the seawater is constant with depth, $\rho_{sw} = 10^3 \text{kg m}^{-3}$.
- Assume that the center of the lift bag will be deployed 100 m above the bottom, and that the lift bag material itself has neutral buoyancy in water. Neglect the weight of the attachment cable, and assume the bag has sufficient volume at 1 atm pressure and that the expansion is adiabatic.

If the initial buoyancy force is 20% greater than the weight of one section, what is the final volume of the fully inflated bag (for Helium and for Nitrogen)? What are the required masses of Helium and Nitrogen?