

QUALIFYING EXAM

Part IA

March 14, 2003

8:30 - 11:30 AM

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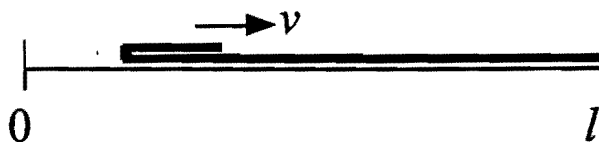
TOTAL _____

INSTRUCTIONS: **CLOSED BOOK.** Integral tables are permitted. **WORK ALL PROBLEMS.** Use back of pages if necessary. Extra pages are available. If you use them, be sure to make reference on the page containing the problem.

PUT YOUR NAME ON ALL THE PAGES!

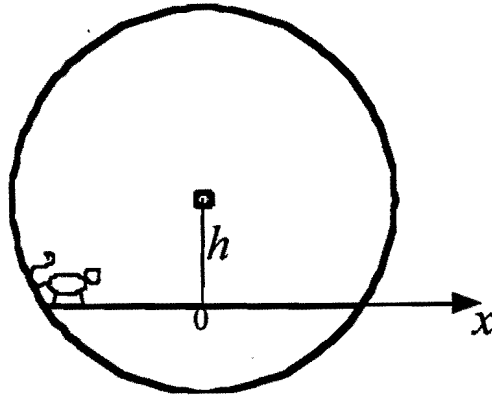
1. Consider the head-on collision of two equal mass particles whose interaction is described by a potential $U(x) = \alpha|x|^2$, where x is the relative distance between the two particles, $\alpha > 0$. At $t = 0$, the first particle is at rest at $x = 0$, the second particle is at $x = -R$ moving with velocity v . In the limit $R \rightarrow \infty$, determine where the second particle comes to rest after the collision. Are there qualitative changes in the solution if the interaction potential is $U(x) = \alpha\sqrt{|x|}$?

2. A long, thick, pliable carpet of length l and mass m is laid on the floor. One end of the carpet is bent back and then pulled backwards with constant velocity v , just above the part of the carpet that is still at rest on the floor.



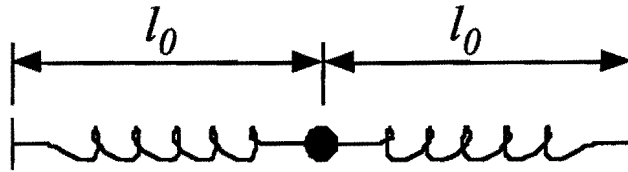
- (a) Find the speed of the center of mass of the moving part.
- (b) Find the force required to pull the carpet.

3. A small hamster is put into a circular wheel-cage, which has a frictionless central axle. A horizontal platform is mounted inside the wheel a distance h below the center. Initially, the hamster is at rest at one end of the platform.



When the wheel is released, the hamster starts running, but because of the hamster's motion the platform and wheel remain stationary. Determine how the hamster moves, i.e., derive an equation of motion for the hamster's position $x(t)$.

4. A small bob joins two light unstretched, identical springs, anchored at their far ends and arranged along a straight line.



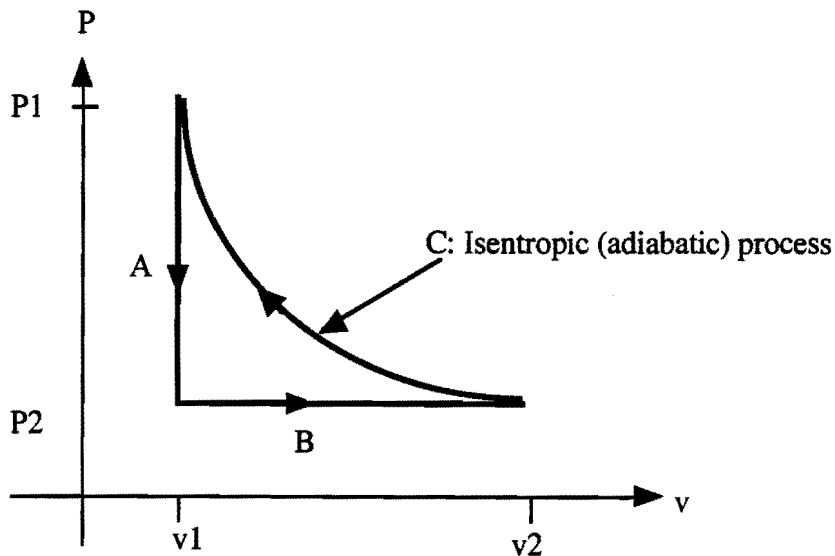
The bob is displaced in a direction perpendicular to the line of the springs by 1 cm and then released. The period of the ensuing vibration of the bob is 2 s. Find the period of the vibration if the bob is displaced by 2 cm before release. (The unstretched length of the springs is $l_0 \gg 1$ cm, and gravity is to be ignored.)

5. You find an abandoned mineshaft and wish to measure its depth. Using an audio oscillator of variable frequency, you note that you can produce successive resonances at frequencies of 63.58 and 89.25 Hz. What is the depth of the shaft?

6. Find the pressure in atmospheres at the center of the Earth assuming that the interior can be treated as a sphere, filled with a incompressible fluid in equilibrium
($R = 6.37 \times 10^6 \text{ m}$, $M = 5.97 \times 10^{24} \text{ kg}$, $g = 9.81 \text{ m/s}^2$).

7. Suppose the internal energy (u) of one mole of a particular substance is given by $u(P, v) = \frac{5}{2} P v$, where P is the pressure and v is the molar volume.

Consider the following quasi-static cycle:



- (a) Fill the following table for Δu (change in internal energy), W =work done by the substance, Q =heat added to the system. Express your answers in terms of P_1, P_2, v_1 , and v_2 :

	Δu	W	Q
Process A			
Process B			
Process C			

- (b) Since (P_1, v_1) and (P_2, v_2) are connected by an adiabatic process, their values are related. Express the ratio of P_2/P_1 to the ratio of v_2/v_1 .

8. Three beads of mass m are connected by three massless springs of spring constant k . Both the beads and the springs are threaded on a frictionless circular ring of radius R . Determine the frequencies of the normal modes of oscillation and describe qualitatively the motion in each normal mode.

QUALIFYING EXAM

Part IB

March 14, 2003

1:30-4:30 PM

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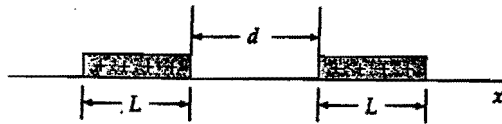
1. A circular loop of wire with mass M and N turns carries a current I in a uniform magnetic field B . It is initially in equilibrium with the loop perpendicular to the magnetic field. The loop is given a small twist about a diameter and then released.

What is the period of the motion (neglect gravity)?

2. A coaxial cable consists of two concentric very thin-walled conducting cylinders of radii r_1 and r_2 . Current I flows in one direction down the inner cylinder and in the opposite direction down the outer cylinder. Assume that the current is distributed uniformly on the cylinders.

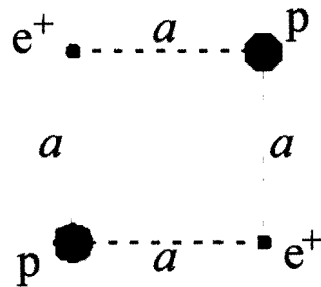
Calculate the self-inductance per unit length of the cable.

3. Two equal uniform line charges of length L , charge density λ , lie on the x -axis a distance d apart as shown in the figure.
- (a) What is the force that one line charge exerts on the other line charge?
- (b) Consider the limit $d \gg L$, show that the force approaches that for two point charges with charge $= \lambda L$.



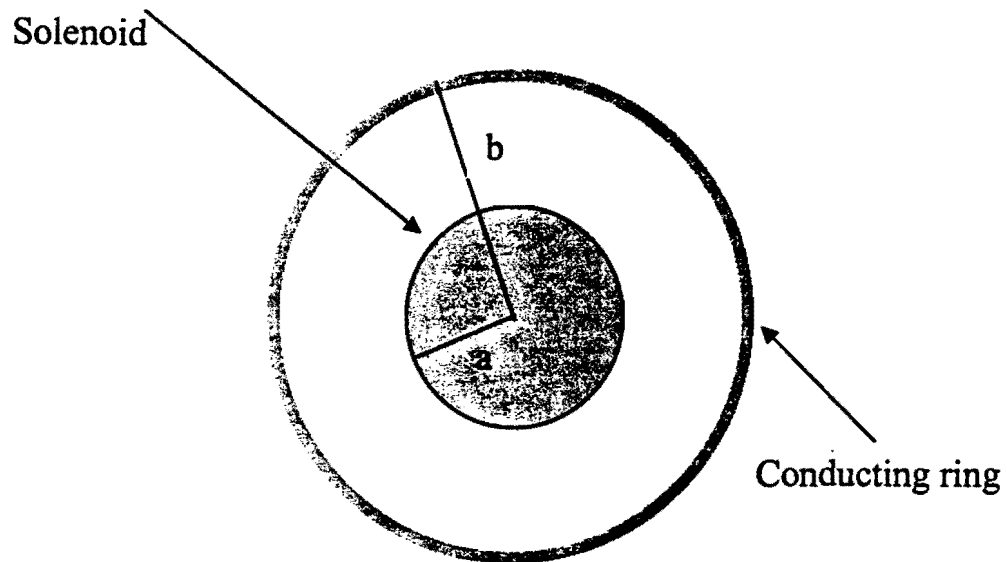
4. It has been suggested that a spacecraft could be propelled by the radiation pressure from the sun.
- (a) What must the surface mass density of a perfectly reflecting sheet be so that the force due to radiation pressure is twice the gravitational attraction of the sun at 1 A.U. (1.5×10^{11} m)? (The power output of the sun is 3.8×10^{26} W. Its mass is 1.99×10^{30} kg).
 - (b) What is the acceleration of this sail?
 - (c) Is this a practical way to travel to the far reaches of the solar system?

5. Two positrons are at opposite corners of a square of side $a = 1$ cm. The other two corners of the square are each occupied by a proton. Initially, the particles are held in these positions, and then released at the same time.

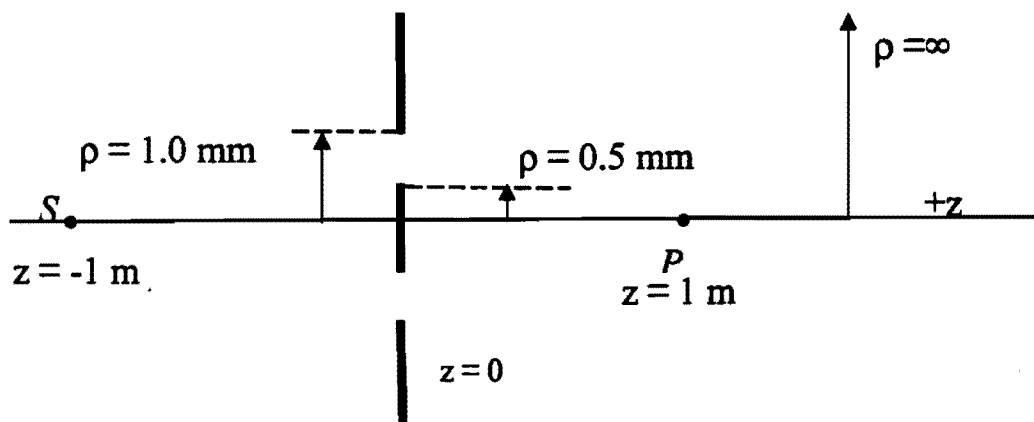


Determine approximately the final speeds of the particles at large distances (ignore gravity). Assume that protons are much more massive than positrons.

6. The current on a infinitely long solenoid of radius a , with n turns per unit length, changes slowly with time at a rate $di/dt < 0$. (a) Find an expression for the direction and magnitude of the induced electric field inside and outside of the solenoid. (b) If a narrow conducting ring of radius b and resistance R is placed coaxially around the solenoid, as shown on the figure, find the induced current on the ring.



7. The plane $z = 0$ is opaque except for the clear circular ring defined by $0.5 \text{ mm} \leq \rho \leq 1.0 \text{ mm}$. A monochromatic point source S ($\lambda = 500 \text{ nm}$) is placed along the z -axis at $z = -1.0 \text{ m}$. What is the irradiance (intensity) at a point P located at $z = 1 \text{ m}$, compared to the irradiance if the opaque plane were not there?



8. The distance between a screen and a light source lined up on an optical bench is 120 cm. When the lens is moved along the line joining them, sharp images of the source can be obtained at two lens positions. The size (*area*) ratio of these two images is 1:9. What is the focal length of the lens (assuming the lenses are thin)?

QUALIFYING EXAM

Part IIA

March 17, 2003

8:30 - 11:30 AM

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1. ${}^8\text{Be}$ nuclei contain 4 protons and 4 neutrons. In principle, these can decay into a pair of α - particles. Given that α - particles are spinless, what can you conclude about the spin of the nuclear states of ${}^8\text{Be}$ for which the decay ${}^8\text{Be} \rightarrow \alpha \alpha$ is observed to occur?

2. A spin $\frac{1}{2}$ particle is prepared with "spin up" along a direction \hat{n}_1 .

If the spin of this particle is measured along \hat{n}_2 which makes an angle of 60° with \hat{n}_1 , what is the probability of getting (a) $-\frac{1}{2}\hbar$? (b) $-\frac{1}{4}\hbar$?

3. Consider a particle of mass m confined to the potential (“quantum-well”):

$$V(x,y,z) = 0 \text{ for } 0 < z < d, \text{ independent of } x \text{ or } y$$
$$= \infty \text{ for } z \leq 0, z \geq d, \text{ independent of } x \text{ or } y$$

- (a) Find the energy eigenfunctions and energy eigenvalues.
- (b) Suppose an EM wave interacts with the particle. Let the interaction Hamiltonian between the particle and an EM wave be

$$H_{\text{int}} = \vec{A} \cdot \vec{P} \cos\omega t$$

Let $\vec{A} = A_z \hat{z}$, where A_z is constant and \hat{z} is a unit vector in the z direction

\vec{P} = momentum operator for that particle

State the selection rules for this interaction in terms of the quantum numbers of the energy eigenfunctions found in part (a).

4. A system is described by a Hamiltonian given by $H = H_0 + H_1$.

There are two states $|+\rangle$ and $|-\rangle$ forming a complete set. The action of H_0 and H_1 on these is described by

$$\begin{aligned} H_0 |\pm\rangle &= \pm E |\pm\rangle \\ \text{and} & \\ H_1 |\pm\rangle &= A |\mp\rangle. \end{aligned} \quad \left. \begin{array}{l} E \text{ and } A \\ \text{are positive.} \end{array} \right\}$$

- (a) Using a trial wave function (normalized)

$$|\Psi_t\rangle = c|+\rangle + s|-\rangle$$

where $c = \cos\theta$ and $s = \sin\theta$; and using the variational principle, show that the ground state energy satisfies the bound

$$E_g \leq -\sqrt{E^2 + A^2}$$

- (b) Calculate the exact value for the ground state energy.
- (c) Give an example of a physical system described by such a Hamiltonian.

5. The Hamiltonian of a spin 1/2 particle in a magnetic field oriented along the z-axis is given by

$$H = \omega S_z = \frac{\hbar\omega}{2} \sigma_z$$

where σ_z is a Pauli matrix. At time $t = 0$, the particle has its spin directed along the x-axis,

$$\text{i.e., } |\Psi(0)\rangle = \frac{1}{\sqrt{2}} (|+\rangle + |-\rangle).$$

- (a) Show that at time t the spin state of the particle is

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}} \left[e^{-i\omega t/2} |+\rangle + e^{i\omega t/2} |-\rangle \right]$$

- (b) Hence, show that the expectation values of the spin components precess around the z-axis with angular velocity ω ,

$$\text{i.e., } \langle S_z \rangle = 0, \langle S_x \rangle = \frac{\hbar}{2} \cos\omega t, \langle S_y \rangle = \frac{\hbar}{2} \sin\omega t.$$

6. A model potential energy, U , for the electron in diatomic molecular hydrogen ion, H_2^+ , is:

$$U(x) = -A \left[\delta \left(x - \frac{R}{2} \right) + \delta \left(x + \frac{R}{2} \right) \right]$$

where $A > 0$ is a constant and R is the fixed separation between the protons. The motion of the electron is assumed one-dimensional (x -axis), and x is the coordinate of the electron.

Write the transcendental expression for the energy levels of the electron for the even parity states.

7. Consider the hydrogen atom. Because of the relativistic relation between electron energy and momentum $E = \sqrt{p^2 + m^2}$, there is a correction to the non-relativistic Hamiltonian $\delta H = -p^4/(8m^3)$ (units $\hbar = 1, c = 1$). Treating δH as a perturbation, find the shift of the energy levels.

8. A particle of energy E scatters off the potential $U(r) = \alpha/r^3$. Determine the dependence of the scattering amplitude on E in the limit $E \rightarrow 0$.

QUALIFYING EXAM

Part IIB

March 17, 2003

1:30-4:30 PM

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1. There are 5×10^9 electrons per meter in a one dimensional infinite square well. Determine the energy (in eV) of the most energetic electron.

2. A gas of photons in a volume, V , is in thermal equilibrium at absolute temperature, T . Calculate

- (a) The energy of the gas
- (b) The entropy of the gas
- (c) The Helmholtz free energy of the gas
- (d) The pressure of the gas
- (e) The average number of photons in the gas

Note: The results should be expressed in terms of V and T .

3. (a) Draw the Feynman diagrams for the decays $\pi^+ \rightarrow e^+ \nu_e$ and $\pi^+ \rightarrow \mu^+ \nu_\mu$.
(The quark content of the charged pion is $(u \bar{d})$).
- (b) The mass of the electron is 0.511 MeV and the mass of the muon is 105.7 MeV. However, the ratio $\Gamma(\pi^+ \rightarrow e^+ \nu_e) / \Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu)$ is found experimentally to be about 1.2×10^{-4} , despite the much larger phase space for the reaction in the numerator. Explain why this is so.

4. Suppose a particle of mass m and of lab-frame energy γmc^2 ($\gamma \gg 1$) collides head-on with a photon of energy E_0 .
- (a) Derive the maximum energy of the scattered photon.
 - (b) If a 10^{20} eV proton collides elastically with a cosmic microwave background photon ($T = 2.7$ K), what is the maximum energy the photon can acquire?

5. The electron and positron can form a bound state, positronium. The ground state of such a system can be either a spin-singlet ($S=0$) [parapositronium] or a spin-triplet state ($S=1$) [orthopositronium].
- (a) Which state has lower energy, singlet or triplet?
(Explain your reasoning.)
- (b) What are the dominant annihilation modes for:
(i) orthopositronium ?
(ii) parapositronium ?

6. (a) List all of the quantum states with radial quantum numbers $n=2$ in the hydrogen atom.
- (b) Indicate on a diagram how the energy levels are shifted by hyperfine splitting.
- (c) Indicate the effect of the Lamb shift.
What is its order of magnitude?
Which state is higher and which state is lower?

7. TRUE (T) OR FALSE (F) (Circle One)

1. T F Crystals of inert gases are transparent insulators.
2. T F The force is zero between two atoms at equilibrium distance.
3. T F The less massive the atom, the smaller the quantum correction to the zero point energy.
4. T F Crystals in a ferroelectric state are also piezoelectric.
5. T F The density of amorphous silicon is less than its crystalline counterpart.
6. T F The nearest neighbors contribute most of the interaction energy of inert gas crystals.
7. T F Hydrogen becomes metallic under high pressure.
8. T F Silicon crystals have a diamond structure.
9. T F Diamagnetism occurs in all materials.
10. T F The spin entropy of a paramagnet is reduced by a magnetic field.
11. T F The two spins in the bonding state of the H₂ molecule are parallel.
12. T F An antiferromagnet is ferromagnetic above the Néel temperature.
13. T F During the motion of a Cooper-pair there is no transfer of energy to the lattice.
14. T F The exciton gas is electrically insulating at low concentration.
15. T F Hydrogen bonds connect only two atoms.
16. T F An electromagnetic wave with imaginary wavevector does not propagate.
17. T F There is an energy gap between optical and acoustical phonon branches.
18. T F There is only one way to define a primitive cell.
19. T F Phonons are fermions.
20. T F The energy gap of a superconductor is temperature dependent.
21. T F An applied magnetic field does not alter the free energy of a superconductor.
22. T F The Einstein model treats the vibrating atoms as coupled quantum oscillators.

8. Neutrinos and anti-neutrinos may annihilate by the reaction $\nu + \bar{\nu} \rightarrow Z^0$ where the Z^0 is the neutral boson of the weak interaction, with a mass energy of about 91 GeV. Determine the threshold energy for this process if the neutrino collides with an anti-neutrino from the relic neutrino background with a temperature of 1.9 K. Assume the neutrinos and anti-neutrinos have mass of 0.03 eV, as indicated by neutrino oscillation results.