QUALIFYING EXAM

Part IA

March 15, 2002

8:30 - 11:30 AM

NAME____

1. ____

2. ____

3. ____

4. ____

5. ____

6. ____

7. ____

8. ____

TOTAL ____________________________________

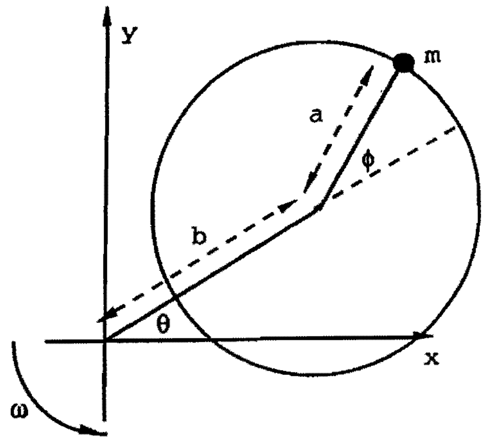
INSTRUCTIONS: CLOSED BOOK. Integral tables are permitted. WORK ALL PROBLEMS. Use back of pages if necessary. Extra pages are available. If you use them, be sure to make reference on the page containing the problem.

PUT YOUR NAME ON ALL THE PAGES!

Physics Qualifying Exam 3/02 Part IA Name______________________

1. A mass m slides without friction along a wire hoop of radius a. The wire hoop rotates with constant angular velocity ω about the origin of the x-y plane in the direction shown. The distance from the origin to the center of the hoop is b (constant).

What are the equations of motion of the system? What is the frequency of small oscillations?

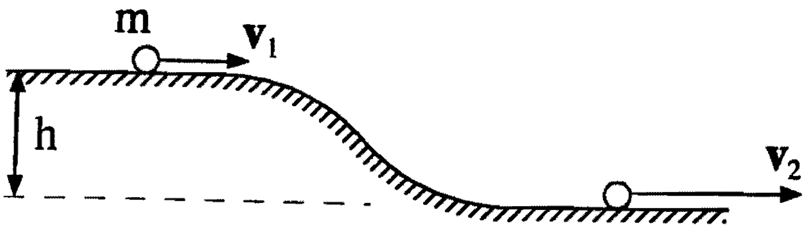


2. A block of wood of mass 1.5 kg floats on water with 68% of its volume submerged. A lead block is placed on the wood and the wood is then fully submerged. Find the mass of the lead block. (density of water: 1000 kg/m^3)

3. A physics student drops a vibrating 440 Hz tuning fork down the elevator shaft of a tall building. When the student hears a frequency of 400 Hz, how far has the tuning fork fallen? (Speed of sound in air is 340 m/s.)

4. A ball of mass m (and negligible moment of inertia) rolls down a hill, and its horizontal speed increases from v_1 to v_2, where $\mathbf{v}_1$ and $\mathbf{v}_2$ are horizontal as shown. In the absence of friction we immediately have

$$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = mgh$$



Now, an observer travelling horizontally with speed v_0 towards the left observes the same ball rolling down the hill, and records the initial and final speeds as v_1' and v_2'. It is clear that

$$v_1' = v_1 + v_0$$

and

$$v_2' = v_2 + v_0$$

but a naive application of conservation of energy in the moving frame reveals that

$$\frac{1}{2}mv_2'^2 - \frac{1}{2}mv_1'^2 \neq mgh$$

Prove analytically that energy is in fact conserved in the frame of the moving observer.

5. A mass *m* slides on a horizontal frictionless track. It is connected to a spring fastened to a wall. Initially, the amplitude of the oscillations is A_1 and the spring constant is k_1. The spring constant then decreases adiabatically at a constant rate until the value k_2 is reached. (For example, suppose the spring is being dissolved by nitric acid.) What is the new amplitude?

Hint: use the adiabatic invariant where by Stokes' theorem:

$$I = \oint \frac{p\,dq}{2\pi} = \int\int \frac{dp\,dp}{2\pi}$$

6. One mole of a non-ideal gas (described by the van der Waals equation) is initially confined to one half of a gas chamber which is insulated from the environment. The partition is removed and the gas finally settles to occupy the entire chamber. If the initial internal energy, temperature, pressure, volume, and entropy are u_i, T_i, P_i, v_i, s_i, find the final internal energy, temperature, pressure and entropy in terms of the initial values and a, b, c, and R. Also state whether the final values are greater, smaller, or equal to the initial values.

van der Waals equation:

$$\frac{P}{T} = \frac{R}{v-b} - \frac{a}{v^2}\frac{1}{T}$$

$$u = cRT - \frac{a}{v}$$

P = Pressure

T = temperature

R = Gas constant

u = internal energy for one mole of gas

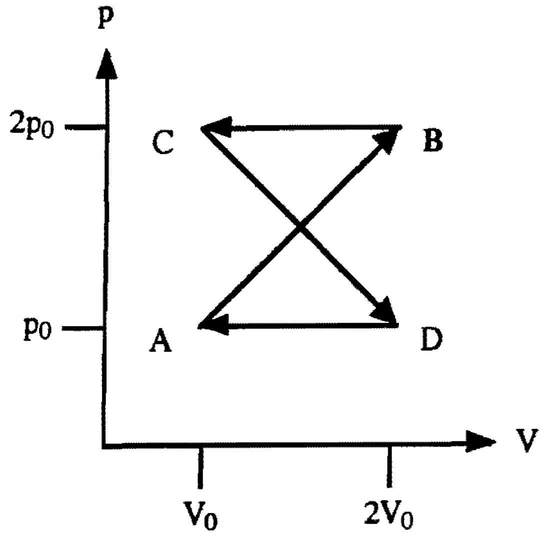
v = molar volume

a, b and c are parameters for the gas.

7. Calculate the efficiency of the heat engine performing the reversible cycle:

$$A \rightarrow B \rightarrow C \rightarrow D \rightarrow A,$$

as shown in the pressure (p) vs. volume (V) diagram below. The working substance is a monatomic, classical ideal gas.



8. Suppose an alien from another universe told you that in her universe, "Kepler's Third Law" of planetary motion is: The period of a planet around the Sun is proportional the square of the orbital radius. Kepler's Second Law remains the same: "Equal area in equal time."

   (a) Deduce the r-dependence of the gravitational force based on this information (you may assume a circular orbit).

   (b) What is the r-dependence of the gravitational potential?

   (c) Would such an r-dependence have allowed life to evolve in our own universe (assuming that planets are required to support life)?

QUALIFYING EXAM

Part IB

March 15, 2002

1:30-4:30 PM

NAME____

1. ____

2. ____

3. ____

4. ____

5. ____

6. ____

7. ____

8. ____

TOTAL ____________________________________

INSTRUCTIONS: CLOSED BOOK. Integral tables are permitted. WORK ALL PROBLEMS. Use back of pages if necessary. Extra pages are available. If you use them, be sure to make reference on the page containing the problem.

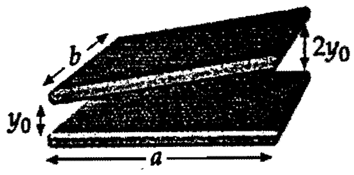
PUT YOUR NAME ON ALL THE PAGES!

1. Three concentric conducting spherical shells have radii a, b, and c such that $a < b < c$. Initially, the inner shell is uncharged, the middle shell has a positive charge, $Q > 0$, and the outer shell has negative charge, $-Q$.

   (a) Find the electric potential of the three shells.

   (b) If the inner and outer shells are now connected by a thin, insulated wire that passes through a tiny hole in the middle shell, what is the electric potential of each of the three shells, and what is the final charge on each shell?

2. A capacitor has rectangular plates of length a and width b. The top plate is inclined at a small angle. The plate separation varies from $d = y_0$ at the left to $d = 2y_0$ at the right where y_0 is much less than a or b. Calculate the capacitance. (Hint: this capacitor is equivalent to many infinitessimal capacitors in parallel).



3. A thin disk of radius R carries a fixed charge density σ (charge per unit area) and rotates with constant angular velocity ω.

   (a) What is the magnetic field at distance x along an axis perpendicular to the center of the disk?

   (b) Does the disk radiate electromagnetic waves?

4. Consider the wave equations of the electromagnetic field ***E***, ***H*** in an infinite dielectric of complex permittivity $\varepsilon^* = \varepsilon' - i\varepsilon''$ and vacuum permeability μ_0:

$$\frac{\partial^2 \boldsymbol{E}}{\partial x^2} = \varepsilon^* \mu_0 \frac{\partial^2 \boldsymbol{E}}{\partial t^2}, \qquad \frac{\partial^2 \boldsymbol{H}}{\partial x^2} = \varepsilon^* \mu_0 \frac{\partial^2 \boldsymbol{H}}{\partial t^2}.$$

A solution to these equations is a plane wave propagating in the $+x$ direction:

$$E_y(\mathrm{x}, \mathrm{t}) = E_{0y} \exp(i\omega t - \gamma x), \qquad H_z(\mathrm{x}, \mathrm{t}) = H_{0z} \exp(i\omega t - \gamma x),$$

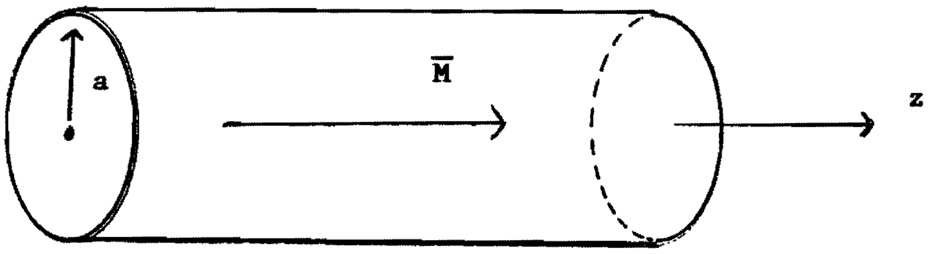
with all other components equal zero.

Here, the complex propagation factor $\gamma = i\omega\sqrt{\varepsilon^* \mu_0}$, $\omega = 2\pi c/\lambda_0$, and $c = (\sqrt{\varepsilon_0 \mu_0})^{-1}$.

Find the observed wavelength λ within the dielectric, and the attenuation length L_a (defined as the distance over which the wave amplitude falls to 1/e of its initial value.) You will not need to solve the wave equations above.

Express the answer in terms of the free-space wavelength λ_0, the real part of the relative dielectric constant $\mathrm{K}' = \varepsilon'/\varepsilon_0$, and the *loss tangent* defined by $\tan \delta = \varepsilon''/\varepsilon' = \mathrm{K}''/\mathrm{K}'$.

5. If a dielectric slab of unknown thickness is inserted over one of the slits in Young's two-slit experiment, the central maximum is observed to shift by 3.4 fringes to one side when illuminated by a He-Ne laser of wavelength 0.6328 μm. If the index of refraction of the slab is 1.467, what is its thickness?



6. An infinitely long cylinder of radius 'a' carries a "frozen-in" magnetization parallel to its axis $\vec{M} = kr(a - r)\,\hat{z}$, where k is a constant, r is the distance from the axis, and $\hat{z}$ is a unit vector directed along the axis of the cylinder. (There are no free currents anywhere.)

Calculate the $\vec{B}$-field both inside and outside of the cylinder.

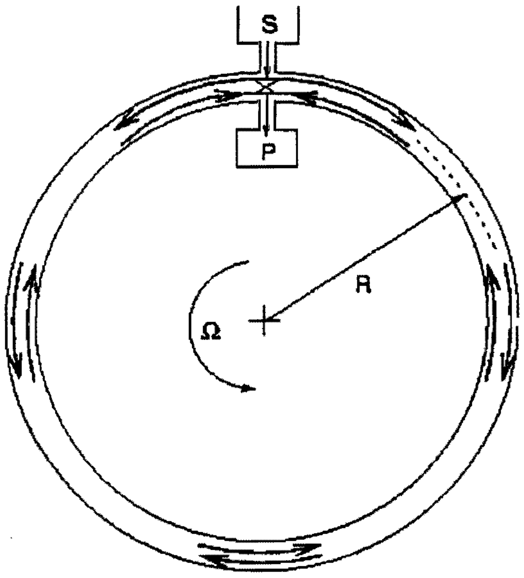
7. If a stellar object collapses, assuming that its mass loss can be neglected, its angular rate of rotation must increase to satisfy conservation of angular momentum, and its "embedded" magnetic field must increase to satisfy conservation of magnetic flux (assuming infinite electrical conductivity).

Calculate the electromotive force $\int \boldsymbol{E} \bullet \mathbf{dl}$ induced by rotation of such a collapsed object, taking the integral from the pole to the equator.

Initial radius = 7×10^5 km
Final radius = 100 km
Initial rotation rate $\omega = 2.8 \times 10^{-6}$ radians/s
Initial embedded magnetic field = 10^{-4} Tesla

HINT: Use the identity $\int \boldsymbol{E} \bullet \mathbf{dl} = \int \mathbf{v} \times \mathbf{B} \bullet \mathbf{dl}$ where **v** is the velocity at the surface.

8.



An electromagnetic wave of wavelength λ is injected at time t=0 by a source S into a coaxial waveguide that forms a circular ring of radius R much larger than either the diameter of the waveguide or λ. The radiation is coupled through a beam splitter which produces two counterpropagating beams in the waveguide. (Assume here that the waves propagate with the free space velocity c, eg., that the permittivity and permeability of the waveguide interior are the same as in free space).

The ring structure rotates around an axis through its center and perpendicular to the plane of the ring with angular velocity Ω. For $\Omega R \ll c$, derive the phase difference $\Delta\phi$ observed in the reference frame of the ring between the two waves when they arrive back at the original beam splitter and are received at point P. Express the result in terms of Ω, λ, and A, the area of the ring. (Assume P and S are very close to the ring). Is Lorentz invariance violated? Why or why not?

QUALIFYING EXAM

Part IIA

March 18, 2002

8:30 - 11:30 AM

NAME____

1. ____

2. ____

3. ____

4. ____

5. ____

6. ____

7. ____

8. ____

TOTAL ____________________________________

INSTRUCTIONS: CLOSED BOOK. Integral tables are permitted. WORK ALL PROBLEMS. Use back of pages if necessary. Extra pages are available. If you use them, be sure to make reference on the page containing the problem.

PUT YOUR NAME ON ALL THE PAGES!

1. Consider a two-particle system with total momentum $\vec{P} = \vec{P}_1 + \vec{P}_2$.
   Let $\vec{r} \equiv \vec{r}_1 - \vec{r}_2$ be the relative co-ordinate.

   (a). A student preparing for the qualifying exam argues that because the simultaneous determination of P_{1x} and r_{1x} (and also P_{2x} and r_{2x}) is limited by Heisenberg's uncertainty principle, there must be a corresponding uncertainty relation between P_x and r_x.

   Is this student correct? If yes, work out the order of magnitude of the minimum value of $\Delta P_x \Delta r_x$. If not, explain why the student is wrong.

   (b). Imagine a world in which every electron e has a partner $\tilde{e}$ with <u>spin zero</u>, and with exactly the same electric charge and mass as the electron. In other words, the electrical potential (and force!) between two electrons is exactly the same as between two $\tilde{e}$'s, and also exactly the same between an e and an $\tilde{e}$.

   The same student argues that because the scattering of one particle off another is completely determined by the interaction potential between them, it must be that

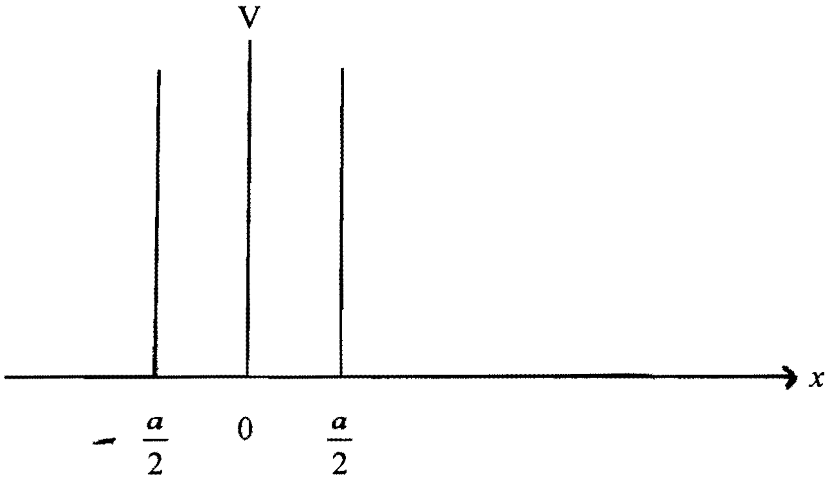
$$\frac{d\sigma^{ee}}{d\cos\theta} = \frac{d\sigma^{e\tilde{e}}}{d\cos\theta} = \frac{d\sigma^{\tilde{e}\tilde{e}}}{d\cos\theta}$$

   where $\frac{d\sigma^{AB}}{d\cos\theta}$ is the differential scattering cross-section for A off B.

Is the student correct this time? Explain why.

2. Consider a particle of mass m in an infinite one-dimensional square-well potential

$$V(x) = 0 \qquad |x| < a/2$$
$$V(x) = \infty \qquad |x| \geq a/2$$



Compute $p_{rms} \equiv \sqrt{\langle p^2 \rangle}$ for the ground state.

3. A rigid rotator with moment of inertia I rotates in a plane.

(i) Write down the Hamiltonian.

(ii) Show that the eigenvalues and eigenfunctions are:
$E_m = \hbar^2 m^2 / 2I, \quad \psi_m(\phi) = \frac{1}{\sqrt{2\pi}} e^{im\phi}; \ (m = 0, \pm 1, \ldots.)$

(iii) If the rotator has an electric dipole moment $\underline{d}$ and is placed in an electric field $\underline{E}$, the interaction energy is $H_I = \underline{d} \bullet \underline{E} = |d||E| \cos\theta$. Calculate the energy shifts up to second order in perturbation theory.

4. The wave function of a particle in a spherically symmetric potential V(r) is

$$\psi(\underline{r}) = (2x + 2y + z)\ f(\underline{r})$$

(i) Is ψ an eigenfunction of $\underline{L}^2$? If yes, what is the eigenvalue?

(ii) Is ψ an eigenfunction of L_z? If not, what are the possible values of m_ℓ that may be observed?

(iii) Show that the relative probability of finding the particle in $m_\ell = 0$ to $m_\ell = \pm 1$ is 1/4.

5. An electron is in the spin state $\frac{1}{\sqrt{3}}\begin{pmatrix}\sqrt{2}\\ 1\end{pmatrix}$ in the S_z basis.

a). What is the probability that a measurement of S_z will give the value $+1/2$?

b). What is the spin state after the measurement in a). ?

c). What is the probability that a measurement of S_x on the original state will give the value $-1/2$?

d). What is the spin state after the measurement in c). ?

6. Given the eigenfunctions ψ_n of a Hamiltonian H, a general state can be expanded in terms of ψ_n. A bound on E_0, the ground state energy of the system, is $\langle H \rangle$.

Consider a particle in the 1-d delta function potential $V(x) = -A\delta(x)$. Use the trial function $\psi = \left(\frac{2b}{\pi}\right)^{1/4} e^{-bx^2}$ to obtain a variational estimate of the ground state energy.

7. A particle of mass M bounces elastically between two infinite plane walls separated by a distance D. The particle is in its lowest possible energy state.

   (a) What is the energy of this state?

   (b) The separation between the walls is slowly (i.e., adiabatically) increased to $2D$. How does the expectation value of the energy change?

   (Note: "Adiabatic" means that a particle initially in the n^{th} state of the original Hamiltonian will remain in the n^{th} state of the new Hamiltonian.)

8. A spin 1/2 particle is governed by the Hamiltonian $H = 2\omega S_x$, where ω is a constant and S_x is the x-component of the spin operator. The particle is initially in the spin state $S_z = +\frac{\hbar}{2}$ at time $t = 0$.

(i) What is the state at a later time t?

(ii) What is the probability to find $S_z = -\hbar/2$?

(iii) What is the probability to find $S_x = +\hbar/2$?

QUALIFYING EXAM

Part IIB

March 18, 2002

1:30-4:30 PM

NAME____

1. ____

2. ____

3. ____

4. ____

5. ____

6. ____

7. ____

8. ____

TOTAL ____________________________________

INSTRUCTIONS: CLOSED BOOK. Integral tables are permitted. WORK ALL PROBLEMS. Use back of pages if necessary. Extra pages are available. If you use them, be sure to make reference on the page containing the problem.

PUT YOUR NAME ON ALL THE PAGES!

Physics Qualifying Exam 3/02 Part IIB Name________________________

1. An antiproton of mass m and velocity v approaches a proton target (also mass m). The radius of proton and anti-proton is b. Assume that the proton and anti-proton touch when they annihilate. Assume they interact only via a Coulomb potential.

   Obtain the cross-section for annihilation in terms of the kinetic energy in the lab system ($E_0 = mv^2$), the proton radius b, and the fine structure constant α.

Physics Qualifying Exam 3/02 Part IIB Name________________________

2. The Σ^+ hyperon can be produced in the reaction

$$\pi^+ + p \rightarrow \Sigma^+ + K^+.$$

What is the minimum π^+ momentum for this reaction to occur if the proton is at rest?

$M_{\pi}^{+} = 139.57$ MeV

$M_{\Sigma}^{+} = 1189.37$ MeV

$M_p = 938.27$ MeV

$M_k^{+} = 493.68$ MeV

3. A car of rest length 20 ft moves at high speed through a garage of rest length 16 ft; the garage doors are open at both ends. According to an observer S at rest with respect to the garage, there is one instant, say t=0, in which the car is entirely inside the garage, the ends of the car coinciding simultaneously with the ends of the garage.

   a) What is the speed of the car?

   b) What is the length of the garage as viewed by the car's driver?

   c) For the driver, will there ever be a time at which the car is entirely inside the garage?

   d) At what times, according to the driver, do the ends of the car coincide with the ends of the garage?

4. A nonrelativistic gas of N (number) ^{3}He atoms is contained in a volume V. The interaction energy between the atoms is weak and may be neglected. Thus, the gas may be considered as "ideal" for all temperatures. The high-temperature limit of the gas has the equation of state:

$$pV = NkT$$

where p is the pressure, T is the absolute temperature, and k is Boltzmann's constant. The low-temperature limit for the equation of state of the ideal gas is different. Calculate the zero temperature (T=0) p vs. V isotherm for this system.

5. Suppose that an object perceived as having 3 translational degrees of freedom has, in fact, 120 additional coordinates.

   (a.) Assume that the motion of such an object along these additional coordinates could be approximated as classical, unconstrained inertial motion analogous to the motion of an ordinary object drifting through 3-space, and that the object has no vibrational or rotational modes of motion.

   What is the specific heat of such an object?

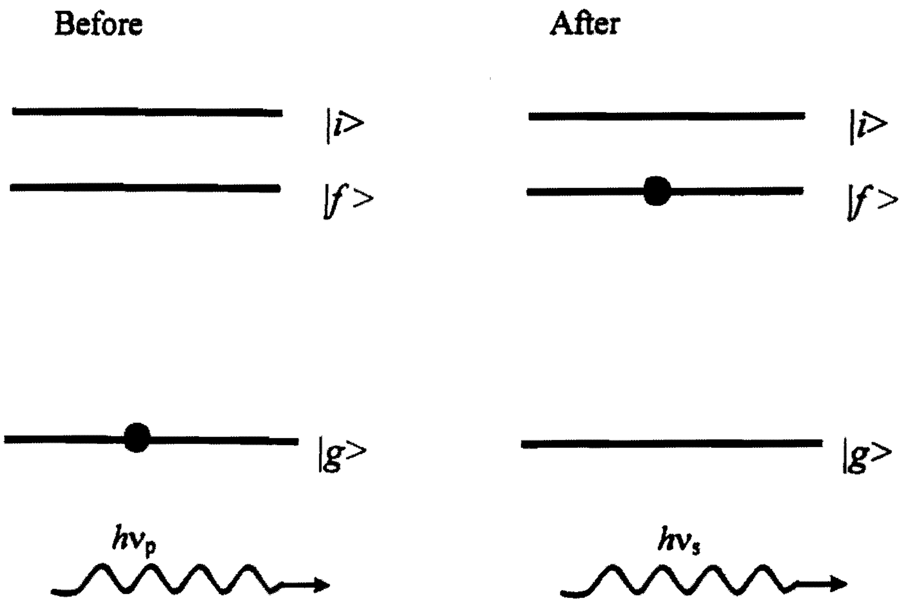
   (b.) How does this specific heat compare to the specific heat of objects that exist only in 3-space?

   (c.) Consider the alternate and more likely possibility that motion along these additional coordinates would be quantized, with level spacing $>> k_B T$. Assuming Boltzmann's statistics and a level spacing $\approx 10^{11}$eV, estimate the effect of these quantized extra dimensions on the specific heat at room temperature.

Physics Qualifying Exam 3/02 Part IIB Name______________________

6. Consider a 1000 Å photon emitted from an excited hydrogen atom initially at rest. Estimate the maximum fractional change in wavelength of this photon when the recoil energy of the hydrogen nucleus is taken into account.

7. In stimulated Raman scattering a photon of energy $h\nu_p$ (pump photon) excites an atom in a vapor cell from the ground state $|g\rangle$ to a short lived intermediate virtual state $|i\rangle$. Thereafter the atom emits a photon of energy $h\nu_s$ (signal photon), leaving the atom in an excited state $|f\rangle$ as shown in the figure.



(a) Assume that the intensity of the signal I_{s0} at the input of the atomic vapor cell is much smaller than the intensity I_{p0} of the pump beam. The intensity of the signal $I_s(z)$ changes as a function of position z according to

$$\frac{dI_s(z)}{dz} = gI_p(z)I_s(z)$$

where $I_p(z)$ is the intensity of the pump beam as a function of position, and g is the Raman amplifier small signal gain. In the small signal regime the pump wave is not intense enough to change significantly the population of the ground state.

Find an approximate expression for the intensity of the signal as function of position.

(b) Use quantum mechanical arguments to explain why the amplifier is tunable in frequency.

HINT: By conservation of photon number:

$$\frac{I_{s0}}{\nu_s} + \frac{I_{p0}}{\nu_p} = \frac{I_s}{\nu_s} + \frac{I_p}{\nu_p}$$

Physics Qualifying Exam 3/02 Part IIB Name____________________

8. TRUE (T) OR FALSE (F) (Circle One)

1. T F The first Brillouin zone of a square lattice is a circle.
2. T F In a germanium crystal each atom has 6 nearest neighbors.
3. T F The group velocity for electrons is zero at the Brillouin zone boundaries.
4. T F The total wavevector of the electrons in a filled band is zero.
5. T F For small k the E(k) diagram follows the free electron pattern.
6. T F Bloch waves whose wave vectors differ by a reciprocal wave vector are identical.
7. T F Semiconductors are transparent to infrared radiation.
8. T F Fermi surfaces for electrons in solids are always spherical.
9. T F A Brillouin zone is a Wigner-Seitz cell in the reciprocal lattice.
10. T F "For a plane that intercepts the axes at 4,1,2, the Miller indices are (1,4,2)."
11. T F The fcc lattice is characterized by ABABAB... stacking of planes.
12. T F Light travels through a solid as a longitudinal wave.
13. T F Thermal expansion is described by anharmonic crystal interactions.
14. T F The free electron Fermi gas is subject to the Pauli principle.
15. T F The cyclotron frequency is due to oscillations of electron spins in a magnetic field.
16. T F The phonon heat capacity is temperature independent at low temperatures.
17. T F In the optical phonon mode positive and negative ions are moving in phase.
18. T F There is an energy gap between optical and acoustical phonon branches.
19. T F Exciton energy levels in semiconductors are equally spaced.
20. T F Alkali metals are transparent for ultraviolet light.
21. T F A polariton is the quantum of the coupled phonon-photon transverse wavefield.
22. T F Electrons in Fermi liquids have higher effective masses than free electrons.
23. T F In superconductors the entropy increases on cooling below the critical temperature.
24. T F At T=0K all the Cooper pairs of a superconductor are in the same quantum state.
25. T F An applied magnetic field does not alter the free energy of a superconductor.