QUALIFYING EXAM

Part Ia

March 16, 2001

8:30 - 11:30 AM

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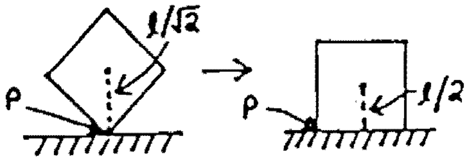
INSTRUCTIONS: CLOSED BOOK. Integral tables are permitted. WORK ALL PROBLEMS. Use back of pages if necessary. Extra pages are available. If you use them, be sure to make reference on the page containing the problem.

PUT YOUR NAME ON ALL THE PAGES!

1. A homogenous cube, each edge of which has length ℓ, is initially in a position of unstable equilibrium with one edge in contact with a horizontal plane. The cube is then released and allowed to fall. The edge cannot slide on the plane.

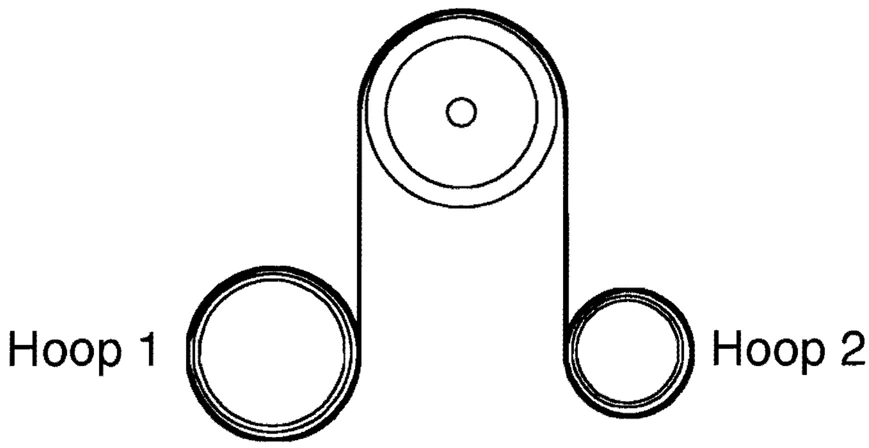


Find the angular velocity when one face hits the plane.
(Hint: $I_{cube} = 1/6\ M\ \ell^2$ for an axis perpendicular to one face and passing through the center.)

2. A mass m is hung by a string wound around a frictionless pulley of mass M and radius r. Assume the pulley is a disk and that the string does not slip.

   a) Find the acceleration of the mass.
   b) How long does it take the mass to fall through 1 meter starting from rest?

3. A massless string is placed over a fixed massless pulley, and each end is wound around separate hoops of masses $M_{1,2}$ and radii $R_{1,2}$ oriented vertically as shown. The apparatus is placed in a uniform gravitational field g, and released with each end of the string aligned along the field. The motion is confined to the vertical plane.



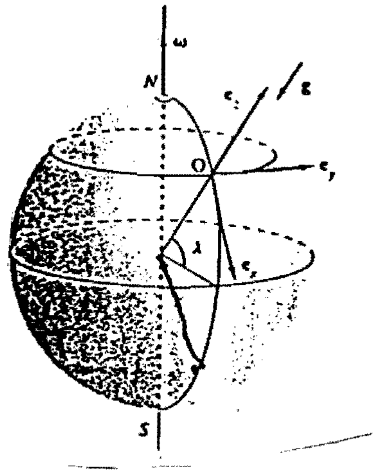
a) Find the tension T in the string in terms of g, M_1, M_2.

b) Show that the acceleration a_1 of the center of mass of hoop 1 is

$$a_1 = \frac{M_1}{M_1 - M_2} a_{1,point}$$

where $a_{1,point}$ is the acceleration when both hoops are replaced with point masses.

4. Consider a particle dropped from a flagpole at a height h above the surface of the earth. The particle is at a latitude λ in the northern hemisphere



a) What is the magnitude of the deflection of the particle when it hits the ground?

b) What is the direction of the deflection?

c) How do these result change in the southern hemisphere?

5. A beam of 10.0 MeV protons is incident on a copper foil (Z = 29, M = 65 amu). Assume that the protons are elastically scattered at an angle of 90°. Determine the distance of closest approach of a proton to a copper nucleus for this scattering.

6. Gas leaks from a balloon through a small hole. How will the rate at which it leaks out change if the temperature of the gas is quadrupled and its pressure increased by a factor of 8? You may assume the gas is close to being an ideal gas.

7. Suppose we treat the universe as an expanding gas of photons. The present temperature of the universe is around 3 K. Further assume that the expansion occurs at constant entropy. What will be the temperature of the universe when its volume is twice the present value?

Given:

$U = bVT^4$

$P = \frac{U}{3V}$

U = energy of the photon gas

V = volume

T = temperature

P = pressure

b = constant

8. Consider a mole of gas which is described by the van der Waals' model:

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

$$U = \frac{3}{2}RT - \frac{a}{V}$$

where:

P = pressure

V = volume

T = temperature

U = internal energy

R = universal gas constant = 8.314J / mole – Kelvin

The parameters, a and b, characterize the gas.

a) Suppose this one mole of gas undergoes an isothermal expansion from V_1 to V_2 at constant temperature T. Calculate the change in its entropy? Is it positive or negative?

b) Suppose this one mole of gas undergoes a quasi-static adiabatic expansion from $(V_{1,} T_1)$ to $(V_{2,} T_2)$. Express V_2 in terms of $(V_{1,} T_{1,} T_2)$ and other parameters if necessary.

c) Deduce the equations for pressure and internal energy for n moles of gas.
(**Hint**: P is intensive, U is extensive.)

QUALIFYING EXAM

Part Ib

March 16, 2001

1:30 - 4:30 PM

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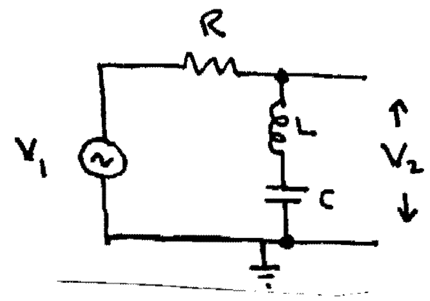
INSTRUCTIONS: CLOSED BOOK. Integral tables are permitted. WORK ALL PROBLEMS. Use back of pages if necessary. Extra pages are available. If you use them, be sure to make reference on the page containing the problem.

PUT YOUR NAME ON ALL THE PAGES!

1. An electron is launched with velocity v at an angle θ with respect to the x - axis. There is a uniform magnetic field B pointing in the +x direction.

   a) Qualitatively describe its motion.
   b) How far does it travel before returning to the x - axis for the first time?

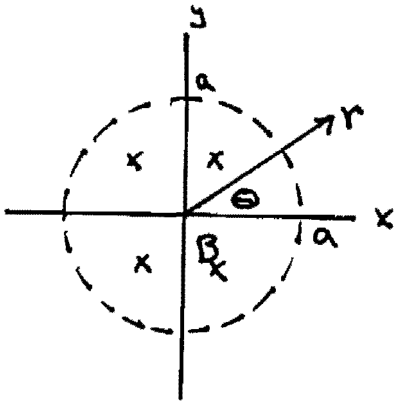
2. The RLC circuit shown is driven by AC voltage V_1 coswt



a) Calculate and sketch $|V_2/V_1|$ versus frequency ω

b) Calculate the phase shift of V_2 with respect to V_1 and sketch it versus frequency ω.

3. The time-dependent magnetic field between the circular pole faces of an electromagnet is; $B(r,t) = \alpha r t^2$. If the radius of the pole faces is a and if we can neglect fringing fields,



a) Find a vector potential **A** for $r < a$. Sketch A vs r.

b) Find a vector potential **A** for $r > a$. Sketch A vs r.

c) Calculate the electric field **E** for $r < a$.

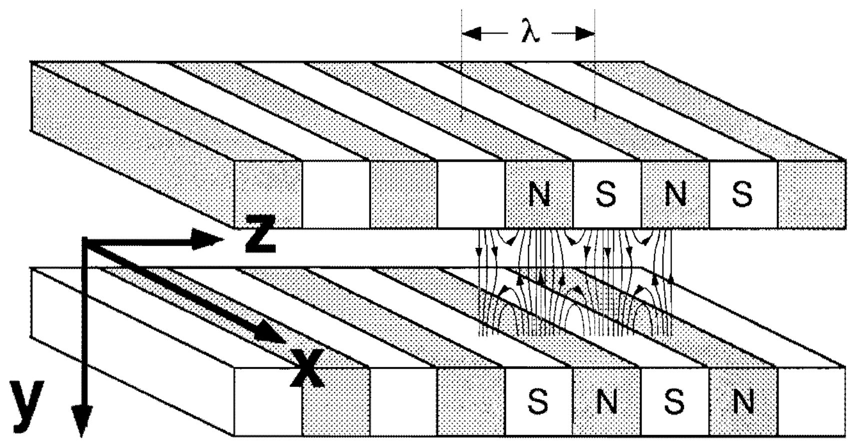
d) Calculate the electric field **E** for $r > a$.

4. A point charge +q is situated halfway between two parallel, grounded metal plates; the dimensions of the plates are much greater than their separation 2d. Use the method of images to find the force of attraction on each plate. (Clearly explain your answer using appropriate diagrams.)

5. The wiggler magnet for a free-electron laser is constructed of alternating {... N, S, N ...} poles and {... S, N, S ...} poles as shown in the diagram. The B-field on axis (0, 0, z) points along the y-direction and has the spatial dependence

$$B_y(0, 0, z) = B \sin kz \ ,$$

where B is the peak field and $k = 2\pi/\lambda$ is the wiggler constant. We assume that the wiggler magnet is sufficiently wide that $B_x \equiv 0$ at all locations, and that the remaining $\vec{B}$-field components have no x-dependence.



But this cannot be the *full expression* for the magnetic field in the vacuum gap at other values of y, because it does not satisfy Maxwell's equations. (There is no current $\vec{J}$ and no time-varying $\vec{E}$-field on axis.)

Assume that the complete $\vec{B}$-field in the gap (both on- and off-axis) is given by

$$B_x(x, y, z) = 0 \ ,$$
$$B_y(x, y, z) = B\, f(y) \sin kz \ ,$$
and
$$B_z(x, y, z) = B\, g(y) \cos kz \ .$$

Problem: Determine the functions f(y) and g(y).

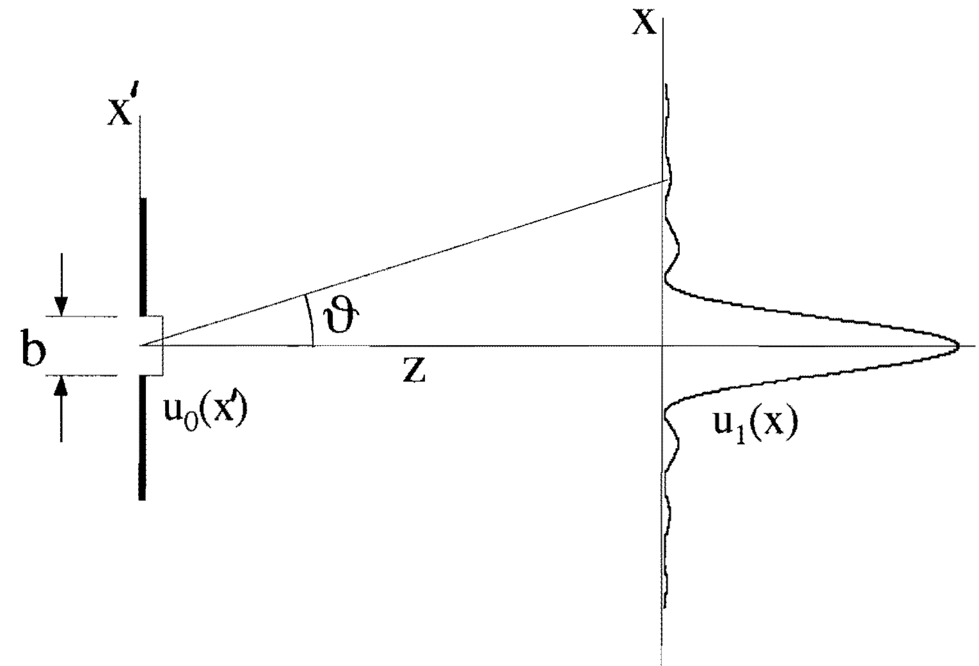
6. A randomly oriented sample of cesium triborate (CBO) is analysed at a wavelength of 0.532 μm using a frequency-doubled Nd:YAG laser, and is found to have the following dielectric tensor:

$$\varepsilon = \begin{bmatrix} 2.455 & -0.100 & 0 \\ -0.100 & 2.420 & 0 \\ 0 & 0 & 2.450 \end{bmatrix}$$

What is the minimum thickness of a film made of CBO, of any orientation, which will act as a quarter-wave plate for a wavelength of 0.532 μm? (Recall that a quarter-wave plate yields a relative phase retardation of $\pi/2$ radians between two orthogonally polarized components of light.)

HINT: The principal dielectric constants can be recovered by a similarity transformation of the dielectric tensor. The refractive indices are the square roots of the principal dielectric constants for light polarized along the principal axes.

7 A. Prove that the secondary maxima of the Fraunhofer intensity diffraction pattern from a single slit occur at the points for which $\beta = \tan(\beta)$, where $\beta = \pi b \nu$ for a slit of width b, and $\nu = \vartheta/\lambda = x/\lambda z$, where ϑ is the diffraction angle.



HINT: Recall that the Fraunhofer amplitude diffraction pattern $u_1(x)$ is the Fourier transform of the input pattern $u_0(x')$, where the output is evaluated at the spatial frequencies $\nu = \nu(x)$ given above.

8. The electric field of a collimated, fundamental mode in a laser beam of wavelength λ has a Gaussian transverse profile given by

$$u_0(x,y) = E_0 \exp\left[-\frac{x^2+y^2}{\omega_0^2}\right]$$

This beam will diffract as it propagates. Show using the Fresnel approximation that the beam will preserve a transverse Gaussian amplitude profile of the form

$$u_1(x,y) = E_1 \exp\left[-\frac{x^2+y^2}{\omega^2(z)}\right]$$

where

$$\omega^2(z) = \omega_0^2\left[1+\frac{z^2}{z_R^2}\right]$$

and the Rayleigh range z_R is given by

$$z_R = \frac{\pi\,\omega_0^2}{\lambda}$$

NOTE:

The Fresnel diffraction formula for propagation over distance z is given by

$$u_1(x,y) = \sqrt{\frac{i}{\lambda z}}\exp\left[i\frac{\pi}{\lambda z}(x^2+y^2)\right]\iint_{-\infty}^{\infty} u_0(x',y')\exp\left[i\frac{\pi}{\lambda z}(x'^2+y'^2)\right]\exp\left[-i\frac{2\pi}{\lambda z}(x'x+y'y)\right]dx'\,dy'$$

and the following integral is always useful for such calculations:

$$\int_{-\infty}^{\infty} e^{-ax^2-2bx}\,dx = \sqrt{\frac{\pi}{a}}\,e^{b^2/a}$$

QUALIFYING EXAM

Part IIa

March 19, 2001

8:30 - 11:30 AM

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1. Two spin 1/2 particles are described by a Hamiltonian $H_0 = A\ \mathbf{S}_1 \cdot \mathbf{S}_2$, where A is a positive constant.

   a) What are its eigenvalues and the eigenstates?

   b) A perturbation $H' = BS_{1z}$ where B is a positive constant is now applied. Find the corrections to the eigenvalues in the lowest non-vanishing order in perturbation theory. Draw an energy level diagram to illustrate.

2. Consider the one-dimensional potential

$$V(x) = \begin{cases} \infty & x < 0 \\ \alpha\,\delta(x-L) & x \geq 0 \end{cases}$$

where $\alpha < 0$.

a) State the boundary conditions on the wavefunction at $x = 0$ and at $x = L$.

b) Find the equation that determines the energies of the bound states in the potential.

c) Determine the number of bound states. Does your answer depend on α?

3. At time t=0, an atomic state with zero total angular momentum decays by emitting two back-to-back photons (traveling in the +z and -z directions).

a) Write down a state vector for these two photons.

b) At some later time t_o, an observer in the +z direction measures the polarization with a circular-light polarimeter and determines that it is right-circularly polarized. What is the state vector of the surviving photon for $t>t_o$?

c) At a time $t>t_o$, a -z observer measures the light polarization with a linear polarizer. What are the relative probabilities for measuring the polarization to be in the x and y directions.

d) Repeat b) and c) for the case where the +z observer measures the polarization with a linear polarizer and finds it to be in the x direction.

4. A beam of particles, each in the state

$$|\chi\rangle = \frac{1}{\sqrt{3}}\,|S_z = 1/2\rangle - \sqrt{\frac{2}{3}}\,|S_z = -1/2\rangle$$

is passed through a Stern-Gerlach apparatus "oriented along the z-direction", i.e. beams emerging from this apparatus have a definite value of S_z.

a) How many beams will emerge from this apparatus, and what is the intensity of each of these beams relative to the initial beam intensity I_0?

b) If <u>instead</u> we pass the same beam through a Stern-Gerlach apparatus "oriented in the x-direction", how many beams will emerge from it? What will be the intensity of each beam?

c) Consider a <u>particular</u> particle that emerges from the "x-apparatus" of part (b) above. What is the probability that a measurement of S_z will give $\hbar/2$? If instead of S, S_y is measured. What is the probability that a measurement of S_y will give zero? Explain your reasoning clearly.

5. The maximum value of the electron energy in the beta decay of Tritium (^3H) is sensitive to neutrino mass and is used to place bounds on the mass of the electron-neutrino.

$$^3H \rightarrow {}^3He + e^- + \bar{\nu}_e$$

This maximum value is also affected by the state of the final ^3He (ionized) i.e. whether it is left in the ground state or one of the excited states.

Using the impulse ("sudden") approximation find the probability that the final ^3He stays in the ground state.

Given: $\Psi_{100}(\mathbf{r}) = \sqrt{\dfrac{Z^3}{\pi a_0^3}}\, e^{-Z r/a_0}$ $\int_0^\infty x^2 e^{-bx}\, dx = \frac{2}{b^3}$

6. A negative pion (spin zero) is captured by a deuteron. Mesonic x-rays show that the $\pi^- d$ system reaches its lowest state before annihilating via $\pi^- + d \rightarrow n\,n$ into two neutrons. Assume that the deuteron is a $^3S_1$ bound state of a neutron and a proton.

a) By what type of interaction [strong, electromagnetic, weak, gravity] does the pion deuteron annihilation dominantly occur?

b) What is the parity of the deuteron?

c) What is the total angular momentum of the two neutron final state?

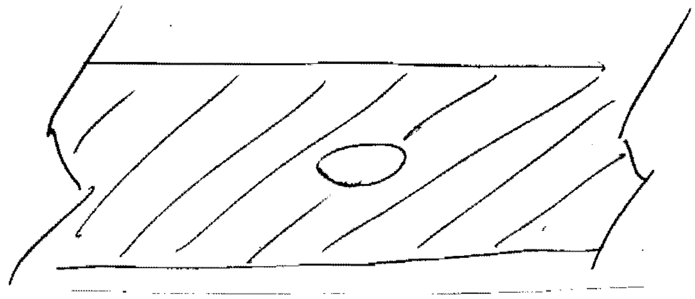
c) Explain clearly why the two neutrons cannot be in the relative L=0 state.

e) Since these neutrons are in the relative L=1 state, what is the parity of the final state two neutron system?

f) Use the results of a)-e) above to deduce the intrinsic parity of the pion.

7. The canonical momentum $\mathbf{p} - \frac{e\mathbf{A}}{c}$ of the charge carriers in superconductors is quantized according to the condition $\oint \left(\mathbf{p} - \frac{e^*\mathbf{A}}{c}\right) \cdot d\boldsymbol{\ell} = n\hbar$

where e* is the effective charge of the carriers. Consider a hole through an infinite slab of such a superconductor:



a) Assuming that the electric current in such a superconductor is confined to the cylindrical surface of the hole, show that the magnetic flux through such a hole is quantized.

b) Show how the measured flux quantum can be used to determine the effective charge of the superconducting charge carriers.

c) The measured flux quantum is approximately 3.3×10^{-8} gauss - cm^2. What is the effective charge of the carriers responsible for this effect? How can this effective charge be reconciled with the known value of the magnitude of the electron charge $e = 4.8 \times 10^{-10}$ stat-coulombs?

8. Consider an isolated pair of parallel square perfectly conducting plates (of side L), separated by a very small distance z. Quantum fluctuation of the electromagnetic field between the plates ranges in wavelength from the longest wave, which fits the boundary condition at the plates, up to some cut-off, which we shall call K.

a) Considering the boundary condition at the plates, what is the largest electromagnetic wave-number of a standing wave between the plates?

b) What is the energy in a single mode of the electromagnetic field of frequency ω?

c) Sum the energy over all modes in the volume between the plates. Approximate this energy as an integral. What is the z-dependence of the result?

d) From c) find the z dependence of the force between the plates.

QUALIFYING EXAM

Part IIb

March 19, 2001

1:30 - 4:30 PM

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1. For the reaction $e^+ + e^- \rightarrow \pi^+ + \pi^-$ with the electron target at rest, what is the minimum energy of the positron for the reaction to take place?
($m(\pi^\pm) = 273.1 m_e$)

2. An atomic clock is carried once around the world by a jet plane (velocity = 1000 m/s) and then compared with a previously synchronized clock that did not travel. The circumference of the earth is 2π x 6400 km.

What is the difference between the two clocks' readings predicted by special relativity? (Neglect the acceleration at the beginning and end of the experiment.)

3. a) Which two hydrogen energy levels are separated by the Lamb Shift?

   b) Explain, qualitatively, what causes the shift and which state is the higher of the two.

   c) What is the order of magnitude of the Lamb Shift?

4. The Mossbauer effect is observed in the photon decay of an excited state of ^{57}Fe solid which is 14.4 keV above the ground state. (The excited state results from the decay of ^{57}Co.)

   a) Calculate the energy of the photon emitted in the decay of an excited isolated atom of $^{57}Fe^{*}$. Give the difference between the energy of the photon and 14.4 eV.

   b) Calculate the photon energy that would be required to cause an upward transition from the ground state of an isolated ^{57}Fe atom to the 14.4 keV excited state. Again, give the difference between the required energy and 14.4 keV.

   c) Calculate the linear velocity of the decaying $^{57}Fe^{*}$ atom that would produce a photon capable of exciting an isolated ^{57}Fe atom that is initially at rest.

   d) Explain how the Mossbauer effect changes that answers to a), b), and c).

   The atomic mass of ^{57}Fe is 56.94 u, u = 931.49 MeV.

5. List the ground-state electronic configurations and the L, S, and J quantum numbers for the following atoms:

   Li(Z = 3), B(Z = 5), N(Z = 7), Na(Z = 11), K(Z = 19).

   Hint: Hund's Rules
   1. maximize S,
   2. maximize L (after maximizing S), and
   3. for a shell labeled by n,
      - minimize J if the shell is less than half full, or
      - maximize J if the shell is more than half full.

6. a) Give an explanation of neutrino oscillations between two species of neutrinos. What is meant by maximal mixing? How does the mixing depend upon distance and neutrino energy?

   b) Do you know of any other systems in nature which exhibit behavior analogous to neutrino oscillations? Give an example and clarify the analogy.

7. The dispersion relation for photons is:

$$\omega = c\,|\mathbf{k}|\,, \quad 0<|\mathbf{k}|<\infty\,,$$

where c is the speed of light in vacuum. For isotropic acoustic phonons in the Debye approximation, the dispersion relation is:

$$\omega = v_s|\mathbf{k}|\,,\ 0<|\mathbf{k}|<k_D\,,$$

where v_s is the speed of sound and k_D is the Debye cutoff.

a) Find general expressions for the thermally averaged number of (i) photons (ii) phonons at absolute temperature T in d-dimensions (d=1, 2, 3). Do not evaluate the integrals.

b) Find and sketch the T-dependence of the expressions (i) and (ii) found in (a) with particular attention to the low-temperature, $T \to 0$, and high-temperature, $T \to \infty$, limits. [Assume the Debye model is valid for all T.]

8. TRUE (T) OR FALSE (F) (Circle One)

1. T F There is only one way to define a primitive cell for a crystal.
2. T F The packing fraction for bcc is smaller than for fcc.
3. T F The BCS theory of superconductivity was put forth by Bardeen, Cooper, and Schrödinger.
4. T F The Bravais space lattice of diamond is fcc.
5. T F There is always one lattice point per primitive cell.
6. T F Icosahedra cannot fill a lattice in three-dimensional space.
7. T F The phonon heat capacity is temperature independent at low temperatures.
8. T F The Einstein model treats the vibrating atoms as coupled quantum oscillators.
9. T F In the k=0 optical phonon mode of an ionic crystal, positive and negative ions are moving together in phase.
10. T F There is an energy gap at k=0 between optical and acoustical phonon branches.
11. T F Each allowable k state is two-fold degenerate for free electrons in solids.
12. T F At T=0, the Fermi sphere contains the occupied electron states of a solid.
13. T F The Fermi temperature is much lower than the melting temperature of a metal.
14. T F Plasmons can be excited by passage of energetic electrons through a metal.
15. T F The Bohr magneton depends on temperature.
16. T F The spin entropy of a paramagnet is reduced by a magnetic field.
17. T F An electric field applied to a metal leads to a shift of the Fermi sphere in k-space.
18. T F The Hall effect is used to measure the electron charge density of a sample.
19. T F The energy levels of excitons are equally spaced.
20. T F The entropy increases when a superconductor is cooled.