# $B^0$-$\bar{B}^0$ ***mixing in unquenched lattice QCD***


Norikazu Yamada

RIKEN BNL Reserch Center

yamada@bnl.gov


for the *JLQCD* Collaboration


Phys.Rev.Lett.91(2003)212001


Super B factory workshop in Hawaii

January 19-22, 2004

# Introduction

In the Standard Model, the mass difference of $B$ and $\bar{B}$ mesons is given by

$$\Delta M_{B_q} = \Big(\text{known factor}\Big) \times |V_{tb}^* V_{tq}|^2 \frac{\langle \bar{B}_q^0 | \bar{b}\gamma_\mu(1-\gamma_5)q \, \bar{b}\gamma_\mu(1-\gamma_5)q \, | B_q^0 \rangle}{M_{B_q}}$$

$(q = d$ or $s)$

$$\delta(|V_{tb}^* V_{tq}|^2) \approx \text{(Error of } \Delta M_{B_q}\text{)} + \text{(Error in M.E.)}$$

- Experiments:
  - $\Delta M_{B_d}$ : well determined ($\sim$ 1%)
  - $\Delta M_{B_s}$ : will be measured soon ($\sim$ a few %)
- Theory:

  $$\langle \bar{B}_q^0 | \, \bar{b}\gamma_\mu(1-\gamma_5)q \, \bar{b}\gamma_\mu(1-\gamma_5)q \, | B_q^0 \rangle = \frac{8}{3} B_{B_q}(\mu_b) f_{B_q}^2 M_{B_q}^2$$

  $\delta(B_{B_q} f_{B_q}^2) \approx$ 20–40 % (lattice QCD)

# Introduction

In the Standard Model, the mass difference of $B$ and $\bar{B}$ mesons is given by

$$\Delta M_{B_q} = \Big(\text{known factor}\Big) \times |V_{tb}^* V_{tq}|^2 \frac{\langle \bar{B}_q^0 | \bar{b}\gamma_\mu(1-\gamma_5)q\, \bar{b}\gamma_\mu(1-\gamma_5)q\, |B_q^0\rangle}{M_{B_q}}$$

$(q = d$ or $s)$

$\delta(|V_{tb}^* V_{tq}|^2) \approx$ (Error of $\Delta M_{B_q}$) $+$ (Error in M.E.) $\approx$ (Error in M.E.)

- Experiments:

  $\Delta M_{B_d}$ : well determined ($\sim$ 1%)

  $\Delta M_{B_s}$ : will be measured soon ($\sim$ a few %)

- Theory:

  $$\langle \bar{B}_q^0 |\, \bar{b}\gamma_\mu(1-\gamma_5)q\, \bar{b}\gamma_\mu(1-\gamma_5)q\, |B_q^0\rangle = \frac{8}{3} B_{B_q}(\mu_b) f_{B_q}^2 M_{B_q}^2$$

  $\delta(B_{B_q} f_{B_q}^2) \approx$ 20–40 % (lattice QCD)

Theory error dominates.

# Sources of uncertainty in the lattice calculation

- Scaling violation $\sim O(a^n m_b^n)$
  currently $am_b \sim$ 2 to 3 $\rightarrow$ serious problem

# Sources of uncertainty in the lattice calculation

- Scaling violation $\sim O(a^n m_b^n)$
  currently $am_b \sim$ 2 to 3 $\rightarrow$ serious problem

  $\Rightarrow$ The use of effective theory (HQET, NRQCD) helps to reduce this to $O(a^2\Lambda_{\rm QCD}^2) \sim$ 5% and $O(\Lambda_{\rm QCD}^2/m_b^2) \lesssim$ 5%

# *Sources of uncertainty in the lattice calculation*

- Scaling violation $\sim O(a^n m_b^n)$
  currently $am_b \sim$ 2 to 3 $\rightarrow$ serious problem
  
  $\Rightarrow$ The use of effective theory (HQET, NRQCD) helps to reduce this to $O(a^2 \Lambda_{\mathrm{QCD}}^2) \sim$ 5% and $O(\Lambda_{\mathrm{QCD}}^2 / m_b^2) \lesssim$ 5%

- Operator matching
  depending on to which order the operator is matched

$$\mathcal{O}^{\mathrm{cont}} = Z_{\mathcal{O}} \mathcal{O}^{\mathrm{latt}}$$

$$Z_{\mathcal{O}} = 1 + z_1 \alpha + z_2 \alpha^2 + \cdots$$

# Sources of uncertainty in the lattice calculation

- Scaling violation $\sim O(a^n m_b^n)$

  currently $am_b \sim$ 2 to 3 $\longrightarrow$ serious problem

  $\Rightarrow$ The use of effective theory (HQET, NRQCD) helps to reduce this to $O(a^2 \Lambda_{\mathrm{QCD}}^2) \sim$ 5% and $O(\Lambda_{\mathrm{QCD}}^2 / m_b^2) \lesssim$ 5%

- Operator matching

  depending on to which order the operator is matched

  $\Rightarrow$ One-loop matching $\rightarrow O(\alpha^2) \sim$ 5%

# Sources of uncertainty in the lattice calculation

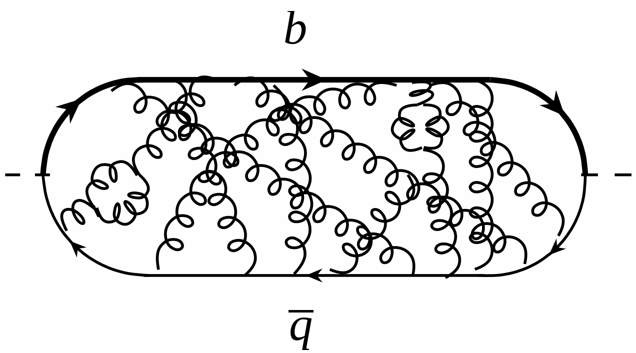
- Scaling violation $\sim O(a^n m_b^n)$
  currently $am_b \sim$ 2 to 3 $\rightarrow$ serious problem

  $\Rightarrow$ The use of effective theory (HQET, NRQCD) helps to reduce this to $O(a^2\Lambda_{\rm QCD}^2) \sim$ 5% and $O(\Lambda_{\rm QCD}^2/m_b^2) \lesssim$ 5%

- Operator matching
  depending on to which order the operator is matched

  $\Rightarrow$ One-loop matching $\rightarrow O(\alpha^2) \sim$ 5%

- Quenching error




Quench ($N_f$=0)

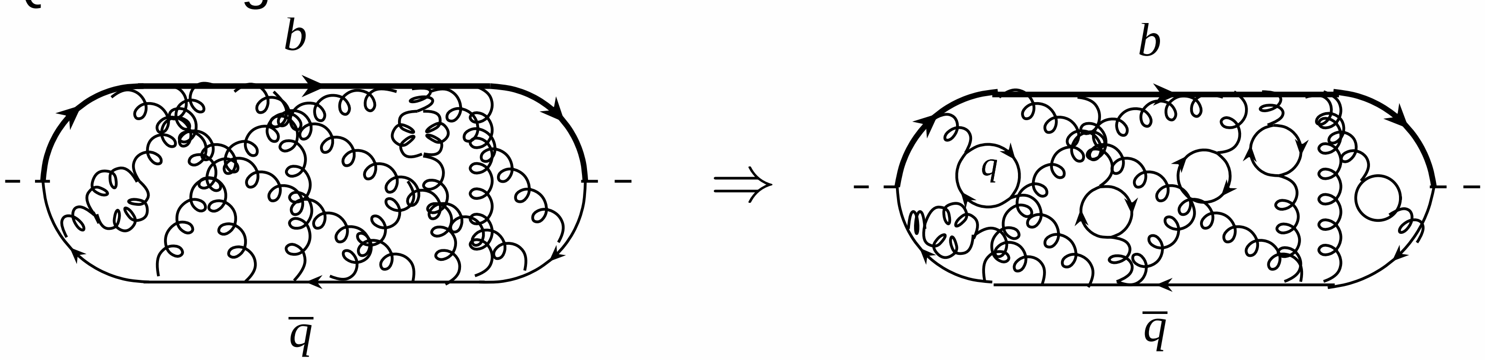
Because of computational cost, the quenched approximation has been mainly studied so far.

# Sources of uncertainty in the lattice calculation

- Scaling violation $\sim O(a^n m_b^n)$
  currently $am_b \sim$ 2 to 3 $\rightarrow$ serious problem
  
  $\Rightarrow$ The use of effective theory (HQET, NRQCD) helps to reduce this to $O(a^2\Lambda_{\rm QCD}^2) \sim$ 5% and $O(\Lambda_{\rm QCD}^2/m_b^2) \lesssim$ 5%

- Operator matching
  depending on to which order the operator is matched
  
  $\Rightarrow$ One-loop matching $\rightarrow O(\alpha^2) \sim$ 5%

- Quenching error




Quench ($N_f$=0) Unquench ($N_f \neq$ 0)

Now we can realize simulations with arbitrary number of $N_f$.

# Sources of uncertainty in the lattice calculation

- Scaling violation $\sim O(a^n m_b^n)$
  currently $am_b \sim$ 2 to 3 $\rightarrow$ serious problem
  $\Rightarrow$ The use of effective theory (HQET, NRQCD) helps to reduce this to $O(a^2\Lambda_{\rm QCD}^2) \sim$ 5% and $O(\Lambda_{\rm QCD}^2/m_b^2) \lesssim$ 5%
- Operator matching
  depending on to which order the operator is matched
  $\Rightarrow$ One-loop matching $\rightarrow O(\alpha^2) \sim$ 5%
- Quenching error
  Now we can realize simulations with arbitrary number of $N_f$.
  $\Rightarrow$ Incorporation of dynamical quarks

# Sources of uncertainty in the lattice calculation

- Scaling violation $\sim O(a^n m_b^n)$
  currently $am_b \sim$ 2 to 3 $\rightarrow$ serious problem

  $\Rightarrow$ The use of effective theory (HQET, NRQCD) helps to reduce this to $O(a^2\Lambda_{\rm QCD}^2) \sim$ 5% and $O(\Lambda_{\rm QCD}^2/m_b^2) \lesssim$ 5%

- Operator matching
  depending on to which order the operator is matched

  $\Rightarrow$ One-loop matching $\rightarrow O(\alpha^2) \sim$ 5%

- Quenching error
  Now we can realize simulations with arbitrary number of $N_f$.

  $\Rightarrow$ Incorporation of dynamical quarks

- Chiral extrapolation
  Most of previous works have missed this problem.

Lattice can not simulate very light quarks ($\sim m_{u,d}$) directly.
$\Rightarrow$ Extrapolations in $m_q$ to $m_q$=$m_{ud}$ is required.

# Sources of uncertainty in the lattice calculation

- Scaling violation $\sim O(a^n m_b^n)$
  currently $am_b \sim$ 2 to 3 $\to$ serious problem

  $\Rightarrow$ The use of effective theory (HQET, NRQCD) helps to reduce this to $O(a^2\Lambda_{\rm QCD}^2) \sim$ 5% and $O(\Lambda_{\rm QCD}^2/m_b^2) \lesssim$ 5%

- Operator matching
  depending on to which order the operator is matched

  $\Rightarrow$ One-loop matching $\to O(\alpha^2) \sim$ 5%

- Quenching error
  Now we can realize simulations with arbitrary number of $N_f$.

  $\Rightarrow$ Incorporation of dynamical quarks

- Chiral extrapolation
  Most of previous works have missed this problem.

Focus on the discussion about the chiral extrapolations.

# *Simulation parameters*

We performed the two-flavor ($u$ and $d$ in the loop) lattice simulation with the following parameters.

| | |
|---|---|
| gauge : | Plaquette |
| light quark : | $O(a)$-improved Wilson |
| heavy quark : | NRQCD with $1/m_Q$ corrections |
| lattice spacing : | $a^{-1}$ = 2.22(4) GeV from $m_\rho$ |
| sea quark mass : | $m_{PS}/m_V$=0.6–0.8 ($m_\pi$=550–1000 MeV) |
| size : | $20^3 \times 48$ |
| # of trajectories : | 12,000 for each sea quark |

# Lattice QCD vs. Chiral Perturbation Theory

ChPT is a low energy effective theory of QCD, and tells us a low energy behavior of physical quantities.

*e.g.)* $m_\pi^2$ dependence of $f_\pi$

$$\frac{f_\pi}{f} = 1 + c_1 m_\pi^2 + c_2 m_\pi^4 + \cdots$$
$$- N_f \frac{m_\pi^2}{(4\pi f)^2} \ln\left(\frac{m_\pi^2}{\mu^2}\right)$$

# Lattice QCD vs. Chiral Perturbation Theory

ChPT is a low energy effective theory of QCD, and tells us a low energy behavior of physical quantities.

*e.g.)* $m_\pi^2$ dependence of $f_\pi$

$$\frac{f_\pi}{f} = 1 + c_1 m_\pi^2 + c_2 m_\pi^4 + \cdots - N_f \frac{m_\pi^2}{(4\pi f)^2} \ln\left(\frac{m_\pi^2}{\mu^2}\right)$$

Coef. of log term is known!

# *Lattice QCD vs. Chiral Perturbation Theory*

ChPT is a low energy effective theory of QCD, and tells us a low energy behavior of physical quantities.

*e.g.)* $m_\pi^2$ dependence of $f_\pi$

$$\frac{f_\pi}{f} = 1 + c_1 m_\pi^2 + c_2 m_\pi^4 + \cdots$$
$$- N_f \frac{m_\pi^2}{(4\pi f)^2} \ln\left(\frac{m_\pi^2}{\mu^2}\right)$$

Coef. of log term is known!

Does the lattice data show the logarithmic dependence?

# Lattice QCD vs. Chiral Perturbation Theory

ChPT is a low energy effective theory of QCD, and tells us a low energy behavior of physical quantities.

*e.g.)* $m_\pi^2$ dependence of $f_\pi$

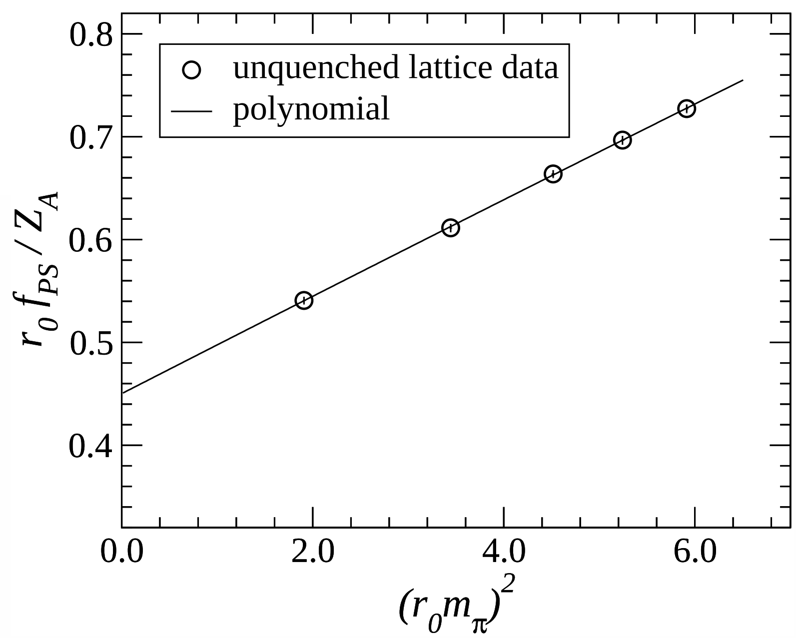
$$\frac{f_\pi}{f} = 1 + c_1 m_\pi^2 + c_2 m_\pi^4 + \cdots - N_f \frac{m_\pi^2}{(4\pi f)^2} \ln\left(\frac{m_\pi^2}{\mu^2}\right)$$

Coef. of log term is known!

Does the lattice data show the logarithmic dependence?

**Lattice data of $m_\pi^2$ vs. $f_\pi$ ($N_f$=2)**

JLQCD@Lattice 2002



# Lattice QCD vs. Chiral Perturbation Theory

ChPT is a low energy effective theory of QCD, and tells us a low energy behavior of physical quantities.

*e.g.)* $m_\pi^2$ dependence of $f_\pi$

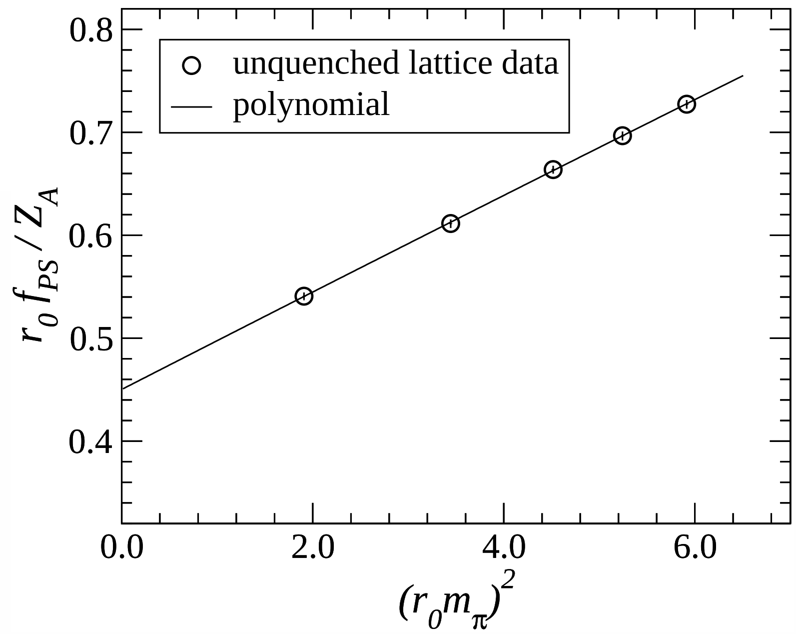
$$\frac{f_\pi}{f} = 1 + c_1 m_\pi^2 + c_2 m_\pi^4 + \cdots - N_f \frac{m_\pi^2}{(4\pi f)^2} \ln\left(\frac{m_\pi^2}{\mu^2}\right)$$

Coef. of log term is known!

Does the lattice data show the logarithmic dependence?

**Lattice data of $m_\pi^2$ vs. $f_\pi$ ($N_f$=2)**

JLQCD@Lattice 2002



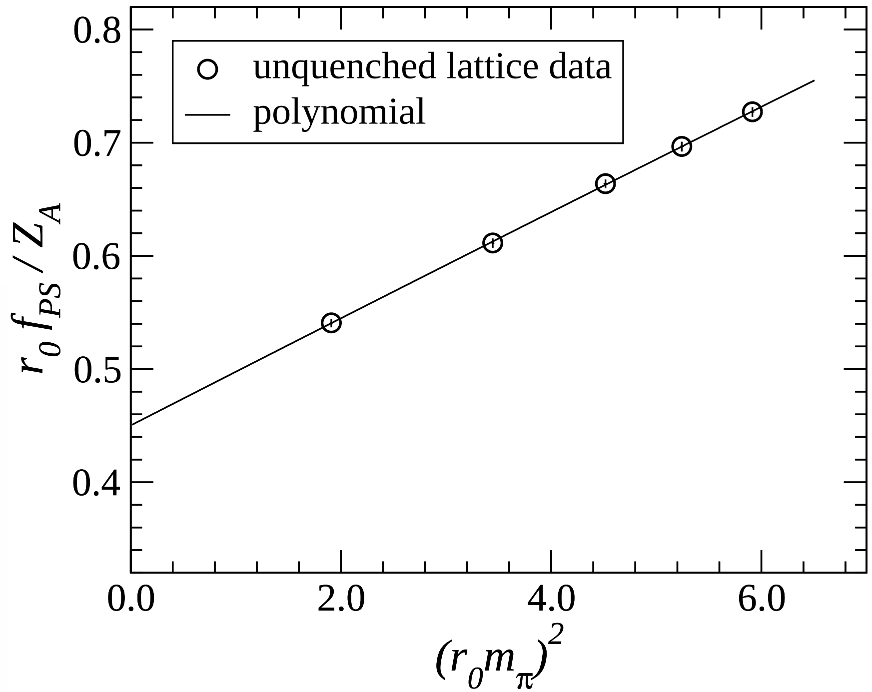

- No curvature ⇔ Lattice data looks inconsistent with ChPT.
- A possible explanation is that the sea quark mass is too heavy to be described by ChPT. ($m_\pi \sim$ 550–1000 MeV)

How to incorporate this inconsistency into the systematic error?

# Strategy

- In most of previous works, the polynomial (quadratic) fit was made in chiral extrapolations.
  ⇔ Inconsistent with ChPT

**Lattice data of $m_\pi^2$ vs. $f_\pi$ ($N_f$=2)**

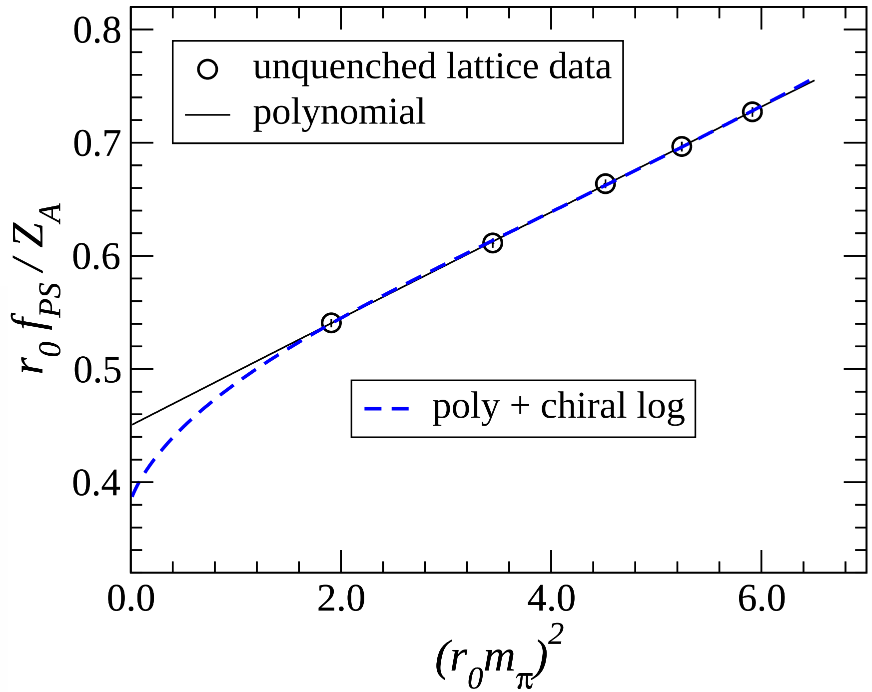



$$f_\pi = c_0 + c_1 m_\pi^2 + c_2 (m_\pi^2)^2$$

# Strategy

- In most of previous works, the polynomial (quadratic) fit was made in chiral extrapolations.
  $\Leftrightarrow$ Inconsistent with ChPT
- Add chiral log with the known coef. to the fit function

**Lattice data of $m_\pi^2$ vs. $f_\pi$ ($N_f$=2)**



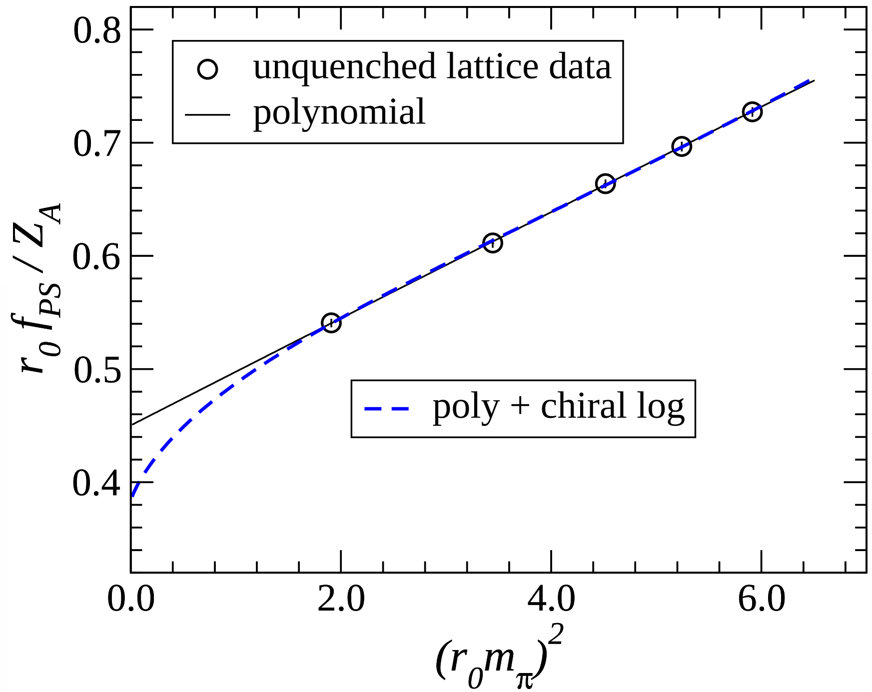

$$f_\pi = c_0 + c_1 m_\pi^2 + c_2 (m_\pi^2)^2 - N_f \frac{m_\pi^2}{(4\pi f)^2} \ln\left(\frac{m_\pi^2}{\mu^2}\right)$$

# *Strategy*

- In most of previous works, the polynomial (quadratic) fit was made in chiral extrapolations.
  $\Leftrightarrow$ Inconsistent with ChPT
- Add chiral log with the known coef. to the fit function

But when $m_\pi \gg \mu$ the effects of pion loops should decouple from the theory.
$\Rightarrow$ Contributions from log term should weaken as $m_\pi \to \infty$.

**Lattice data of $m_\pi^2$ vs. $f_\pi$ ($N_f$=2)**




$$f_\pi = c_0 + c_1 m_\pi^2 + c_2 (m_\pi^2)^2 - N_f \frac{m_\pi^2}{(4\pi f)^2} \ln\left(\frac{m_\pi^2}{\mu^2}\right)$$

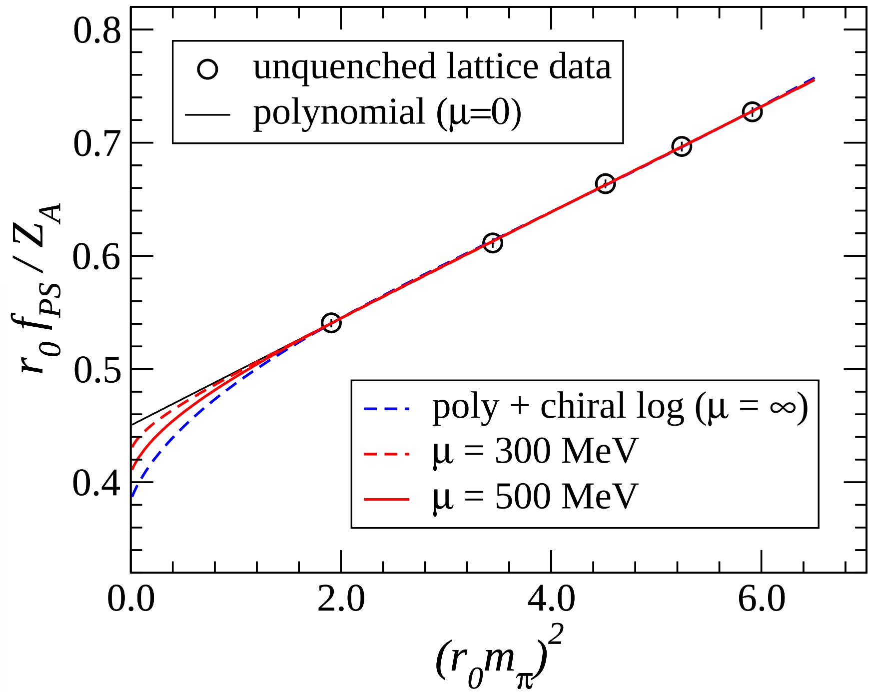
# Strategy

- In most of previous works, the polynomial (quadratic) fit was made in chiral extrapolations.
  $\Leftrightarrow$ Inconsistent with ChPT
- Add chiral log with the known coef. to the fit function

But when $m_\pi \gg \mu$ the effects of pion loops should decouple from the theory.
$\Rightarrow$ Contributions from log term should weaken as $m_\pi \to \infty$.

- Modify the argument of log and make $\mu$ variable
  Detmold et al., PRL87, 172001 (2001)

**Lattice data of $m_\pi^2$ vs. $f_\pi$ ($N_f$=2)**




$$f_\pi = c_0 + c_1 m_\pi^2 + c_2 (m_\pi^2)^2 - N_f \frac{m_\pi^2}{(4\pi f)^2} \ln \left( \frac{m_\pi^2}{m_\pi^2 + \mu^2} \right)$$

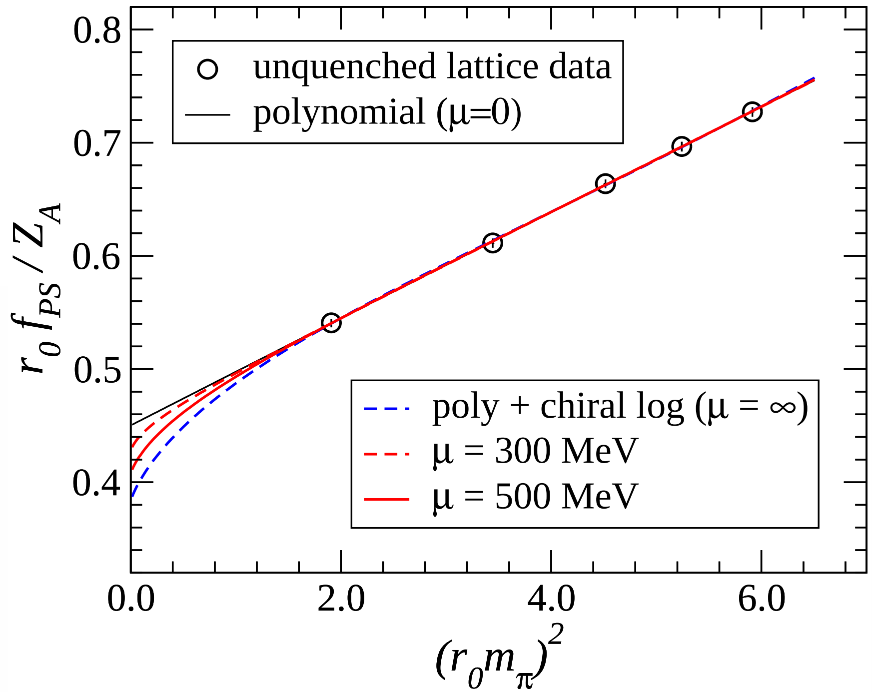
# Strategy

- In most of previous works, the polynomial (quadratic) fit was made in chiral extrapolations.
  $\Leftrightarrow$ Inconsistent with ChPT
- Add chiral log with the known coef. to the fit function

But when $m_\pi \gg \mu$ the effects of pion loops should decouple from the theory.
$\Rightarrow$ Contributions from log term should weaken as $m_\pi \to \infty$.

- Modify the argument of log and make $\mu$ variable
  Detmold et al., PRL87, 172001 (2001)

**Lattice data of $m_\pi^2$ vs. $f_\pi$ ($N_f$=2)**




- Take variance of $f_\pi$ with $\mu$ as the systematic uncertainty.

*e.g.)* $\mu \in [0,\infty] \Rightarrow f_\pi^{\rm phys}$ = 147–128 MeV

$$f_\pi = c_0 + c_1 m_\pi^2 + c_2 (m_\pi^2)^2 - N_f \frac{m_\pi^2}{(4\pi f)^2} \ln\left(\frac{m_\pi^2}{m_\pi^2 + \mu^2}\right)$$

# Summary of strategy

- We take "a modified (poly+chiral log) function",

$$f_\pi = c_0 + c_1 m_\pi^2 + c_2 (m_\pi^2)^2 - N_f \frac{m_\pi^2}{(4\pi f)^2} \ln\left(\frac{m_\pi^2}{m_\pi^2 + \mu^2}\right)$$

  when extrapolating data to the chiral limit.
- Change $\mu$ and take the variance as the sys error in the chiral extrapolation.
- We do not know the definitely appropriate range of $\mu$. So we choose $\mu \in$ [0,$\infty$] because this choice gives the largest (most conservative) error.
- This strategy introduces a model into lattice calculations. But this is the best way we can do right now.

This strategy is applied to the $f_B$ and $B_B$.

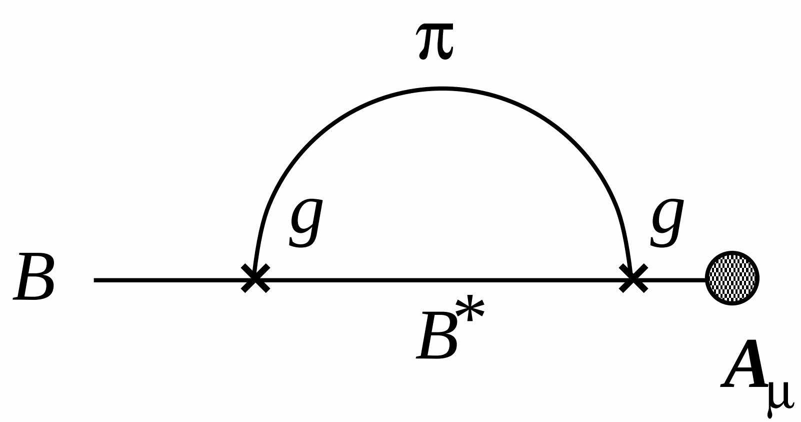
# *Chiral perturbation theory with $B$ mesons*

Wise(1992), Burdman and Donoghu(1992), Grinstein *et al.*(1992), Goity(1992), Booth(1995), Sharpe and Zhang(1996)

Chiral log in $f_{B_{(s)}}$ ($N_f$=2 case)

$$f_B : -\frac{3}{4}(1+3g^2)\frac{m_\pi^2}{(4\pi f)^2}\ln\left(\frac{m_\pi^2}{m_\pi^2+\mu^2}\right)$$

$$f_{B_s} : -(1+3g^2)\frac{m_K^2}{(4\pi f)^2}\ln\left(\frac{m_K^2}{m_K^2+\mu^2}\right)$$




Log term $\rightarrow$ smaller effect for $f_{B_s}$, because $m_\pi^2 < m_K^2$.

$g$ : $BB^*\pi$ coupling

$$g = 0.6 \text{ ($D^*$ width, CLEO('02))}$$
$$= 0.27 \text{ (One-loop ChPT+$D^* \rightarrow D\pi(\gamma)$, Stewart('98))}$$

# Chiral perturbation theory with $B$ mesons

Wise(1992), Burdman and Donoghu(1992), Grinstein *et al.*(1992), Goity(1992), Booth(1995), Sharpe and Zhang(1996)

**Chiral log in $f_{B_{(s)}}$ ($N_f$=2 case)**

$$f_B : -\frac{3}{4}(1+3g^2)\frac{m_\pi^2}{(4\pi f)^2}\ln\left(\frac{m_\pi^2}{m_\pi^2+\mu^2}\right)$$

$$f_{B_s} : -(1+3g^2)\frac{m_K^2}{(4\pi f)^2}\ln\left(\frac{m_K^2}{m_K^2+\mu^2}\right)$$

**Chiral log in $B_{B_{(s)}}$ ($N_f$=2 case)**

$$B_B : -\frac{1}{2}(1-3g^2)\frac{m_\pi^2}{(4\pi f)^2}\ln\left(\frac{m_\pi^2}{m_\pi^2+\mu^2}\right)$$

$B_{B_s}$ : No log term

Log term → smaller effect for $f_{B_s}$, because $m_\pi^2 < m_K^2$.

Log term → smaller effect for $B_B$, because $(1-3g^2)$ is small.

$$g = 0.6 \text{ ($D^*$ width, CLEO('02))}$$
$$= 0.27 \text{ (One-loop ChPT+$D^* \to D\pi(\gamma)$, Stewart('98))}$$

The relevant coefs. are all present.

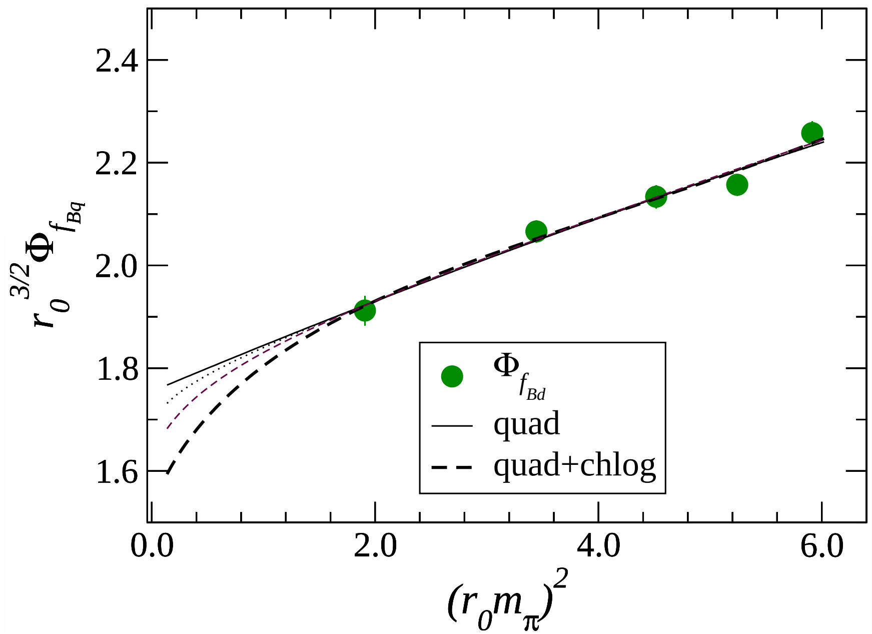
# Results of $f_B$

Fitting data of $f_{B_{(s)}}$ to the modified function, we obtain

$$f_B = 191(10)(^{+\ 0}_{-19})(12)(0) \text{ MeV}$$
$$\text{error} \quad (\text{stat.})(\text{chiral})(\text{sys.})(\text{m}_\text{s})$$

(chiral) is sizable in $f_B$.

$m_\pi^2$ **vs.** $f_B$



# Results of $f_B$

Fitting data of $f_{B_{(s)}}$ to the modified function, we obtain

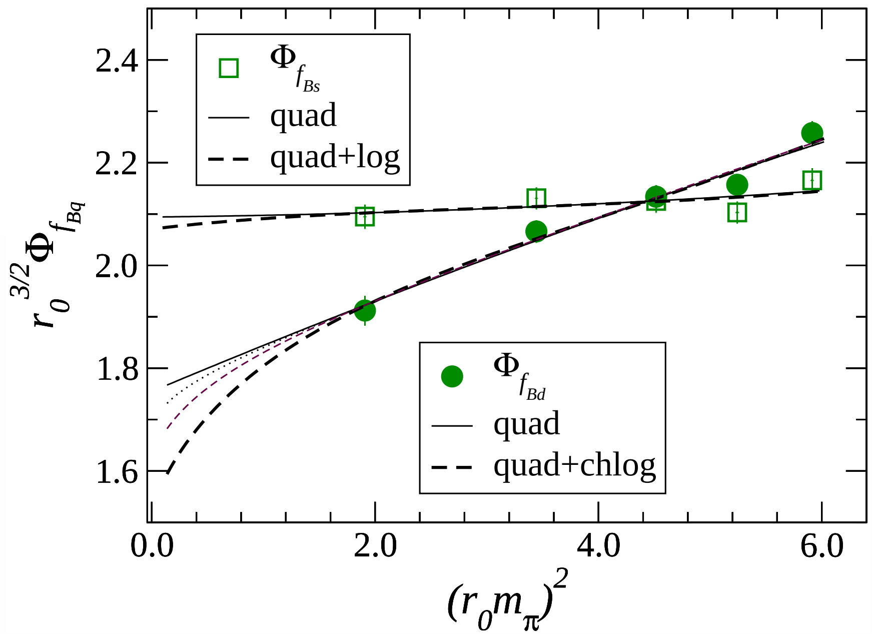
$$f_B \quad = \quad 191(10)(^{+\ 0}_{-19})(12)(0)\ \text{MeV}$$

$$\text{error} \qquad (\text{stat.})(\text{chiral})(\text{sys.})(\text{m}_\text{s})$$

(chiral) is sizable in $f_B$.

$$f_{B_s} \quad = \quad 215(9)(^{+0}_{-2})(13)(^{+6}_{-0})\ \text{MeV}$$

$m_\pi^2$ **vs.** $f_B$



# Results of $f_B$

Fitting data of $f_{B_{(s)}}$ to the modified function, we obtain

$$f_B = 191(10)(^{+\ 0}_{-19})(12)(0)\ \text{MeV}$$

$$\text{error} \quad (\text{stat.})(\text{chiral})(\text{sys.})(\text{m}_\text{s})$$

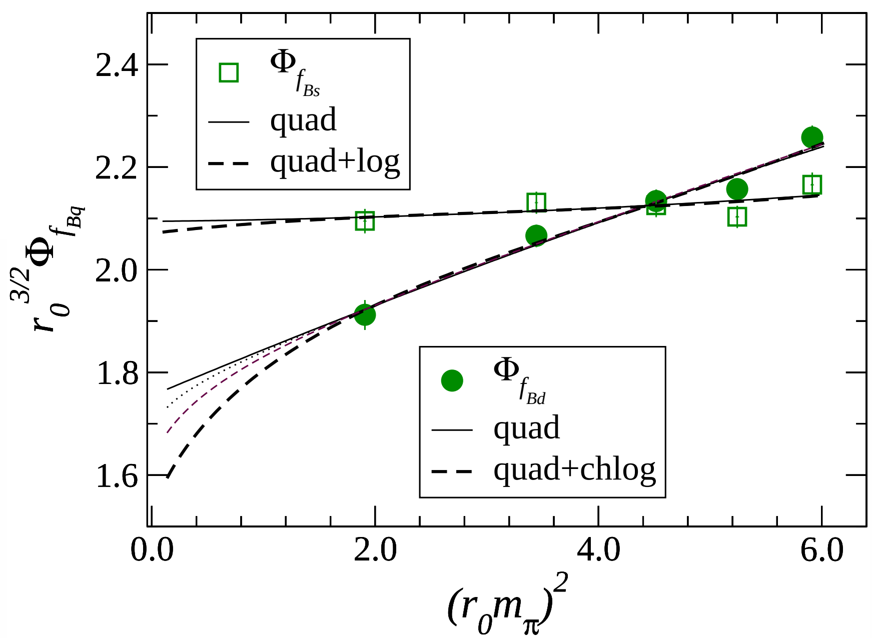
(chiral) is sizable in $f_B$.

$$f_{B_s} = 215(9)(^{+0}_{-2})(13)(^{+6}_{-0})\ \text{MeV}$$

$$\Downarrow$$

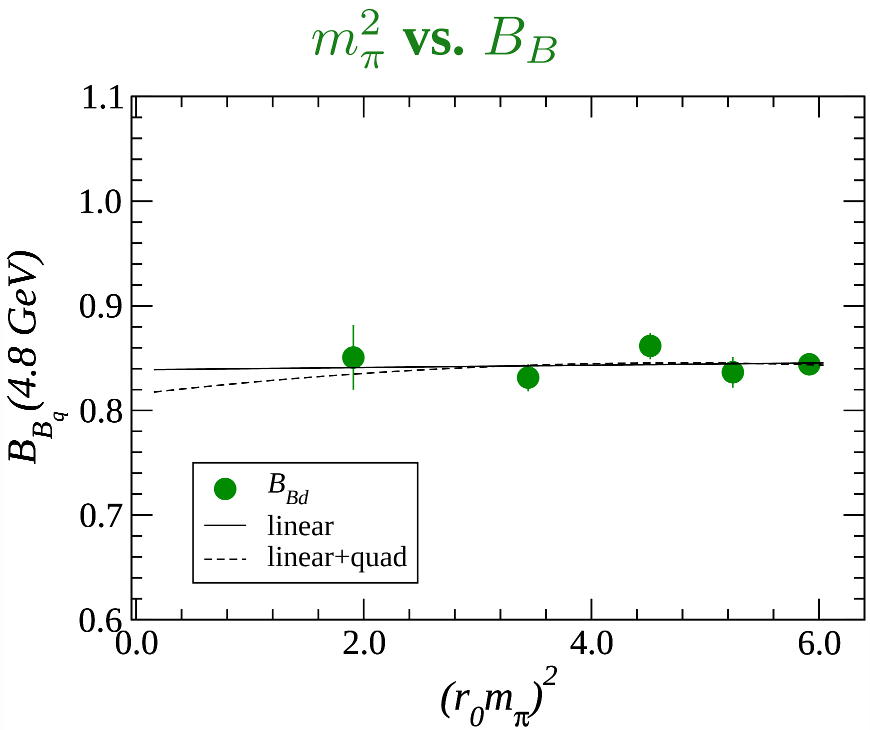
$$f_{B_s}/f_B = 1.13(3)(^{+12}_{-\ 0})(2)(^{+3}_{-0})$$

$m_\pi^2$ vs. $f_B$




$f_{B_s}/f_B$ has a sizable uncertainty.

# Results of $B_B$



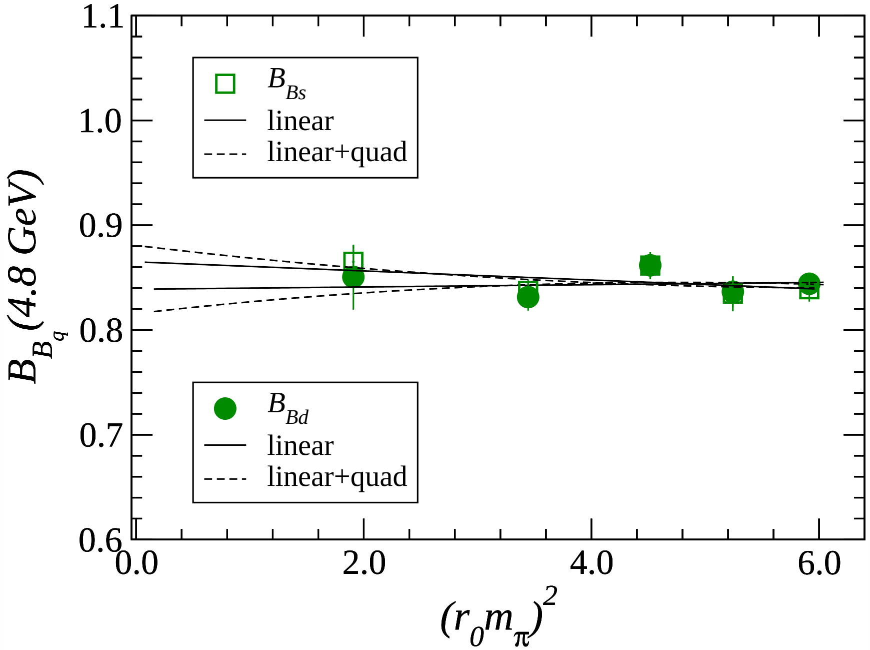

Fitting data of $B_{B_{(s)}}$ to the modified function, we obtain

$$B_B(m_b) = 0.836(27)(^{+\ 0}_{-27})(56)(0) \text{ MeV}$$

$$\text{error} \quad (\text{stat.})(\text{chiral})(\text{sys.})(\text{m}_\text{s})$$

# Results of $B_B$

$m_\pi^2$ **vs.** $B_B$



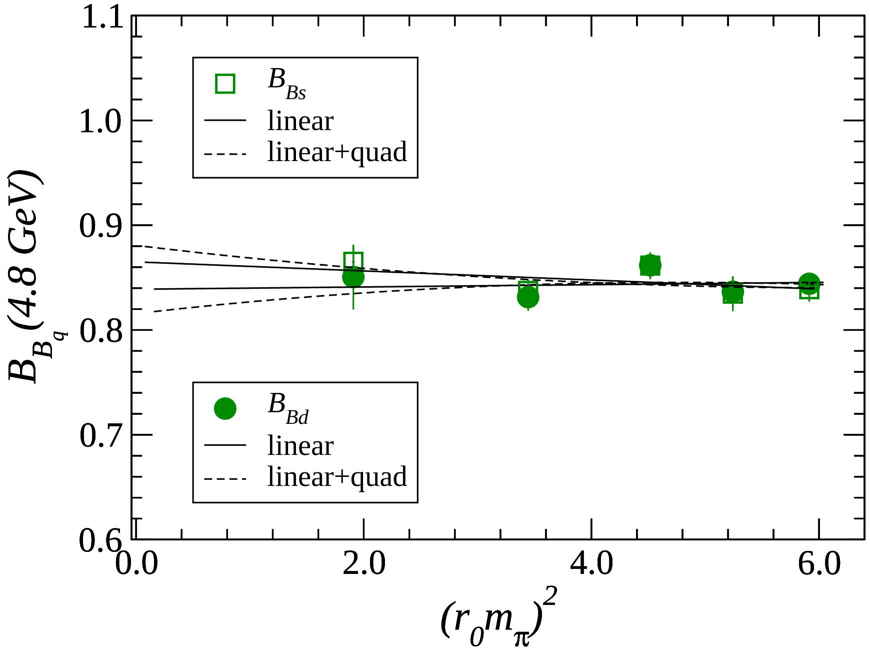

Fitting data of $B_{B_{(s)}}$ to the modified function, we obtain

$$B_B(m_b) = 0.836(27)(^{+\ 0}_{-27})(56)(0) \text{ MeV}$$
$$\text{error} \quad (\text{stat.})(\text{chiral})(\text{sys.})(\text{m}_\text{s})$$

$$B_{B_s}(m_b) = 0.850(22)(^{+18}_{-\ 0})(57)(^{+5}_{-0}) \text{ MeV}$$

# Results of $B_B$

$m_\pi^2$ **vs.** $B_B$




Fitting data of $B_{B_{(s)}}$ to the modified function, we obtain

$$B_B(m_b) \;=\; 0.836(27)(^{+\ 0}_{-27})(56)(0) \text{ MeV}$$

$$\text{error} \qquad (\text{stat.})(\text{chiral})(\text{sys.})(\text{m}_\text{s})$$

$$B_{B_s}(m_b) \;=\; 0.850(22)(^{+18}_{-\ 0})(57)(^{+5}_{-0}) \text{ MeV}$$

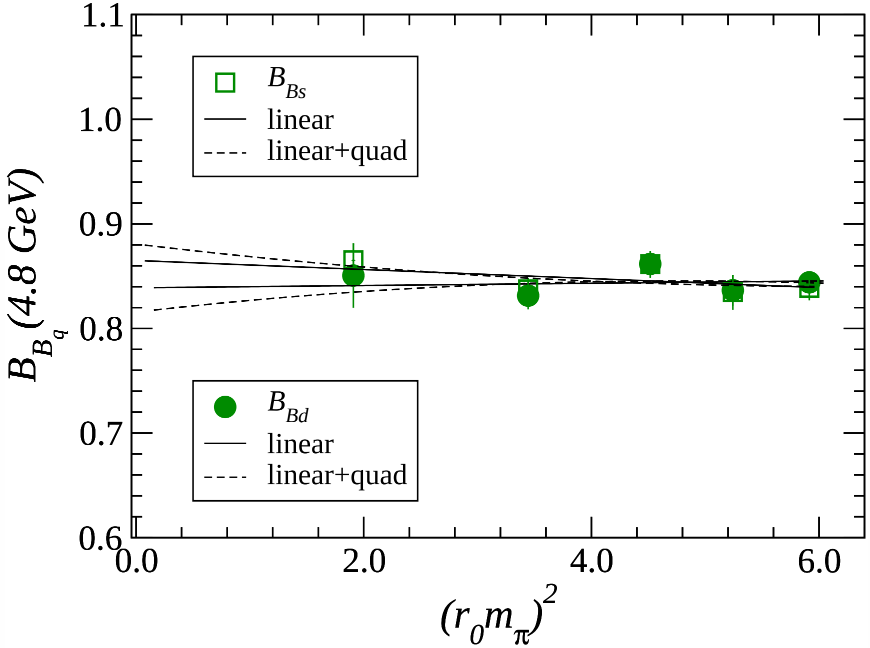
$$\Downarrow$$

$$B_{B_s}/B_B \;=\; 1.017(16)(^{+53}_{-\ 0})(17)(^{+6}_{-0})$$

$B_{B_s}/B_B$ is determined with better precision than $f_{B_s}/f_B$.

# Results of $B_B$

$m_\pi^2$ **vs.** $B_B$




Fitting data of $B_{B_{(s)}}$ to the modified function, we obtain

$$B_B(m_b) = 0.836(27)(^{+\ 0}_{-27})(56)(0) \text{ MeV}$$

error (stat.)(chiral)(sys.)($\mathrm{m_s}$)

$$B_{B_s}(m_b) = 0.850(22)(^{+18}_{-\ 0})(57)(^{+5}_{-0}) \text{ MeV}$$

$\Downarrow$

$$B_{B_s}/B_B = 1.017(16)(^{+53}_{-\ 0})(17)(^{+6}_{-0})$$

$B_{B_s}/B_B$ is determined with better precision than $f_{B_s}/f_B$.

$\Downarrow$

$$\xi = (f_{B_s}\sqrt{B_{B_s}})/(f_B\sqrt{B_B}) = 1.14(3)(^{+13}_{-0})(2)(^{+3}_{-0})$$

$\Downarrow$

Once $\Delta M_s$ has been obtained in experiments,
$|V_{ts}/V_{td}|$ can be determined with $\sim \pm$ 5 %.

# Summary

- We performed the first unquenched calculation of the $B^0$-$\bar{B}^0$ transition amplitude, focusing on the chiral extrapolations.
- Using data obtained with relatively heavy dynamical quarks, we explored the range of errors associated with the chiral extrapolation.
- While $B$-parameters and $f_{B_s}$ are not affected with chiral logs by much, $f_B$ is.
- As a consequence, the error of $f_{B_s}/f_B$ is sizable, while that of $B_{B_s}/B_B$ remains small.

By the way, there is another way to extract $|V_{ts}/V_{td}|$ with better accuracy.

# *Outlook : Grinstein ratio*

B. Grinstein, Phys. Rev. Lett. **71** (1993) 3067

Grinstein ratio of decay constants : $R_1 = (f_{B_s}/f_B)/(f_{D_s}/f_D)$

$$f_{B_s,D_s} = f_{P_d}^{\rm static} \times \left(1 + O(\Lambda_{\rm QCD}/M_{B,D}) + O(m_K^2/(4\pi f_\pi)^2) + \cdots\right)$$

Its deviation from 1 is doubly suppressed:

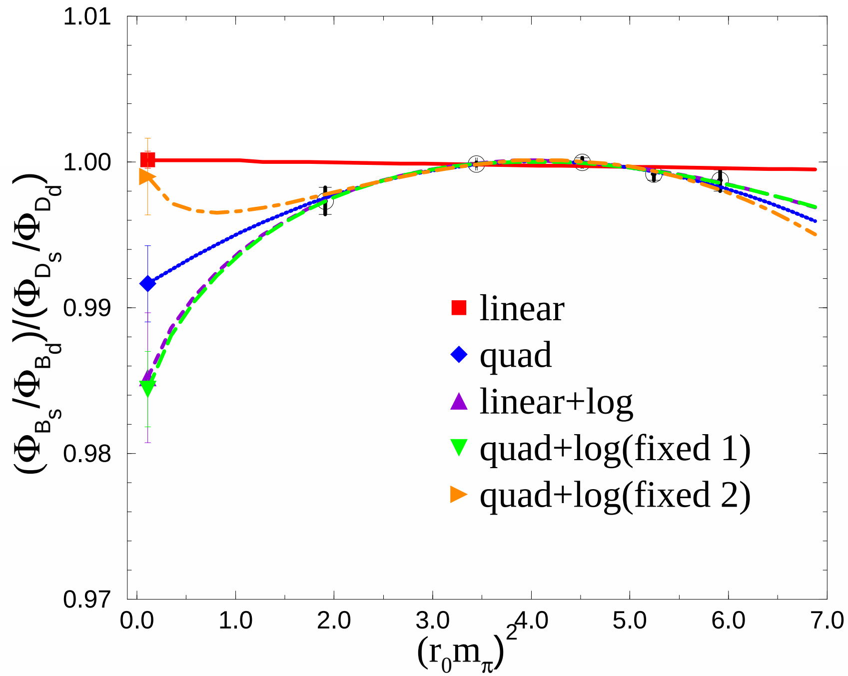
$$R_1 - 1 \sim \left(\frac{\Lambda_{\rm QCD}}{M_B} - \frac{\Lambda_{\rm QCD}}{M_D}\right) \times \frac{m_K^2}{(4\pi f)^2} \ \ln\left(\frac{m_K^2}{(4\pi f)^2}\right)$$

- uncertainty in chiral extrapolation $\Rightarrow$ small
- Lattice QCD can calculate $R_1$ very precisely!

# Numerical results of the GR

## Chiral extrapolation of the GR ($N_f$=2)

JLQCD@Latice 2003




where $\Phi_P = f_P\sqrt{M_P}$

$R_1 = 1.010(3)(8)(5)$ (preliminary)

**1 % determination is possible.**

# SU(3) flavor breaking ratio (revisit)

$$\frac{\Delta M_{B_s}}{\Delta M_{B_d}} = \frac{|V_{ts}|^2}{|V_{td}|^2} \frac{M_{B_s}}{M_{B_d}} \frac{f_{B_s}^2 \hat{B}_{B_s}}{f_{B_d}^2 \hat{B}_{B_d}}$$

$$\Downarrow$$

With the Grinstein Ratio,

$$\left(\frac{\Delta M_{B_s}}{\Delta M_{B_d}}\right)_{\mathrm{RunII}} = \frac{|V_{ts}|^2}{|V_{td}|^2} \left(\frac{M_{B_s}}{M_{B_d}}\right) \left(\frac{f_{D_s}^2}{f_D^2}\right)_{\mathrm{CLEO-c}} \left(\frac{\hat{B}_{B_s}}{\hat{B}_{B_d}} R_1^2\right)_{\mathrm{Lattice}}$$

Once $f_{D_s}$, $f_D$ and $\Delta M_{B_s}$ have been determined in on-going experiments,

$$\Downarrow$$

$|V_{ts}/V_{td}|$ can be extracted very precisely!

The Grinstein ratio will give the first precise determination of $|V_{ts}/V_{td}|$.

# *Outlook2 : Domain-Wall Fermion for light quarks*

Clearly, having smaller dynamical quark masses improves the uncertainty in the chiral extrapolations.

Ginsberg-Wilson Fermions $\Rightarrow$ very light quarks ($\lesssim 1/4 m_s$)
In particular, the domain-wall fermions (DWF) have been successfully used in large scale dynamical simulations of light hadrons (RBC Collaboration).

# Outlook2 : Domain-Wall Fermion for light quarks

Clearly, having smaller dynamical quark masses improves the uncertainty in the chiral extrapolations.

Ginsberg-Wilson Fermions $\Rightarrow$ very light quarks ($\lesssim 1/4 m_s$)
In particular, the domain-wall fermions (DWF) have been successfully used in large scale dynamical simulations of light hadrons (RBC Collaboration).

$\Downarrow$

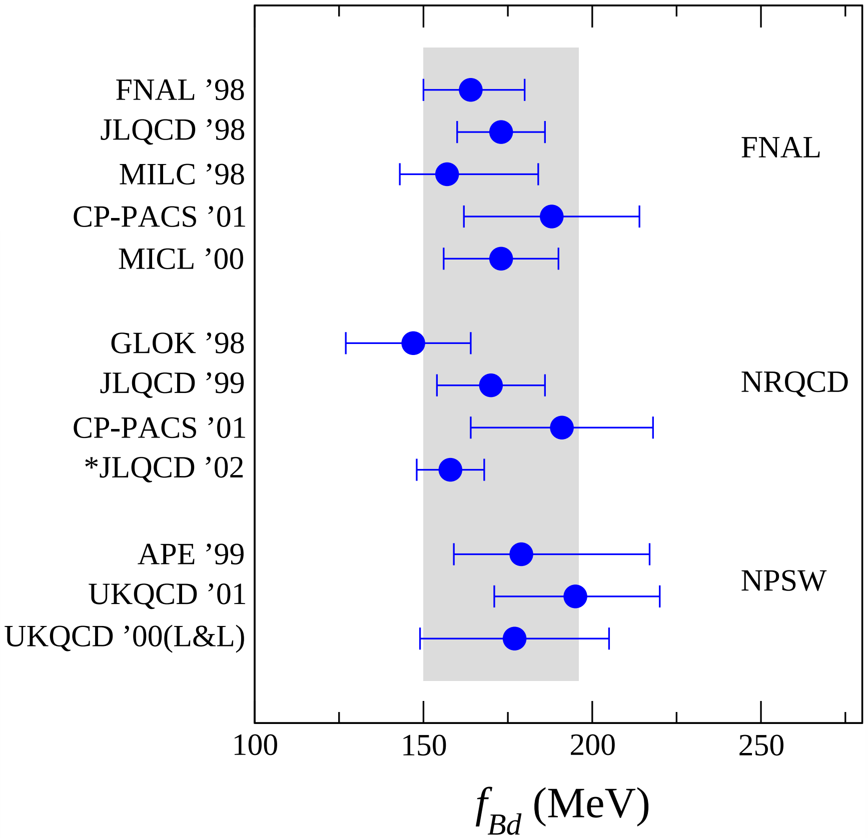
Please have their works in your mind!

# Backup Slides

# Previous results of $f_B$

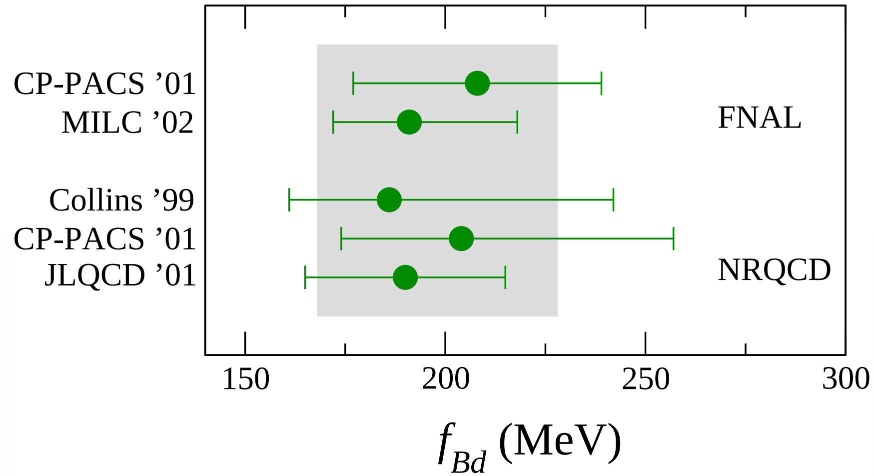
S. Ryan@Latice 2001

**Quench ($N_f$=0)**




$$f_B = 173(23)\text{ MeV}$$
$$f_{B_s} = 200(20)\text{ MeV}$$

**Unquench ($N_f$=2)**




$$f_B = 198(30)\text{ MeV}$$
$$f_{B_s} = 230(30)\text{ MeV}$$

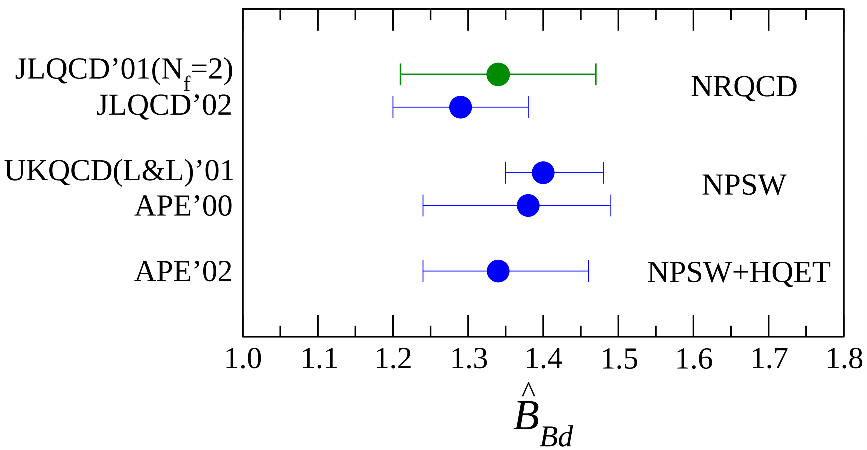
It has been expected that

- $f_B^{\rm unq} > f_B^{\rm que}$ by $\sim$15 %
- $f_{B_s}/f_B \sim 1.16(5)$ in both

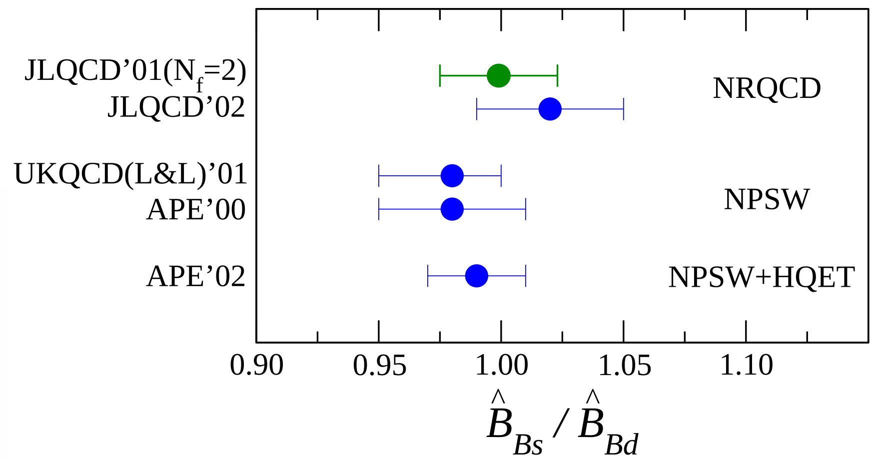
# Present status of $B_B$

N. Yamada@Latice 2002

$\hat{B}_B$ ($N_f$=0, 2)




$\hat{B}_{B_s}/\hat{B}_B$ ($N_f$=0, 2)




$$\hat{B}_B \quad = \quad 1.33(12)$$

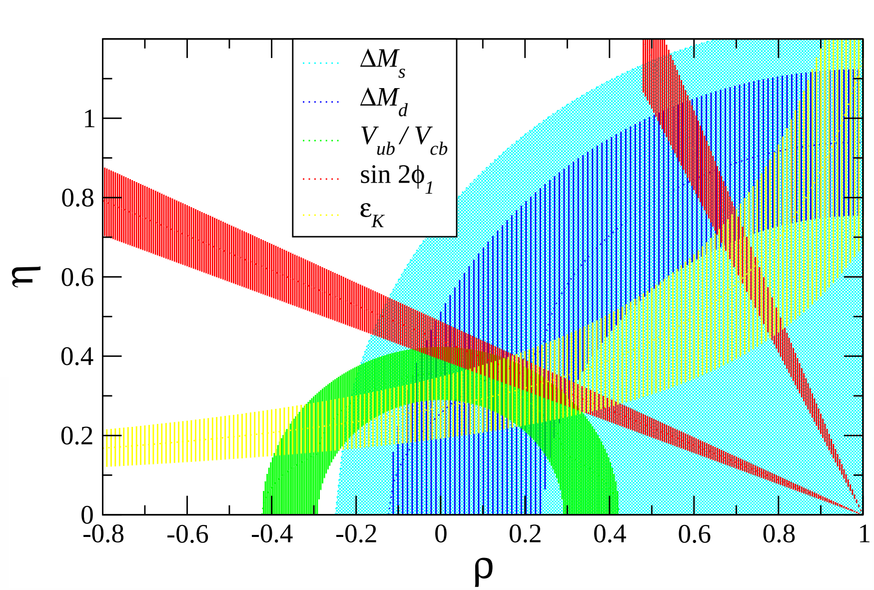
$$\hat{B}_{B_s}/\hat{B}_B \quad = \quad 1.00(3)$$

- Unquench results
- Quenched results

**The ratio is determined well already.**

# Constraint on Unitarity Triangle

Current status with
Theo err.$\sim$ 10–20%,




Assume
Theo err.$\rightarrow$ 5%,
Exp err.$\rightarrow$ unchanged