

---

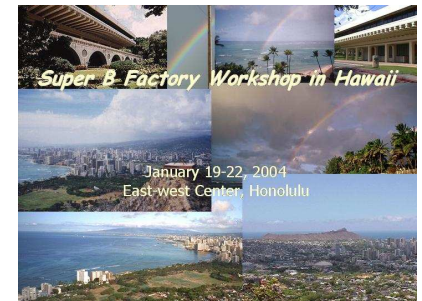
# An Analysis of Supersymmetric Effects on $B \rightarrow \phi K$ Decays in the PQCD Approach

Satoshi Mishima (Nagoya Univ.)

S. M. and A. I. Sanda, accepted in PRD. [hep-ph/0311068]

S. M. and A. I. Sanda, Prog. Theor. Phys. 110 (2003) 549 [hep-ph/0305073]

Super  $B$  Factory Workshop in Hawaii, Jan. 21, 2004



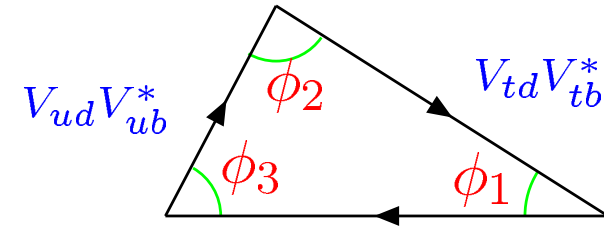
1. Introduction
2. Perturbative QCD Approach in Exclusive  $B$  Decays
  - Review of PQCD
3. MSSM Contribution in  $B$  Decays
  - Mass Insertion Approximation
  - Constraint from  $\text{Br}(B \rightarrow X_s \gamma)$
4. MSSM Effects on  $B \rightarrow \phi K$  Decays
  - Branching Ratio and CP Asymmetry  
for  $B_d \rightarrow \phi K_S$  and  $B^\pm \rightarrow \phi K^\pm$
5. Summary

# 1. Introduction

## The time-dependent CP asymmetry

$$a_{f_{CP}}(t) \equiv \frac{\Gamma(\overline{B^0}(t) \rightarrow f_{CP}) - \Gamma(B^0(t) \rightarrow f_{CP})}{\Gamma(\overline{B^0}(t) \rightarrow f_{CP}) + \Gamma(B^0(t) \rightarrow f_{CP})}$$

$$\equiv A_{f_{CP}} \cos(\Delta M_B t) + S_{f_{CP}} \sin(\Delta M_B t)$$



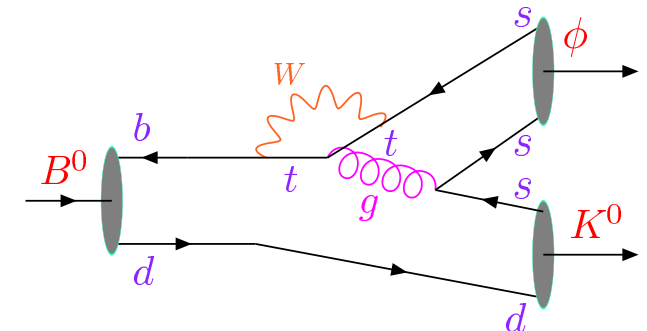
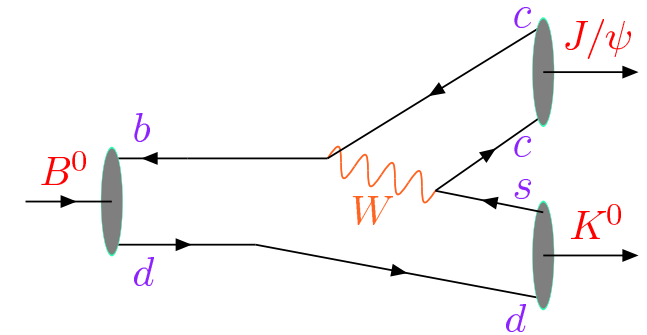
$$A_{f_{CP}} \equiv \frac{|\lambda_{f_{CP}}|^2 - 1}{|\lambda_{f_{CP}}|^2 + 1}, \quad S_{f_{CP}} \equiv \frac{2\text{Im}\lambda_{f_{CP}}}{|\lambda_{f_{CP}}|^2 + 1}, \quad \lambda_{f_{CP}} = e^{-2i\phi_1} \frac{\mathcal{A}(\overline{B^0} \rightarrow f_{CP})}{\mathcal{A}(B^0 \rightarrow f_{CP})}$$

●  $B^0 \rightarrow J/\psi K_S$  : Tree dominant,  $\mathcal{A}_{J/\psi K_S} = V_{cb}^* V_{cs} A_1$

$$\implies S_{J/\psi K_S} = \sin(2\phi_1), \quad A_{J/\psi K_S} = 0$$

●  $B^0 \rightarrow \phi K_S$  : Pure penguin,  $\mathcal{A}_{\phi K_S} = V_{tb}^* V_{ts} A_2$

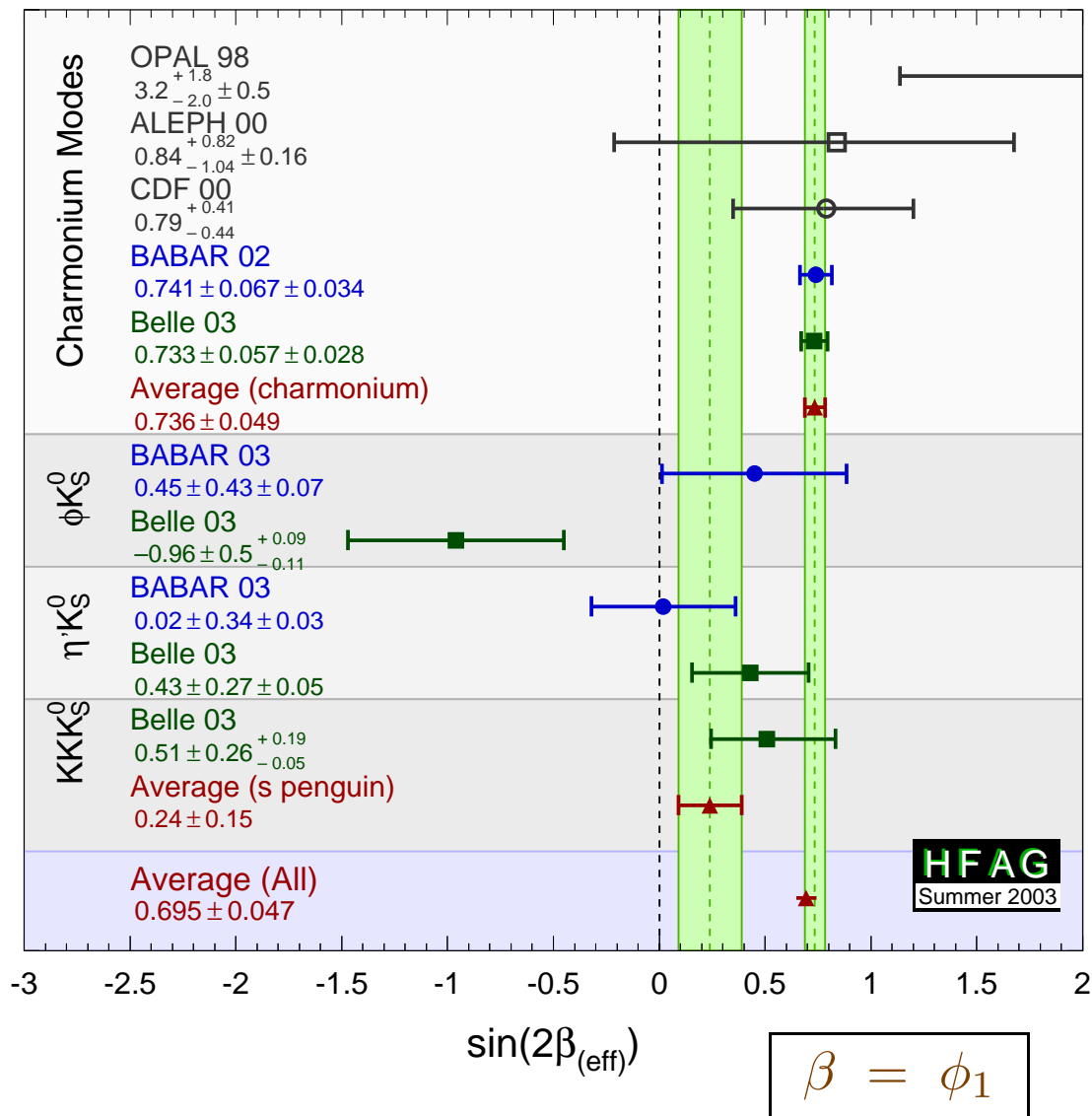
$$\implies S_{\phi K_S} = \sin(2\phi_1), \quad A_{\phi K_S} = 0$$



$$|S_{J/\psi K_S} - S_{\phi K_S}| \gg \mathcal{O}(\lambda^2)$$

$\implies$  New Physics in  $\phi K_S$

$$\lambda = \sin \theta_c \simeq 0.22$$



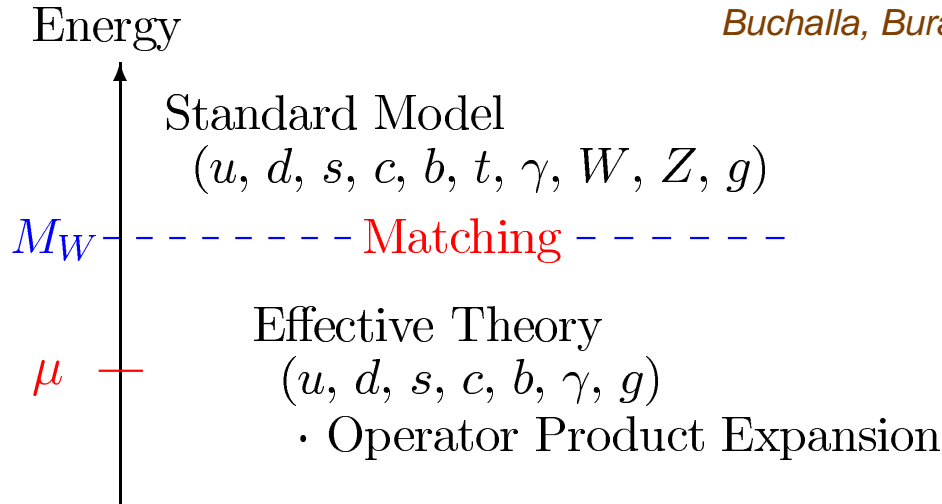
- In the SM,  
 $S_{J/\psi K_S} = S_{\phi K_S} + \mathcal{O}(\lambda^2)$ .
- Belle :  $S_{\phi K_S}$  indicates a  $3.5\sigma$  deviation from the SM prediction.
- There is a  $2.1\sigma$  discrepancy between BABAR and Belle.



We consider New Physics contributions in  $B \rightarrow \phi K$ .

# Effective Hamiltonian for $B$ Decays

Buchalla, Buras, Lautenbacher, Rev.Mod.Phys.68,1125,1996



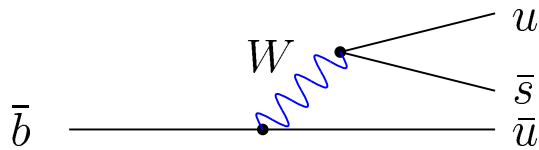
$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_i V_{CKM}^i C_i(\mu) O_i(\mu)$$

$\mu$  : Factorization scale

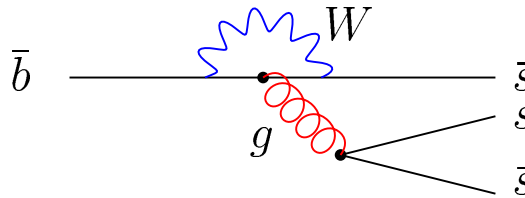
New Physics  $\implies C_{3-6}, C_{7\gamma}, C_{8g}$

## Standard Model

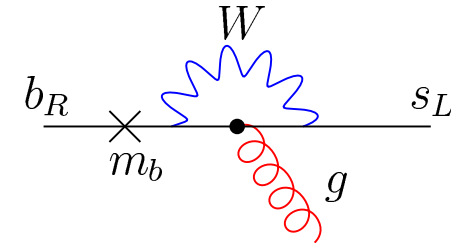
### Tree



### Penguin



### Magnetic Penguin



## Effective Theory

$$O_1^{(q)} = (\bar{s}_i q_j)_{V-A} (\bar{q}_j b_i)_{V-A},$$

$$O_2^{(q)} = (\bar{s}_i q_i)_{V-A} (\bar{q}_j b_j)_{V-A},$$

$$O_{3,5} = (\bar{s}_i b_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V \mp A}, \quad O_{4,6} = (\bar{s}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V \mp A},$$

$$O_{7\gamma} = \frac{e}{8\pi^2} m_b (\bar{s} \sigma^{\mu\nu} (1 + \gamma_5) b) F_{\mu\nu}, \quad O_{8g} = -\frac{g_s}{8\pi^2} m_b (\bar{s}_i \sigma^{\mu\nu} (1 + \gamma_5) T_{ij}^a b_j) G_{\mu\nu}^a$$

# Strong Phases and CP Asymmetries

$$a_{\phi K_S}(t) = A_{\phi K_S} \cos(\Delta M t) + S_{\phi K_S} \sin(\Delta M t)$$

If there is any new physics in  $B \rightarrow \phi K_S$ ,

$$\begin{aligned} A(\phi K_S) &= |A^{\text{SM}}| e^{i\delta_{\text{SM}}} + |A^{\text{NP}}| e^{i\theta_{\text{NP}}} e^{i\delta_{\text{NP}}} \\ \bar{A}(\phi K_S) &= |\bar{A}^{\text{SM}}| e^{i\delta_{\text{SM}}} + |\bar{A}^{\text{NP}}| e^{-i\theta_{\text{NP}}} e^{i\delta_{\text{NP}}} \end{aligned}$$

$\delta_{\text{SM}(\text{NP})}$  : Strong phase,  $\theta_{\text{NP}}$  : CP violating phase  
(Final-state interaction phase)

$$\begin{aligned} S_{\phi K_S} &= \frac{\sin 2\phi_1 + 2 \left| \frac{A^{\text{NP}}}{A^{\text{SM}}} \right| \cos \delta \sin(\theta_{\text{NP}} + 2\phi_1) + \left| \frac{A^{\text{NP}}}{A^{\text{SM}}} \right|^2 \sin(2\theta_{\text{NP}} + 2\phi_1)}{1 + 2 \left| \frac{A^{\text{NP}}}{A^{\text{SM}}} \right| \cos \delta \cos \theta_{\text{NP}} + \left| \frac{A^{\text{NP}}}{A^{\text{SM}}} \right|^2} \\ A_{\phi K_S} &= \frac{2 \left| \frac{A^{\text{NP}}}{A^{\text{SM}}} \right| \sin \delta \sin \theta_{\text{NP}}}{1 + 2 \left| \frac{A^{\text{NP}}}{A^{\text{SM}}} \right| \cos \theta_{\text{NP}} \cos \delta + \left| \frac{A^{\text{NP}}}{A^{\text{SM}}} \right|^2} \quad \delta \equiv \delta_{\text{SM}} - \delta_{\text{NP}} \end{aligned}$$

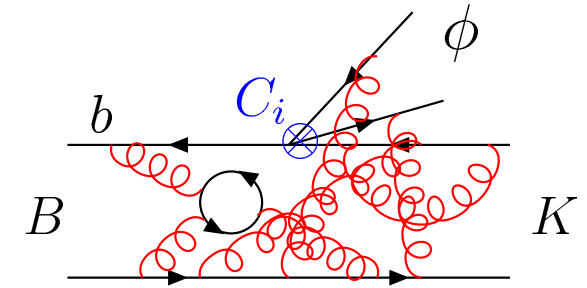
$\implies$

$\delta$  and  $|A^{\text{NP}}/A^{\text{SM}}|$  are needed.

# How to Calculate Matrix Elements

$$\langle \phi K | O_i(\mu) | B \rangle = ?$$

Generalized factorization, QCD factorization,  
Perturbative QCD, Light-cone QCD sum rules,  
Lattice QCD, Soft-collinear effective theory, etc.



- Generalized Factorization Approach

- QCD Factorization (QCDF or BBNS)

(See e.g. Beneke, Buchalla, Neubert, Sachrajda, NPB606,245,2001)

⇒ These approaches were used for estimating new physics contributions  
in  $B \rightarrow \phi K_S$  decay.

(Khalil, Kou, PRD67,055009,2003, Kane et al. PRL94,141803,2003, ...)

In this study, we use another approach :

- **Perturbative QCD (PQCD)** (See e.g. Keum, Li, Sanda, PRD63,054008,2001)

A dominant source of strong phase in PQCD is different from in QCDF.

Although it is difficult to discover NP by PQCD, it can be applied as a guide  
for NP search.

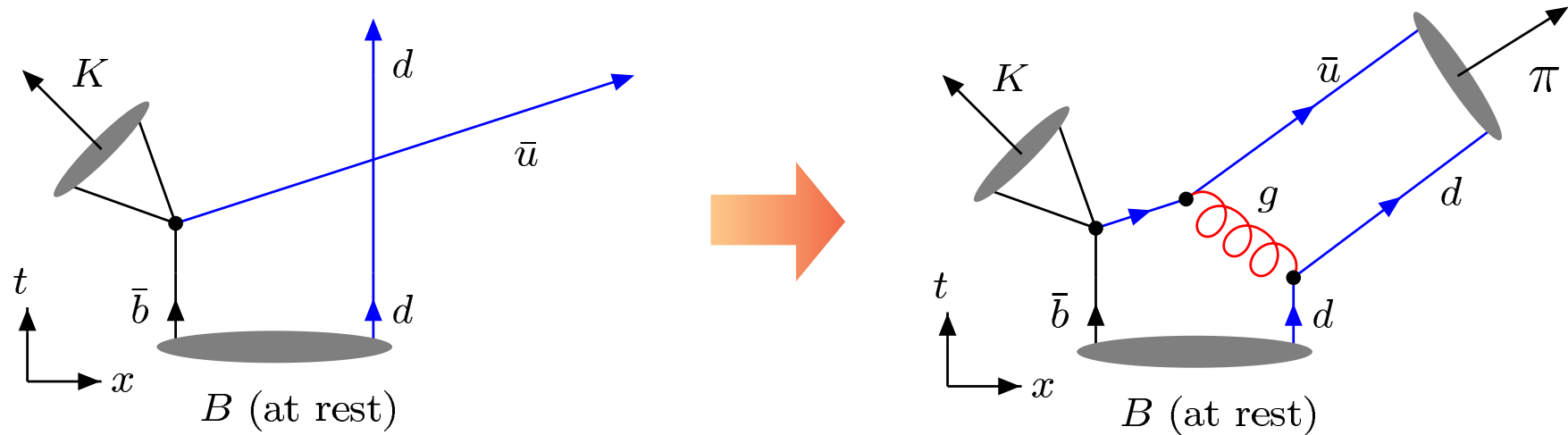
---

## 2. PQCD Approach in $B$ Meson Decays

---



# An Intuitive Picture of the PQCD Approach



The spectator quark ( $d$  quark) **exchanges a hard gluon** and it lines itself with the fast  $\bar{u}$  quark so that **it forms  $\pi^-$  meson**.

- We consider the form factor is dominated by hard gluon exchanges.
- Are contributions from many soft gluon exchanges actually small ?  
⇒ We calculate other contributions, which are calculable perturbatively, and compare the predictions with data.

# Review of PQCD

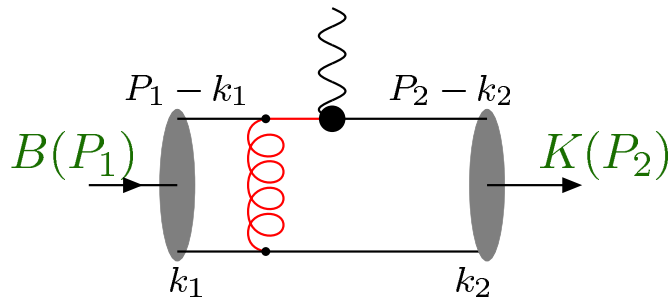
- Factorization Theorem : (Brodsky, Lepage, PRD22,2157,1980)

$$\langle M_2(P_2)M_3(P_3)|H_{\text{eff}}|B(P_1)\rangle$$

$$= \int [dx_i] \Phi_{M_2}(x_2, P_2) \Phi_{M_3}(x_3, P_3) C(t) H(x_1, x_2, x_3, t) \Phi_B(x_1, P_1)$$

$t \sim O(M_B)$ ,  $x_i$  are the momentum fractions of partons.

- End-Point Singularity in  $H$



$$\begin{aligned} &\propto \frac{1}{(k_1 - k_2)^2} \cdot \frac{1}{(P_1 - k_2)^2} \\ &= \frac{1}{-x_1 x_2 M_B^2 - |\mathbf{k}_{1T} - \mathbf{k}_{2T}|^2} \cdot \frac{1}{-x_2 M_B^2 - |\mathbf{k}_{2T}|^2} \\ &\simeq \frac{1}{x_1 x_2^2 M_B^2} \quad \Phi_K(x_2) \propto x_2(1 - x_2) \end{aligned}$$

## Light-Cone Coordinate

$$P = (P^+, P^-, \mathbf{P}_T)$$

$$P^\pm = \frac{1}{\sqrt{2}}(P^0 \pm P^3)$$

$$\mathbf{P}_T = (P^1, P^2)$$

$$P_1 = \frac{M_B}{\sqrt{2}}(1, 1, \mathbf{0}_T) \quad P_2 = \frac{M_B}{\sqrt{2}}(0, 1, \mathbf{0}_T)$$

$$k_1 = (x_1 P_1^+, 0, \mathbf{k}_{1T}) \quad k_2 = (0, x_2 P_2^-, \mathbf{k}_{2T})$$

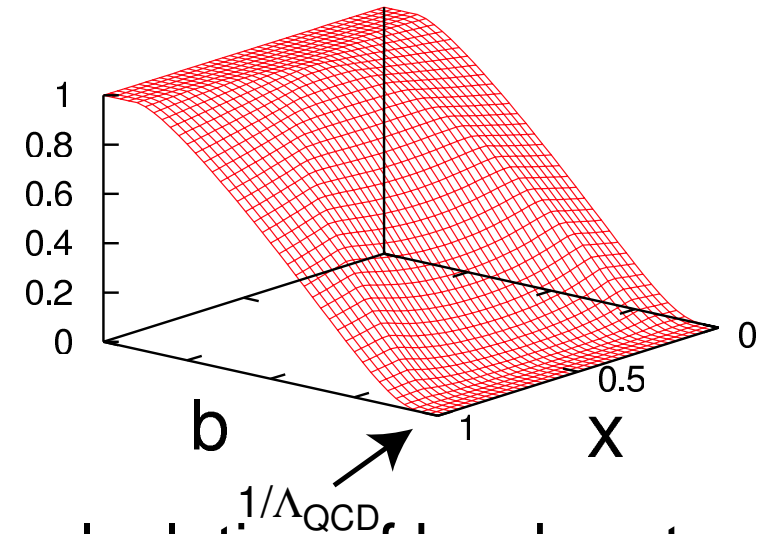
$$0 \leq x_{1,2} \leq 1$$

When we retain  $k_T$ , the large double logarithms are generated from the overlap of collinear and soft divergence in radiative corrections to meson wave functions.

⇒ **Sudakov factor**

⇒ Large  $b$  (small  $k_T$ ) is suppressed.

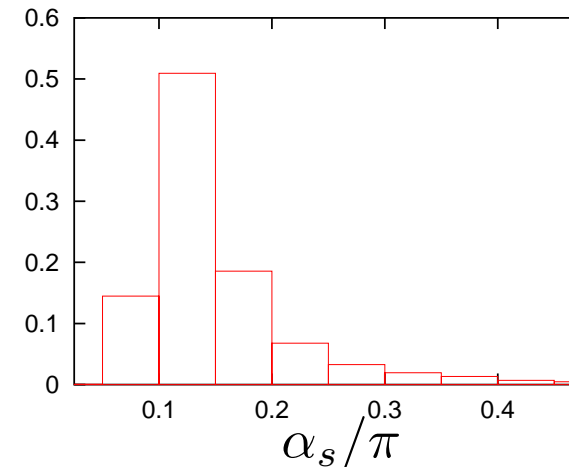
$$\Psi(P, \mathbf{k}_T) = \int d^2\mathbf{b} e^{i\mathbf{k}_T \cdot \mathbf{b}} \Psi(P, \mathbf{b})$$



● Sudakov factor ensures a perturbative calculation of hard part.

$$\frac{1}{q^2} = \frac{1}{-x_1 x_2 M_B^2 - |\mathbf{k}_{1T} - \mathbf{k}_{2T}|^2}$$

**No End-Point Singularity !!**



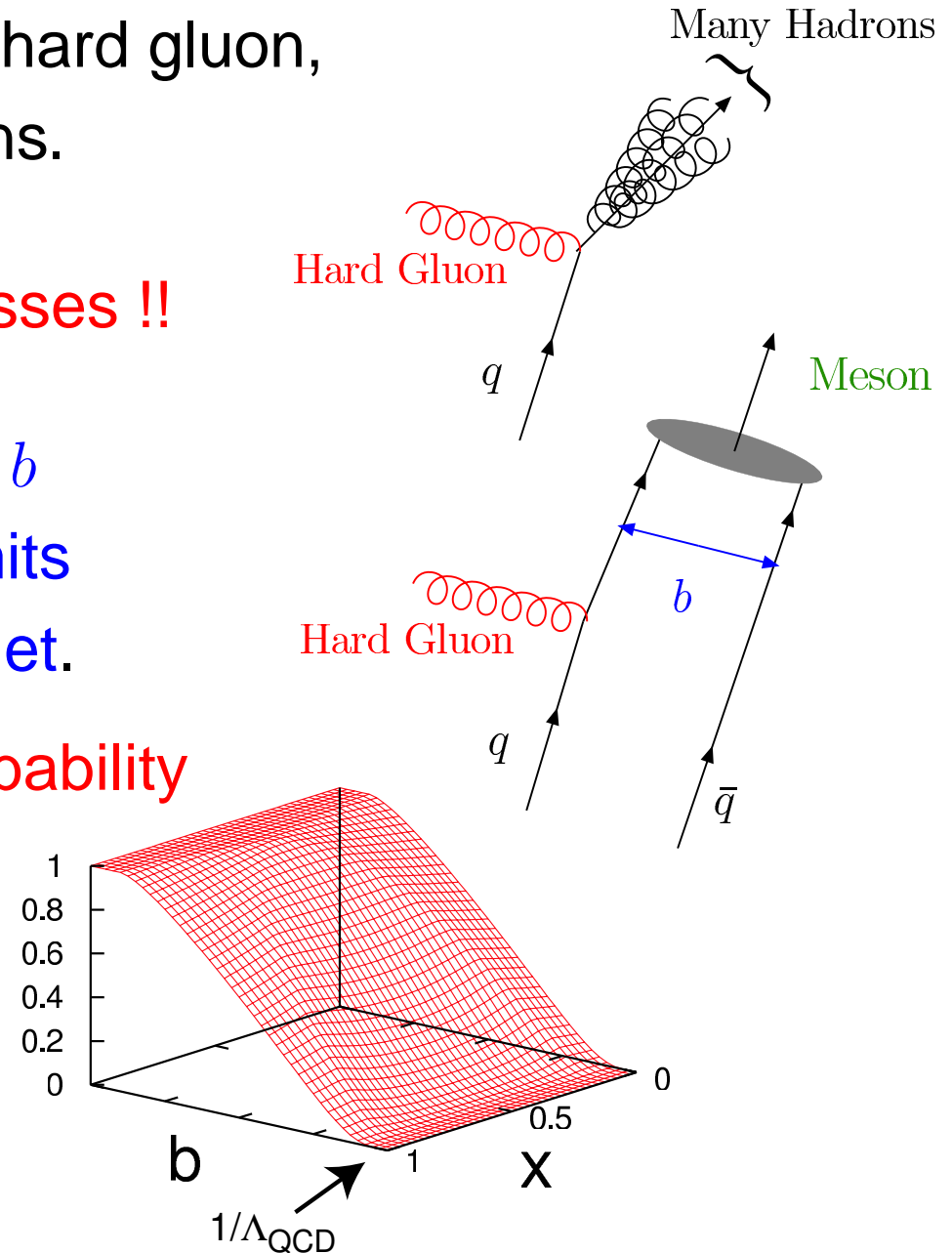
⇒ We have checked self-consistency in our calculations.

# Physical Meaning of Sudakov Factor in $b$ space

- If a single quark interacts with a hard gluon, it must emit many collinear gluons.

⇒ Not exclusive processes !!

- If a  $q\bar{q}$  pair with small separation  $b$  interacts with a hard gluon, it emits no gluons since it is a color singlet.
- The Sudakov factor gives a probability for no emitted gluons.



# Review of PQCD

- $k_T$  Factorization Theorem : (Li, Sterman, NPB381,129,1992)

$$\text{Amp.} = \int [dx_i][db_{iT}] \Phi_K(x_2, b_{2T}) \Phi_\phi(x_3, b_{3T}) C(t) H(x_i, b_{iT}, t) \Phi_B(x_1, b_{1T}) e^{-S}$$

- Non-perturbative part  $\implies$  Meson wave functions :

- $K, \phi \iff$  Light-cone QCD sum rules

(Ball, JHEP01,010,1999; Ball, Braun, Koike, Tanaka, NPB529,323,1998)

- $B \iff$  Model function

$$\Phi_B \sim (\not{P} + M_B) \gamma_5 x^2 (1-x)^2 \exp \left[ -\frac{1}{2} \left( \frac{x M_B}{\omega_B} \right)^2 - \frac{\omega_B^2 b^2}{2} \right], \quad \omega_B = 0.36 \sim 0.44 \text{ GeV}$$

- Theoretical Errors

- Large theoretical uncertainty comes from  $\Phi_B$ .

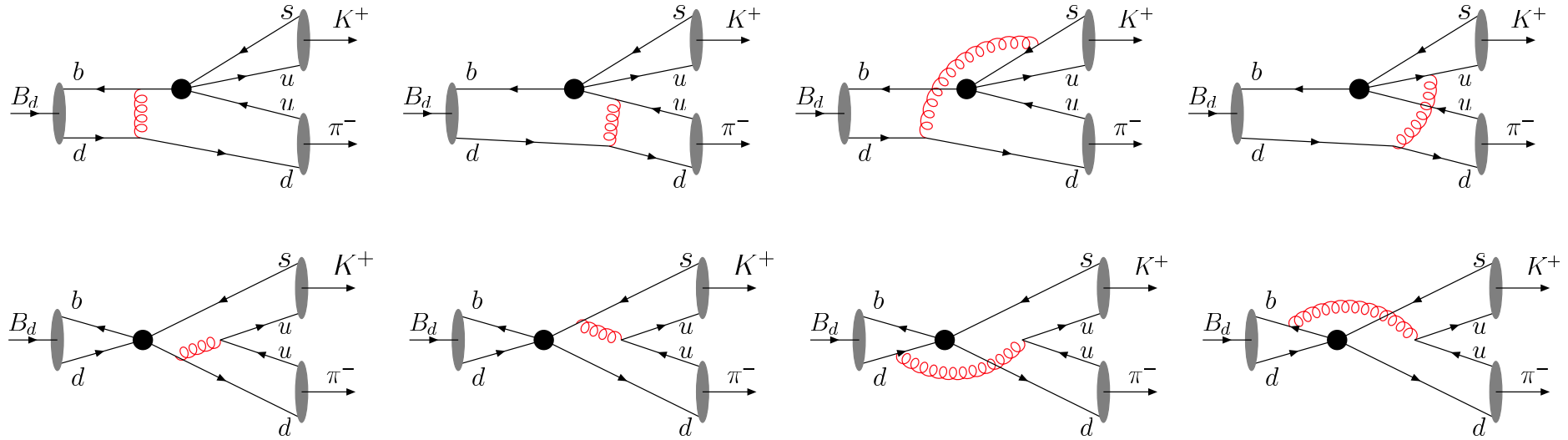
$\implies$  These errors are reduced in CP asymmetries.

- We expect that higher-order corrections are about 30%.

$\implies$  We suppose these corrections are reduced in CP asymmetries.

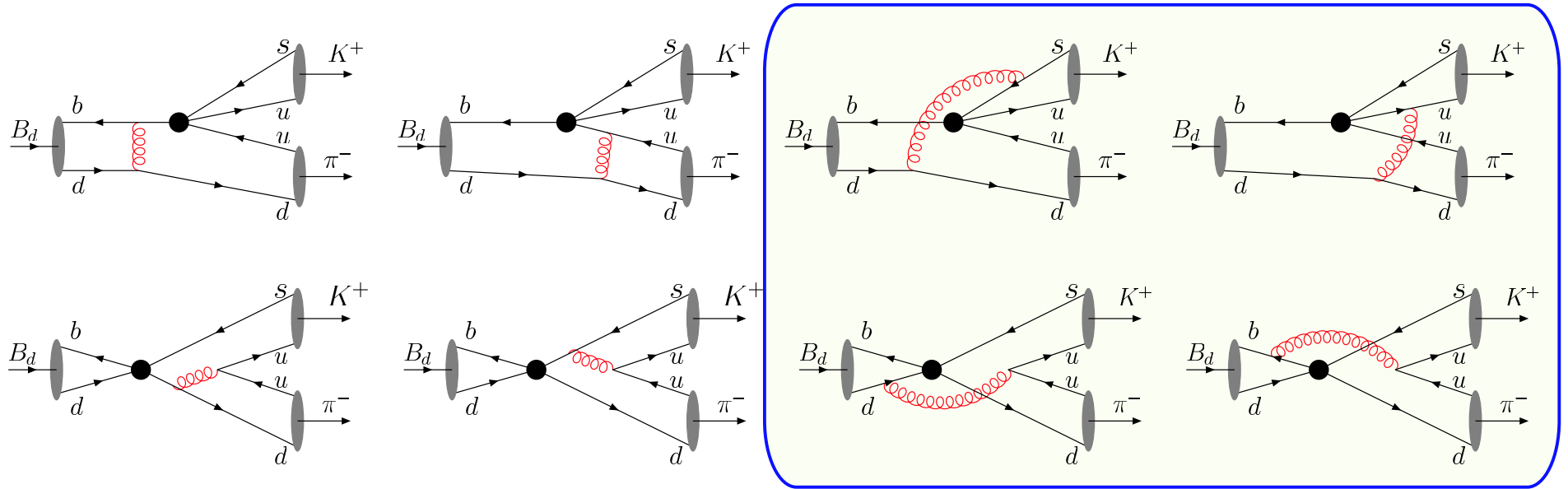
# Review of PQCD

The leading order diagrams in PQCD:



# Review of PQCD

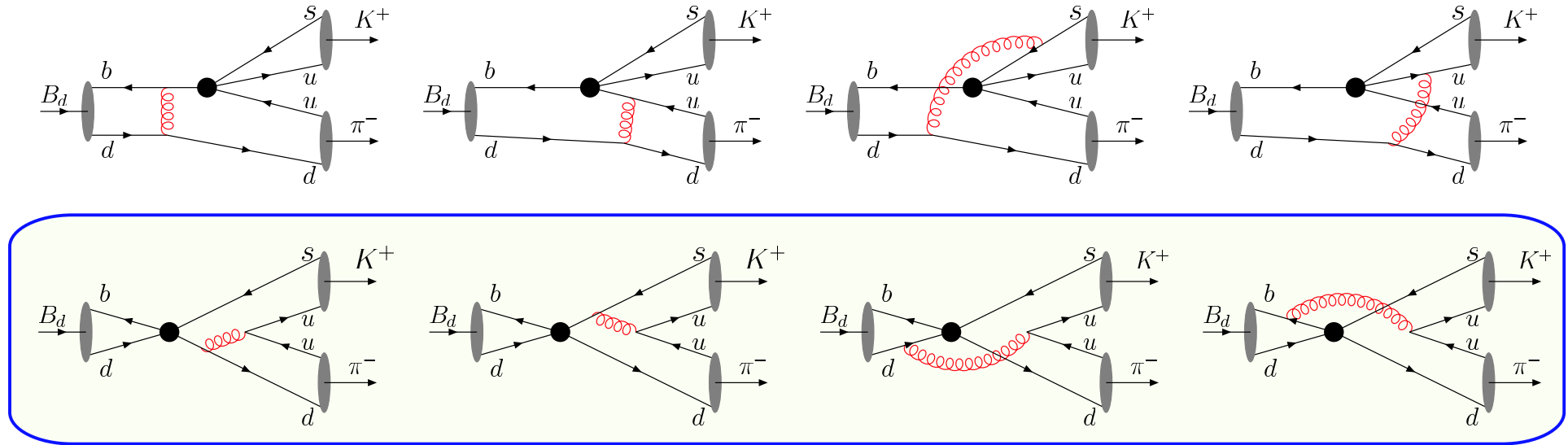
The leading order diagrams in PQCD:



● Non-factorizable diagrams are calculable.

# Review of PQCD

The leading order diagrams in PQCD:

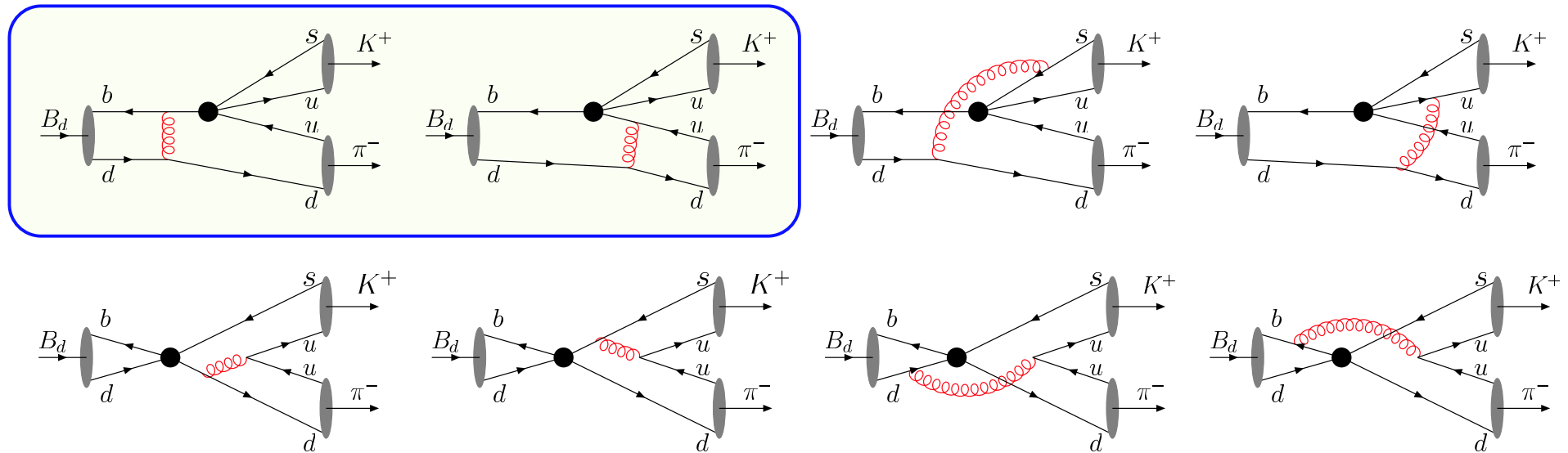


- Non-factorizable diagrams are calculable.
- Annihilation diagrams are also calculable.



# Review of PQCD

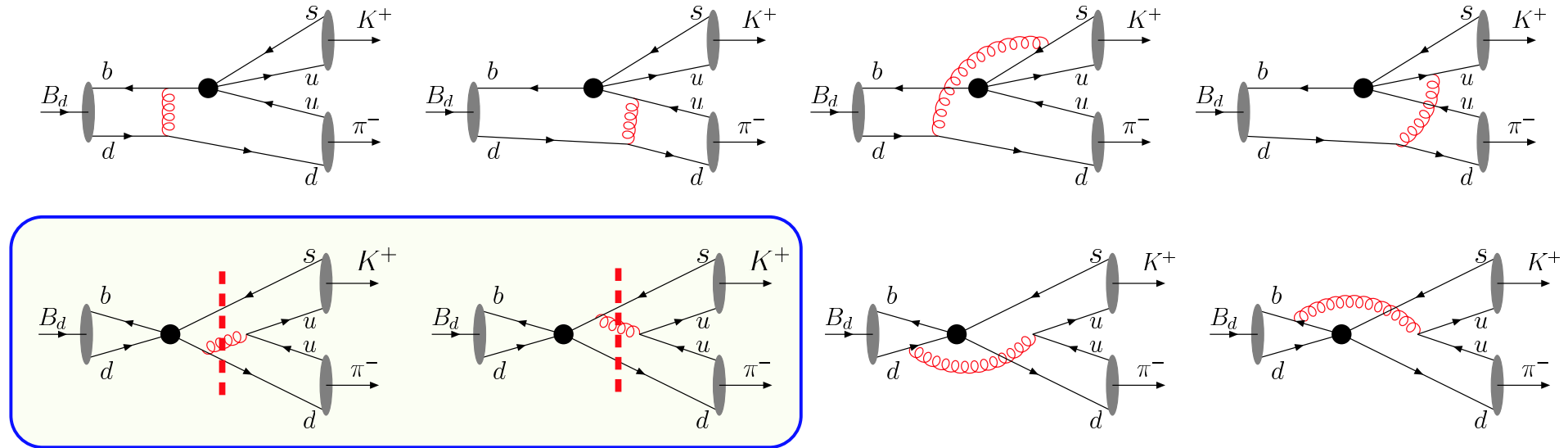
The leading order diagrams in PQCD:



- Non-factorizable diagrams are calculable.
- Annihilation diagrams are also calculable.
- Factorizable diagrams are dominant.

# Review of PQCD

The leading order diagrams in PQCD:



- Non-factorizable diagrams are calculable.
- Annihilation diagrams are also calculable.
- Factorizable diagrams are dominant.
- Factorizable annihilation diagrams generate a large strong phase.

$$\frac{1}{(1-x_2)x_3 M_B^2 - |\mathbf{k}_{2T} - \mathbf{k}_{3T}|^2} = (\text{Pri. val.}) - i\pi\delta((1-x_2)x_3 M_B^2 - |\mathbf{k}_{2T} - \mathbf{k}_{3T}|^2)$$

# Applications of the PQCD approach

---

PQCD has been applied to various charmless  $B$  decays:

- $K\pi$       *Keum, Li, Sanda, Phys.Lett.B504,6,2001:Phys.Rev.D63,054008,2001*
- $\pi\pi$       *Lu, Ukai, Yang, Phys.Rev.D63,074009,2001*
- $KK$       *Chen, Li, Phys.Rev.D63,014003,2001*
- $K\eta^{(1)}$       *Kou, Sanda, Phys.Lett.B525,240,2002*
- $\rho\pi, \omega\pi$       *Lu, Yang, Eur.Phys.J.C23,275,2002*
- $\rho K, \omega K$       *Chen, Phys.Lett.B525,56,2002*
- $\phi K$       *Mishima, Phys.Lett.B521, 252,2001: Chen, Keum, Li, Phys.Rev.D64,112002,2001*
- $\phi\pi$       *Melic, Phys.Rev.D59,074005,1999*
- $K^*\pi$       *Keum, hep-ph/0210127*
- $\phi K^*$       *Chen, Keum, Li, Phys.Rev.D66,054013,2002*
- $K^*\gamma$       *M. Matsumori will talk on this workshop.*

# Branching ratios ( $10^{-6}$ ) and Direct CP Asymmetries

Y. Y. Keum and A. I. Sanda, eConf C0304052, WG420 (2003).

	BABAR	Belle	PQCD
$B \rightarrow \pi^+ \pi^-$	$4.7 \pm 0.6 \pm 0.2$	$4.4 \pm 0.6 \pm 0.3$	$5.93 - 10.99$
$B^\pm \rightarrow \pi^\pm \pi^0$	$5.5^{+1.0}_{-0.9} \pm 0.6$	$5.0 \pm 1.2 \pm 0.5$	$2.72 - 4.79$
$B \rightarrow \pi^0 \pi^0$	$2.1 \pm 0.6 \pm 0.3$	$1.7 \pm 0.6 \pm 0.2$	$0.33 - 0.65$
$B \rightarrow K^\pm \pi^\mp$	$17.9 \pm 0.9 \pm 0.7$	$18.5 \pm 1.0 \pm 0.7$	$12.67 - 19.30$
$B^\pm \rightarrow K^\pm \pi^0$	$12.8^{+1.2}_{-1.1} \pm 1.0$	$12.0 \pm 1.3^{+1.3}_{-0.9}$	$7.87 - 14.21$
$B^\pm \rightarrow K^0 \pi^\pm$	$22.3 \pm 1.7 \pm 1.1$	$22.0 \pm 1.9 \pm 1.1$	$14.43 - 26.26$
$B \rightarrow K^0 \pi^0$	$11.4 \pm 1.7 \pm 0.8$	$11.7 \pm 2.3^{+1.2}_{-1.3}$	$7.92 - 14.27$
$A_{CP}(\pi^\pm \pi^\mp)$	$0.19 \pm 0.19 \pm 0.05$	$0.77 \pm 0.27 \pm 0.08$	$0.160 \sim 0.300$
$A_{CP}(\pi^\pm \pi^0)$	$-0.03^{+0.18}_{-0.17} \pm 0.02$	$-0.14 \pm 0.24^{+0.05}_{-0.04}$	$0.0$
$A_{CP}(K^\pm \pi^\mp)$	$-0.107 \pm 0.041 \pm 0.013$	$-0.088 \pm 0.035 \pm 0.018$	$-0.219 \sim -0.129$
$A_{CP}(K^\pm \pi^0)$	$-0.09 \pm 0.09 \pm 0.01$	$0.23 \pm 0.11^{+0.01}_{-0.04}$	$-0.173 \sim -0.100$
$A_{CP}(K^0 \pi^\pm)$	$-0.05 \pm 0.08 \pm 0.01$	$0.07^{+0.09}_{-0.08} \begin{matrix} +0.01 \\ -0.03 \end{matrix}$	$-0.015 \sim -0.006$

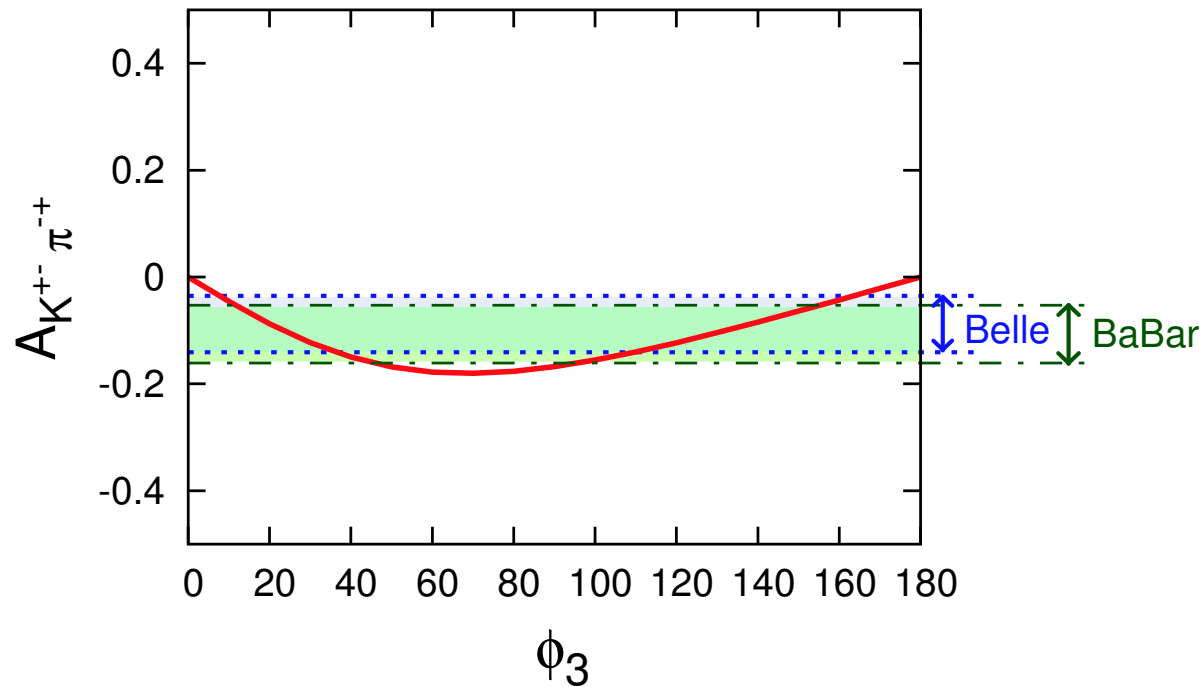
# PQCD Prediction for $A_{K^\mp\pi^\pm}$

## Direct CP asymmetry for $B \rightarrow K^\mp\pi^\pm$

Exp. :

BABAR	Belle	CLEO	Average
$-0.107 \pm 0.041 \pm 0.013$	$-0.088 \pm 0.035 \pm 0.018$	$-0.04 \pm 0.16 \pm 0.02$	$-0.095 \pm 0.028$

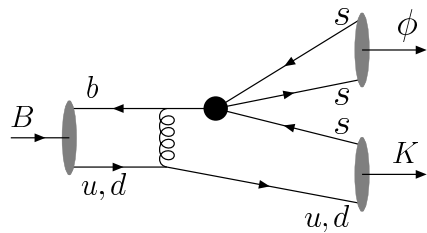
PQCD :



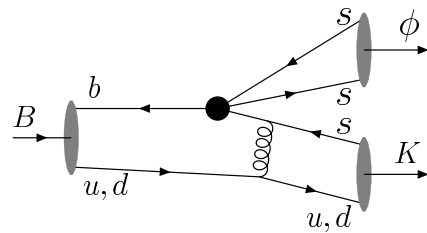
PQCD prediction is negative and in agreement with exp. data.

# $B \rightarrow \phi K$ Decays in the PQCD Approach

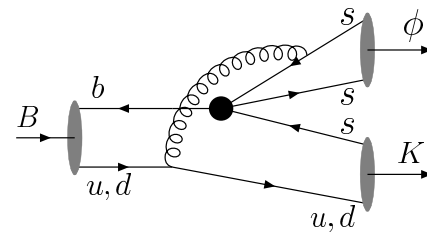
Mishima, Phys.Lett.B521,252,2001  
Chen, Keum, Li, PRD64,112002,2001



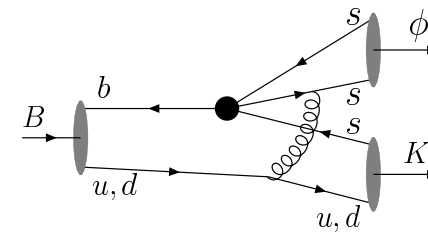
(a)



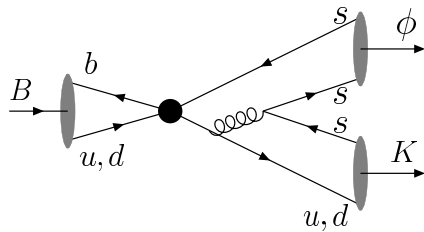
(b)



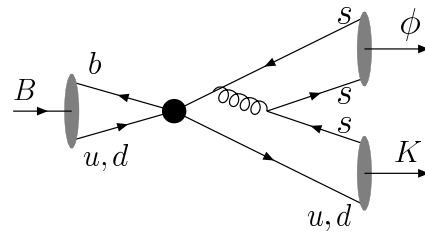
(c)



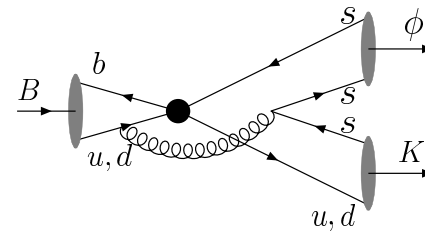
(d)



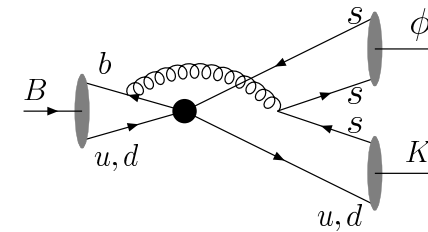
(e)



(f)



(g)



(h)

- (a) and (b) are dominant.
- Branching ratios are larger than those in factorization approaches.

Mode	Exp.(BABAR)	Exp.(Belle)	Result
$B^0 \rightarrow \phi K^0$	$(8.4_{-1.3}^{+1.5} \pm 0.5) \times 10^{-6}$	$(9.0_{-1.8}^{+2.2} \pm 0.7) \times 10^{-6}$	$(8.5_{-2.0}^{+3.0}) \times 10^{-6}$
$B^\pm \rightarrow \phi K^\pm$	$(10.0_{-0.8}^{+0.9} \pm 0.5) \times 10^{-6}$	$(9.4 \pm 1.1 \pm 0.7) \times 10^{-6}$	$(9.3_{-2.1}^{+3.1}) \times 10^{-6}$

$\Rightarrow$  PQCD predictions are in agreement with exp. data.

---

# 3. MSSM Contribution in $B$ Decays

# SUSY Contributions in $B$ Decays

---

We consider **supersymmetric contribution (MSSM)** in  $B$  decays.

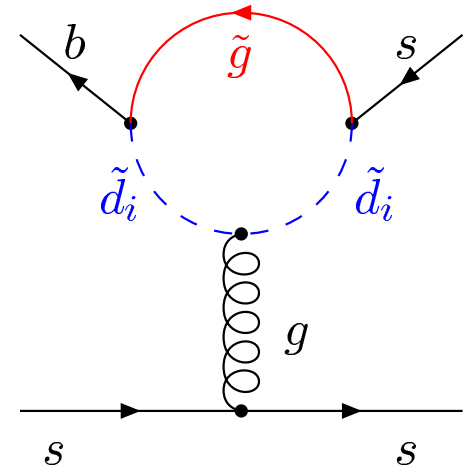
- There are new sources of **CP Violation** and **Flavor Changing Neutral Current**.
  - Gluino contributions
  - Chargino contributions
  - Neutralino contributions



# SUSY Contributions in $B$ Decays

We consider **supersymmetric contribution (MSSM)** in  $B$  decays.

- There are new sources of **CP Violation** and **Flavor Changing Neutral Current**.
  - **Gluino contributions**
  - ~~Chargino contributions~~
  - ~~Neutralino contributions~~

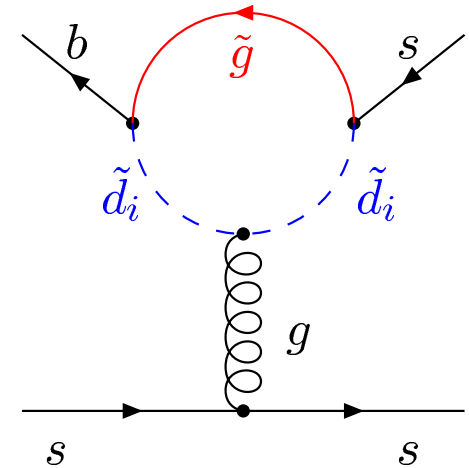


# SUSY Contributions in $B$ Decays

We consider **supersymmetric contribution (MSSM)** in  $B$  decays.

- There are new sources of **CP Violation** and **Flavor Changing Neutral Current**.

- **Glino contributions**
- ~~Chargino contributions~~
- ~~Neutralino contributions~~



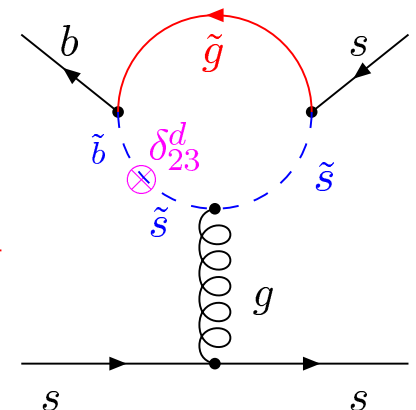
- We use **Mass Insertion Approximation**. (Hall, Kostelecky, Raby, NPB267,415,1986)

Off-diagonal elements in the squark mass matrix produce FCNC.

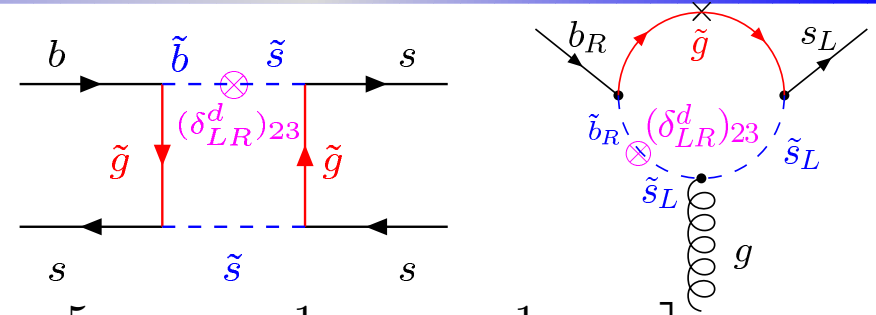
Squark mass matrix :

$$M_{\tilde{d}}^2 = \begin{pmatrix} m_{\tilde{d},LL}^2 & m_{\tilde{d},LR}^2 \\ m_{\tilde{d},RL}^2 & m_{\tilde{d},RR}^2 \end{pmatrix} \Rightarrow \delta_{LL,ij}^d \equiv \frac{(V_d^\dagger m_{\tilde{d},LL}^2 V_d)_{ij}}{m_{\tilde{q}}^2} \ll 1$$

$(i \neq j) \quad LL \rightarrow LR, RL, RR$



Glino contributions  
in Penguin and Magnetic Penguin :



$$\begin{aligned}
 C_3(M_S) &\simeq -\frac{\sqrt{2}\alpha_s^2}{4G_F V_{tb} V_{ts}^* m_{\tilde{q}}^2} (\delta_{LL}^d)_{23} \left[ -\frac{1}{9} B_1(x) - \frac{5}{9} B_2(x) - \frac{1}{18} P_1(x) - \frac{1}{2} P_2(x) \right] \\
 C_4(M_S) &\simeq -\frac{\sqrt{2}\alpha_s^2}{4G_F V_{tb} V_{ts}^* m_{\tilde{q}}^2} (\delta_{LL}^d)_{23} \left[ -\frac{7}{3} B_1(x) + \frac{1}{3} B_2(x) + \frac{1}{6} P_1(x) + \frac{3}{2} P_2(x) \right] \\
 C_5(M_S) &\simeq -\frac{\sqrt{2}\alpha_s^2}{4G_F V_{tb} V_{ts}^* m_{\tilde{q}}^2} (\delta_{LL}^d)_{23} \left[ \frac{10}{9} B_1(x) + \frac{1}{18} B_2(x) - \frac{1}{18} P_1(x) - \frac{1}{2} P_2(x) \right] \\
 C_6(M_S) &\simeq -\frac{\sqrt{2}\alpha_s^2}{4G_F V_{tb} V_{ts}^* m_{\tilde{q}}^2} (\delta_{LL}^d)_{23} \left[ -\frac{2}{3} B_1(x) + \frac{7}{6} B_2(x) + \frac{1}{6} P_1(x) + \frac{3}{2} P_2(x) \right] \\
 C_{7\gamma}(M_S) &\simeq \frac{\sqrt{2}\alpha_s \pi}{6G_F V_{tb} V_{ts}^* m_{\tilde{q}}^2} \left[ (\delta_{LL}^d)_{23} \frac{8}{3} M_3(x) + (\delta_{LR}^d)_{23} \frac{m_{\tilde{g}}}{m_b} \frac{8}{3} M_1(x) \right] \\
 C_{8g}(M_S) &\simeq \frac{\sqrt{2}\alpha_s \pi}{2G_F V_{tb} V_{ts}^* m_{\tilde{q}}^2} \left[ (\delta_{LL}^d)_{23} \left( \frac{1}{3} M_3(x) + 3M_4(x) \right) + (\delta_{LR}^d)_{23} \frac{m_{\tilde{g}}}{m_b} \left( \frac{1}{3} M_1(x) + 3M_2(x) \right) \right]
 \end{aligned}$$

$$x \equiv m_{\tilde{g}}^2 / m_{\tilde{q}}^2$$

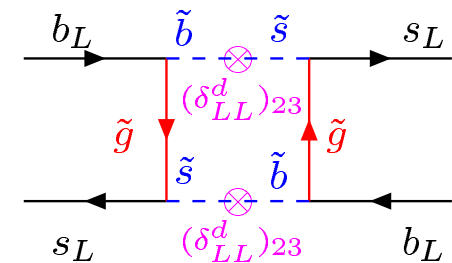
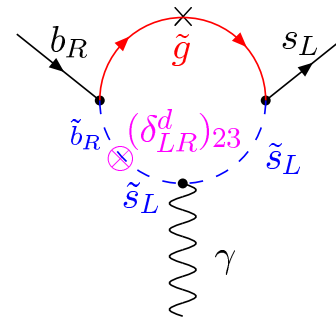
New Operators:  $C_i O_i \xrightarrow{L \leftrightarrow R} \tilde{C}_i \tilde{O}_i$

We consider single mass insertions:

- LR insertion :  $(\delta_{LR}^d)_{23}$
- RL insertion :  $(\delta_{RL}^d)_{23}$
- LL insertion :  $(\delta_{LL}^d)_{23}$
- RR insertion :  $(\delta_{RR}^d)_{23}$

and constrain them from  $b \rightarrow s$  FCNC processes:

- $\text{Br}(B \rightarrow X_s \gamma)$
- $A_{\text{CP}}(B \rightarrow X_s \gamma)$
- $B_s - \bar{B}_s$  Mixing:  $\Delta M_s$



$\implies$  The constraint from  $\text{Br}(B \rightarrow X_s \gamma)$  is strong.

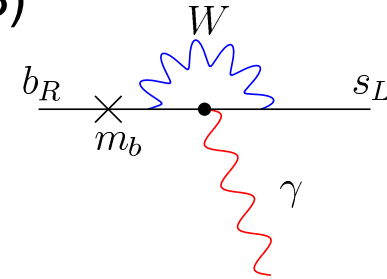
$$\text{Br}(B \rightarrow X_s \gamma) = (3.3 \pm 0.4) \times 10^{-4} \quad (\text{PDG2003})$$

$$\text{Br}(B \rightarrow X_s \gamma) \propto |C_{7\gamma}(m_b)|^2 + |\tilde{C}_{7\gamma}(m_b)|^2$$

$$C_{7\gamma}(m_b) = C_{7\gamma}^{\text{SM}}(m_b) + C_{7\gamma}^{\text{NP}}(m_b)$$

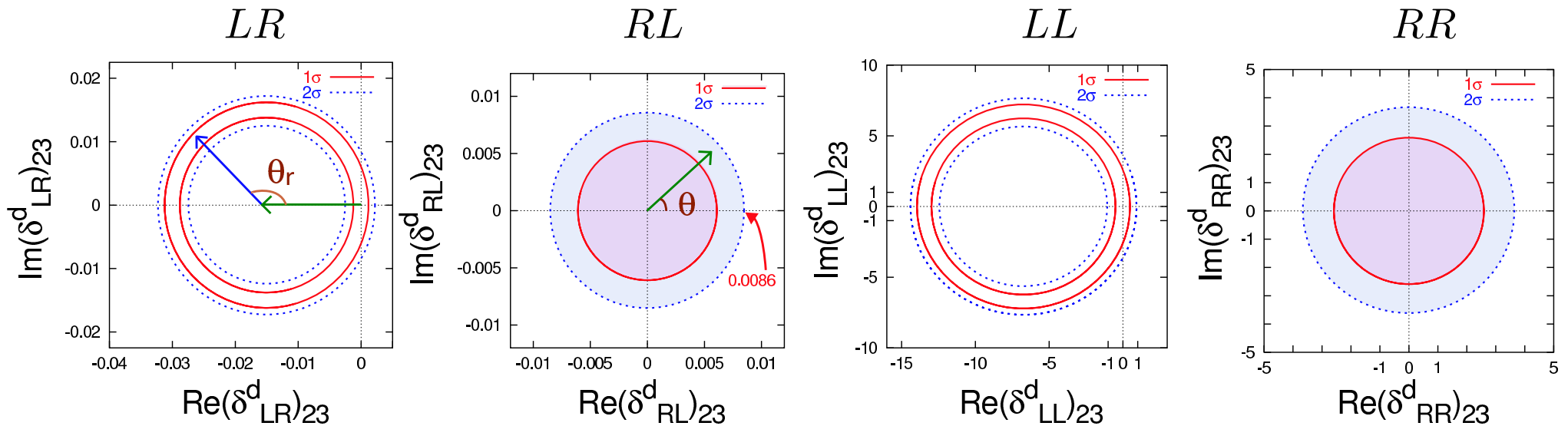
$$\tilde{C}_{7\gamma}(m_b) = \tilde{C}_{7\gamma}^{\text{NP}}(m_b)$$

$$C_{7\gamma}^{\text{NP}}(M_S) \simeq \frac{\sqrt{2}\alpha_s\pi}{6G_F V_{tb} V_{ts}^* m_{\tilde{q}}^2} \left[ (\delta_{LL}^d)_{23} \frac{8}{3} M_3(x) + (\delta_{LR}^d)_{23} \frac{m_{\tilde{g}}}{m_b} \frac{8}{3} M_1(x) \right]$$



$m_{\tilde{g}} = m_{\tilde{q}} = 500 \text{ GeV}$

We take  $(\delta^d)_{23}$  in  $2\sigma$  region.



---

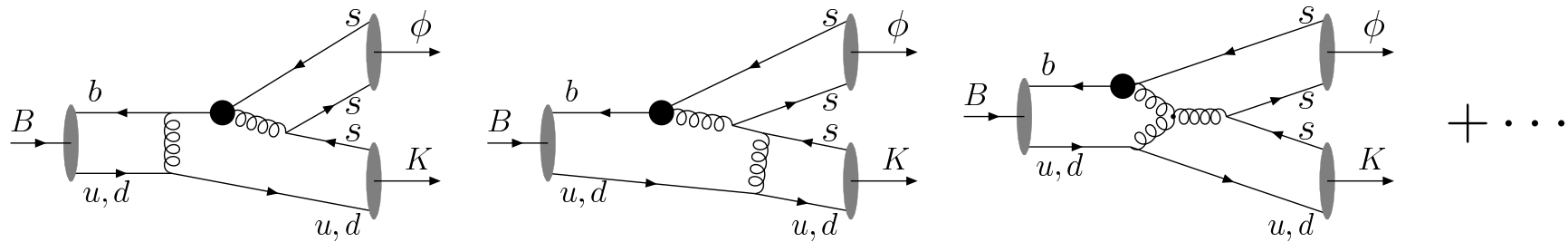
# 4. MSSM Effects

on  $B \rightarrow \phi K$  Decays

---

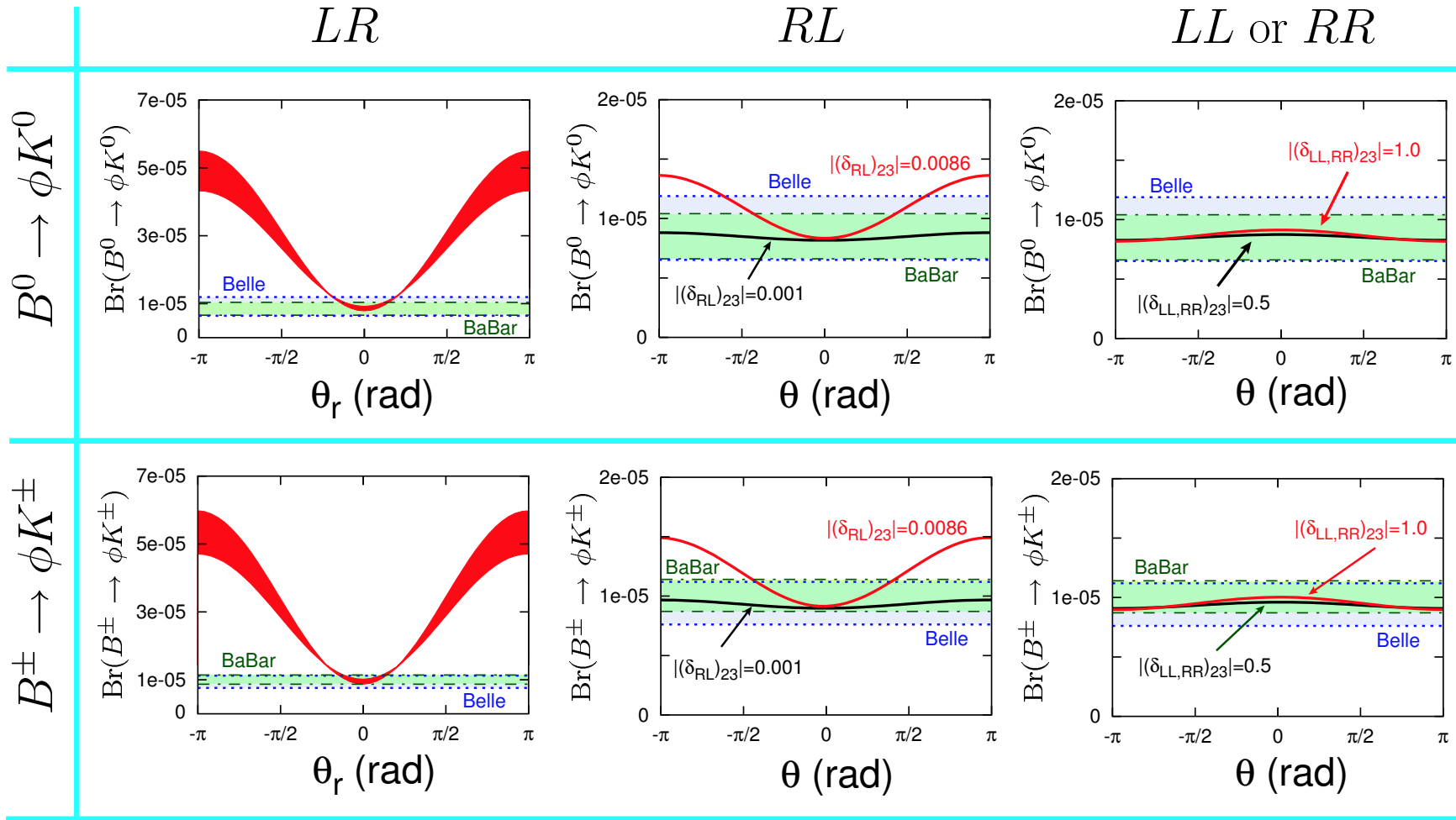
Magnetic Penguin is important to a search for New Physics.

$$O_{8g} = -\frac{g_s}{8\pi^2} m_b (\bar{s}_i \sigma^{\mu\nu} (1 + \gamma_5) T_{ij}^a b_j) G_{\mu\nu}^a$$



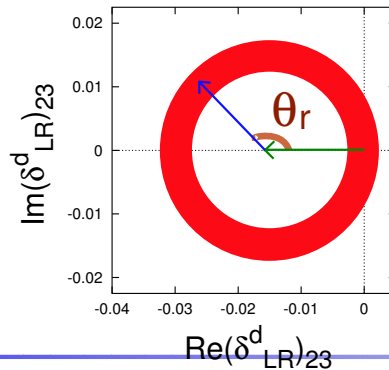
- Above diagrams are dominant.
  - Strong phase comes from their absorptive part.
  - A relative strong phase between Penguin and MP is large.
- ⇒ If there is a new CP-violating phase in MP, then  $A_{\phi K}$  might change significantly from the SM prediction.

# Branching Ratios



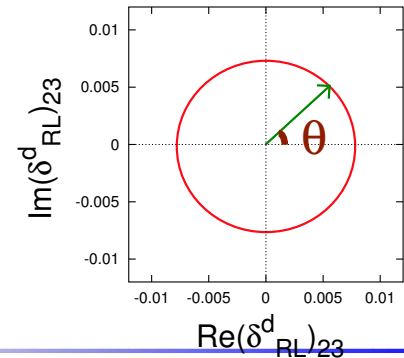
$LR$ :

$$(\delta_{LR}^d)_{23} = -0.015 + r e^{i\theta_r}$$



$RL, LL, RR$ :

$$(\delta^d)_{23} = |(\delta^d)_{23}| e^{i\theta}$$





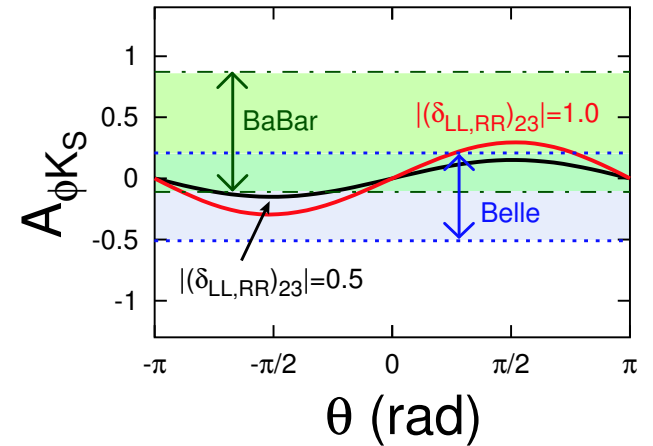
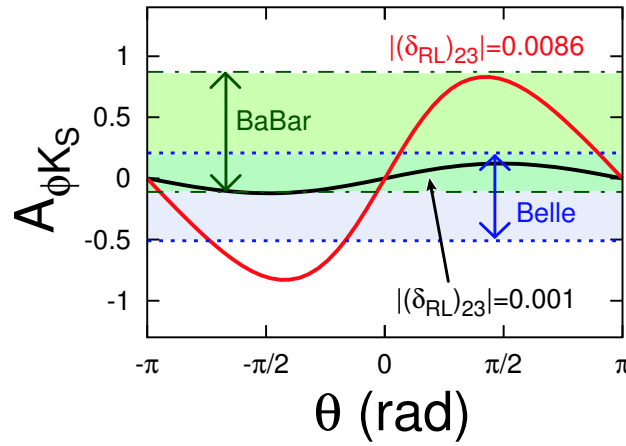
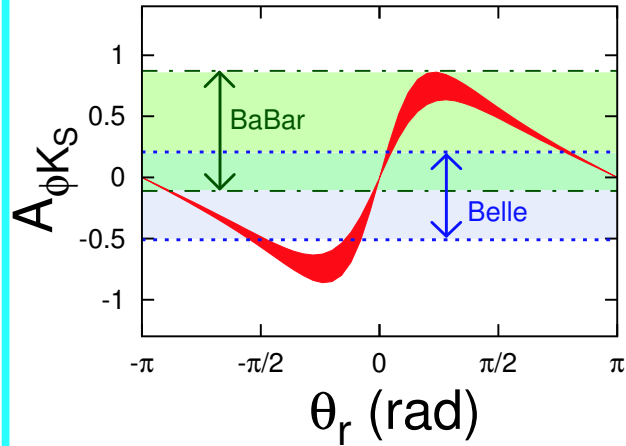
# Direct CP Asymmetries $A_{\phi K}$

$LR$

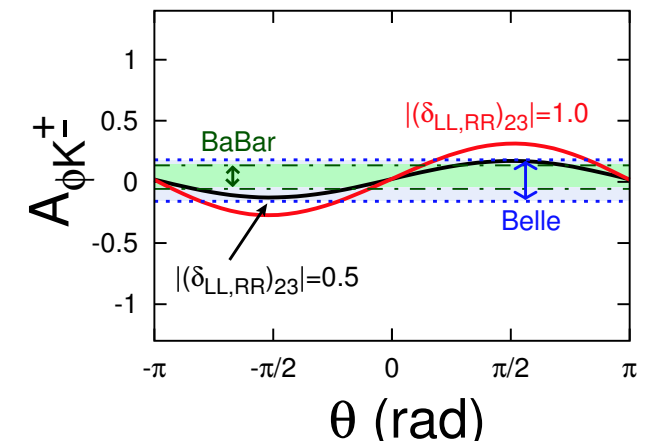
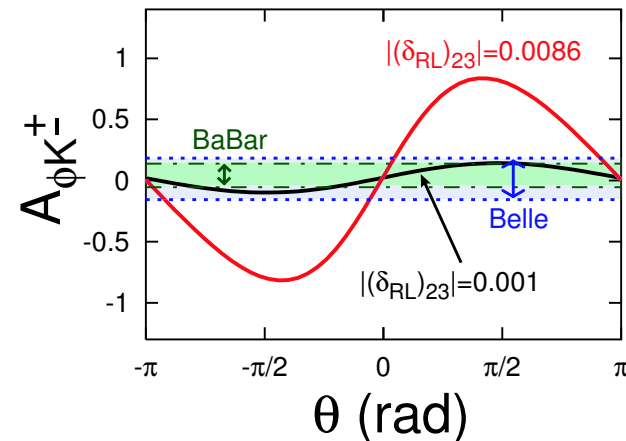
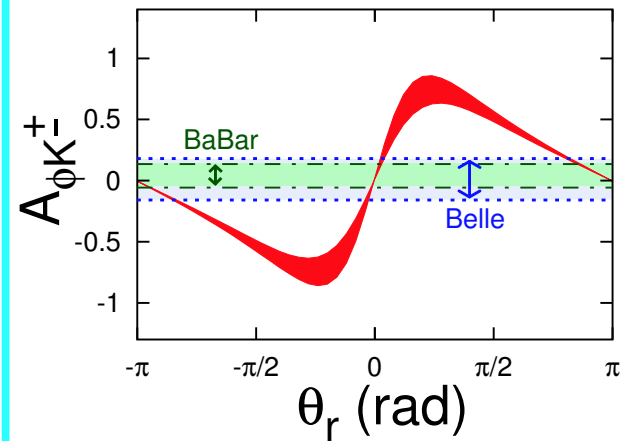
$RL$

$LL$  or  $RR$

$B^0 \rightarrow \phi K^0$



$B^\pm \rightarrow \phi K^\pm$



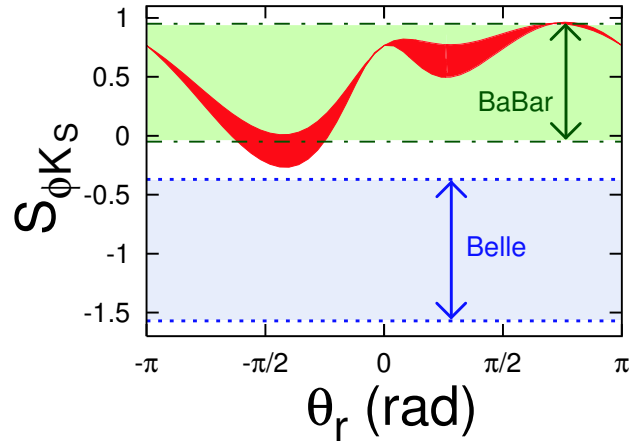
●  $-0.85 \lesssim A_{\phi K} \lesssim 0.85$

●  $A_{\phi K_S} \simeq A_{\phi K^\pm}$

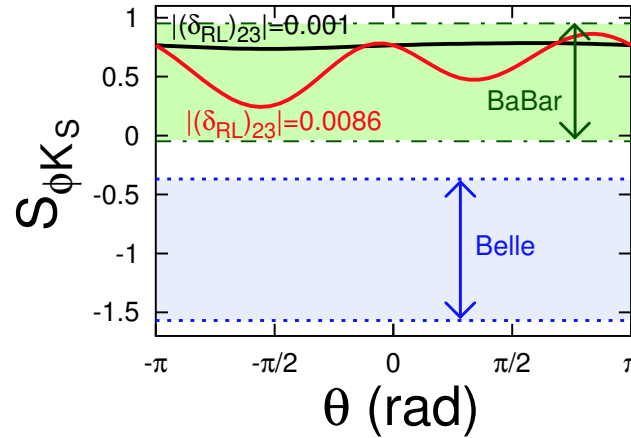
$A_{\phi K}$  are generated from the interference between penguin and magnetic-penguin.

# Indirect CP Asymmetries $S_{\phi K}$

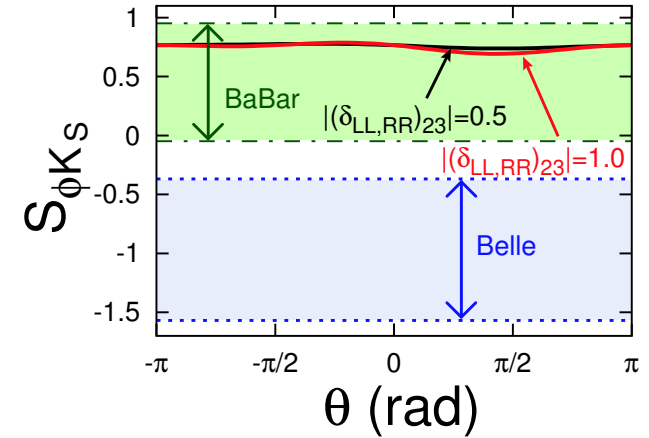
$LR$



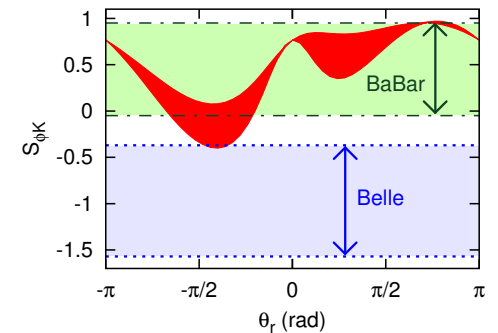
$RL$



$LL$  or  $RR$



- $S_{\phi K_S} \gtrsim -0.28$
- It is difficult to explain the current Belle data.
- We attempted to maximize MSSM effects,  $S_{\phi K_S}$  had been larger than about  $-0.4$ .



$$m_{\tilde{g}} = 200 \text{ GeV}$$

$$m_{\tilde{q}} = 2 \text{ TeV}$$

## 5. Summary

---

- We calculated MSSM contributions with a mass insertion in  $B \rightarrow \phi K$  decays using the PQCD approach.
  - $-0.85 \lesssim A_{\phi K} \lesssim 0.85$
  - $A_{\phi K_S} \simeq A_{\phi K^\pm}$
  - $S_{\phi K_S} \gtrsim -0.28$
- It is difficult to explain the current Belle data.
- Experimental Error (only statistical)
  - 2003 :  $140 \text{ fb}^{-1}$   $\delta S_{\phi K_S} \sim 0.50$
  - $\sim 2005$  :  $300 \text{ fb}^{-1}$   $\delta S_{\phi K_S} \sim 0.34$
  - 2008  $\sim$  :  $1 \text{ ab}^{-1}$   $\delta S_{\phi K_S} < 0.1$
- We need more theoretical study in  $B \rightarrow \phi K$  decays.

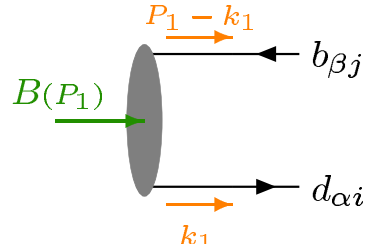


---

# Backup

# Meson Wave Functions

$B$  meson wave function:

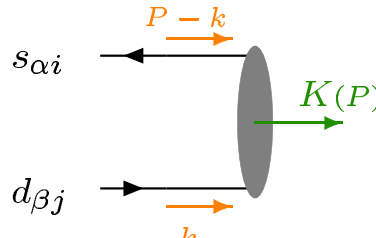


$$\equiv \langle 0 | \bar{b}_{\beta j}(0) d_{\alpha i}(z) | B(P_1) \rangle$$

$i, j$  : color indices  
 $\alpha, \beta$  : spinor indices

$$= \frac{\delta_{ij}}{\sqrt{2N_c}} \int dx_1 d^2 \mathbf{k}_{1T} e^{-i(x_1 P_1^- z^+ - \mathbf{k}_{1T} \mathbf{z}_T)} [(\not{P}_1 + M_B) \gamma_5 \phi_B(x_1, \mathbf{k}_{1T})]_{\alpha\beta}$$

$K$  meson wave function:



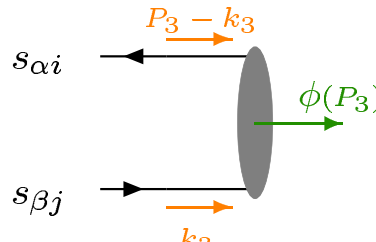
$$\equiv \langle K(P_2) | \bar{d}_{\beta j}(0) s_{\alpha i}(z) | 0 \rangle$$

$$m_{0K} = \frac{M_K^2}{m_d + m_s} = 1.7 \text{ GeV},$$

$$n_\mu \equiv z_\mu / z^-, \quad v_\mu \equiv P_{2\mu} / P_2^+$$

$$= \frac{-i\delta_{ij}}{\sqrt{2N_c}} \int_0^1 dx_2 e^{ix_2 P_2 \cdot z} \gamma_5 \left[ \not{P}_2 \phi_K^A(x_2) + m_{0K} \phi_K^P(x_2) + m_{0K} (\not{v} \not{h} - 1) \phi_K^T(x_2) \right]_{\alpha\beta}$$

$\phi$  meson wave function:



$$\equiv \langle \phi(P_3) | \bar{s}_{\beta j}(0) s_{\alpha i}(z) | 0 \rangle$$

$$= \frac{\delta_{ij}}{\sqrt{2N_c}} \int_0^1 dx_3 e^{ix_3 P_3 \cdot z} \left[ M_\phi \not{\epsilon}_\phi \phi_\phi(x_3) + \not{\epsilon}_\phi \not{P}_3 \phi_\phi^t(x_3) + M_\phi \phi_\phi^s(x_3) \right]_{\alpha\beta}$$

# Meson Distribution Amplitudes

$$\phi_B(x, b) = N_B x^2 (1-x)^2 \exp \left[ -\frac{1}{2} \left( \frac{x M_B}{\omega_B} \right)^2 - \frac{\omega_B^2 b^2}{2} \right], \quad \omega_B = 0.36 \sim 0.44 \text{ GeV}$$

$$\phi_K^A(x) = \frac{f_K}{2\sqrt{2N_c}} 6x(1-x) \left[ 1 + a_1 C_1^{\frac{3}{2}}(1-2x) + a_2 C_2^{\frac{3}{2}}(1-2x) \right], \quad \rho_K = (m_d + m_s)/M_K$$

$$\phi_K^P(x) = \frac{f_K}{2\sqrt{2N_c}} \left[ 1 + \left( 30\eta_3 - \frac{5}{2}\rho_K^2 \right) C_2^{\frac{1}{2}}(1-2x) - 3 \left\{ \eta_3\omega_3 + \frac{9}{20}\rho_K^2(1+6a_2) \right\} C_4^{\frac{1}{2}}(1-2x) \right]$$

$$\phi_K^T(x) = \frac{f_K}{2\sqrt{2N_c}} (1-2x) \left[ 1 + 6 \left( 5\eta_3 - \frac{1}{2}\eta_3\omega_3 - \frac{7}{20}\rho_K^2 - \frac{3}{5}\rho_K^2 a_2 \right) (1-10x+10x^2) \right]$$

$C_n^\nu(x)$  : Gegenbauer polynomial

$$a_1 = -0.18, \quad a_2 = 0.16, \quad \eta_3 = 0.015, \quad \omega_3 = -3.0$$

Light-cone QCD sum rules

Ball, JHEP.01:010,1999

$$\phi_\phi(x) = \frac{f_\phi}{2\sqrt{2N_c}} 6x(1-x) + \frac{3}{2}\delta_+ \left\{ 1 - (1-2x) \log \frac{1-x}{x} \right\}$$

$$\phi_\phi^t(x) = \frac{f_\phi^T}{2\sqrt{2N_c}} \left[ 3(1-2x)^2 + \frac{35}{4}\zeta_3^T \{ 3 - 30(1-2x)^2 + 35(1-2x)^4 \} \right]$$

$$\phi_\phi^s(x) = \frac{f_\phi^T}{4\sqrt{2N_c}} \left[ (1-2x) \left\{ 6 + 9\delta_+ + 140\zeta_3^T (1-10x+10x^2) \right\} + 3\delta_+ \log \frac{x}{1-x} \right]$$

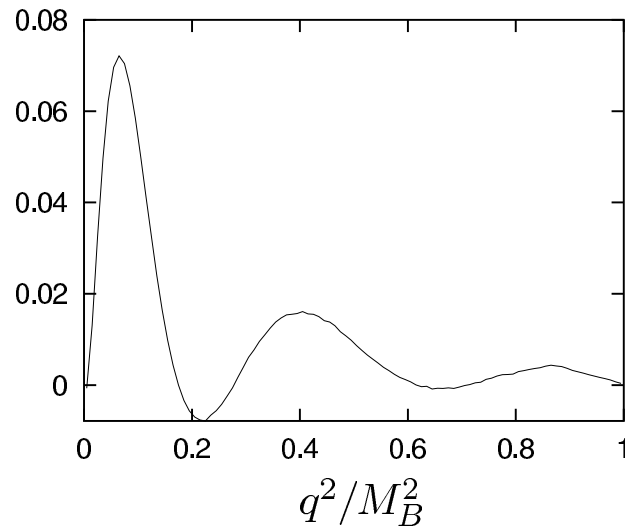
$$\zeta_3^T = 0.024, \quad \delta_+ = 0.46$$

Light-cone QCD sum rules

Ball, Braun, Koike, Tanaka, Nucl.Phys.B529:323,1998

# $q^2$ -dependence of Magnetic Penguin Amplitudes

The distribution of  $q^2$  for  $\text{Re}\mathcal{M}_a^{MP}$  :



$$\Rightarrow \langle q^2 \rangle = 6.3 \text{ GeV}^2 \\ \sim M_B^2/4$$

The shape of this graph is not simple.