SM Uncertainties in the B->X_s+gamma Branching Fraction and Spectrum

Alexander Mitov
University of Hawaii

Outline

- The process B->X_s+gamma.
- Branching Fraction: application, status and future prospects.
- The photon spectrum: application, status.
- New NNLO results for the photon spectrum.
- Conclusions.
- Work in progress. In collaboration with Kirill Melnikov.

The process B->X_s+gamma

We can split the information about this process in two (not uncorrelated) pieces:

The Branching Fraction BR[B->X_s+gamma]. It is:

- Loop induced,
- Sensitive to Physics at the electroweak scale,
- An important constraint for many models of New Physics.

The shape of the photon spectrum. It is:

- Largely insensitive to New Physics,
- Crucial for relating the “theory” BF to the experimental measurements.
- Opens a window for determining various pert. and non-pert. parameters.

The Branching Fraction BR[B->X_s+gamma].

Current status:

Currently known to NLO:

(1998) ”Practically complete” (Chetyrkin, Misiak and Munz).

(2001) ”Essentially” known (Gambino and Misiak).

(2002) ”Strictly complete” (Buras, Czarnecki, Misiak and Urban).

Current developments – inclusion of the NNLO effects:

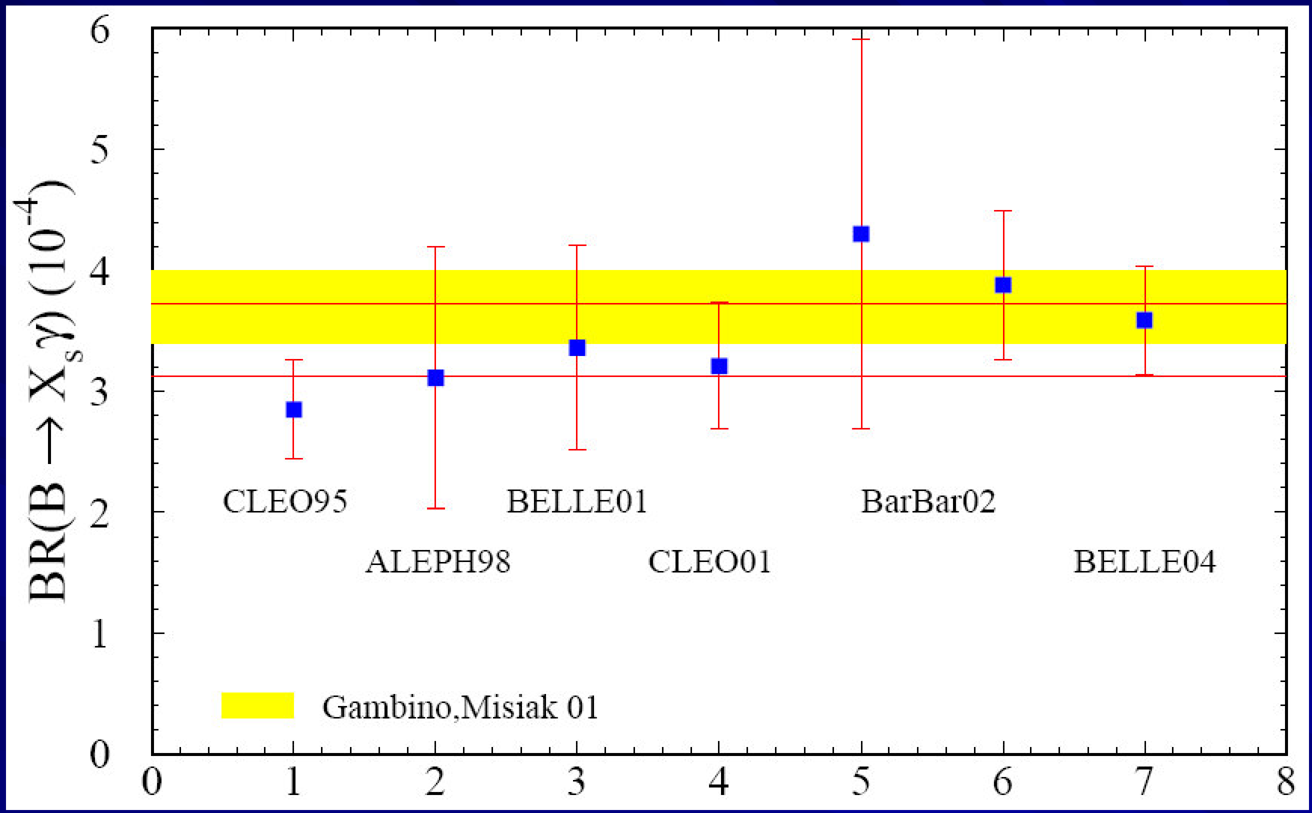
@ Evaluation of the anomalous dimensions and running of the Wilson coefficients to scale ~ m_b.

@ Evaluation of the matching conditions at a scale ~ M_W

@ Evaluation of the matrix elements.

! The NNLO inclusions are needed to reduce large uncertainties due to charm mass effects induced from the four-fermion operators.

The Branching Fraction BR[B->X_s+gamma].



M. Steinhauser , hep-ph/0406240.

At present very good agreement between theory and experiment!

The photon spectrum in B->X_s+gamma.

What is the useful information in this observable?

1) It relates the values of the BF's measured (or evaluated) with different cuts on the energy of the measured photon.

Define the ratio:
$$R(E_{cut}) = \frac{BR[E>E_{cut}]}{BR[E>m_b/20]}$$

That is very important, since:

The experimental measurements are always with a lower cut: E_cut=1.8--2.1 GeV.

The theory predictions are usually needed for the fully inclusive Branching Fraction.

The photon spectrum in B->X_s+gamma.

What is the useful information in this observable (cont.)?

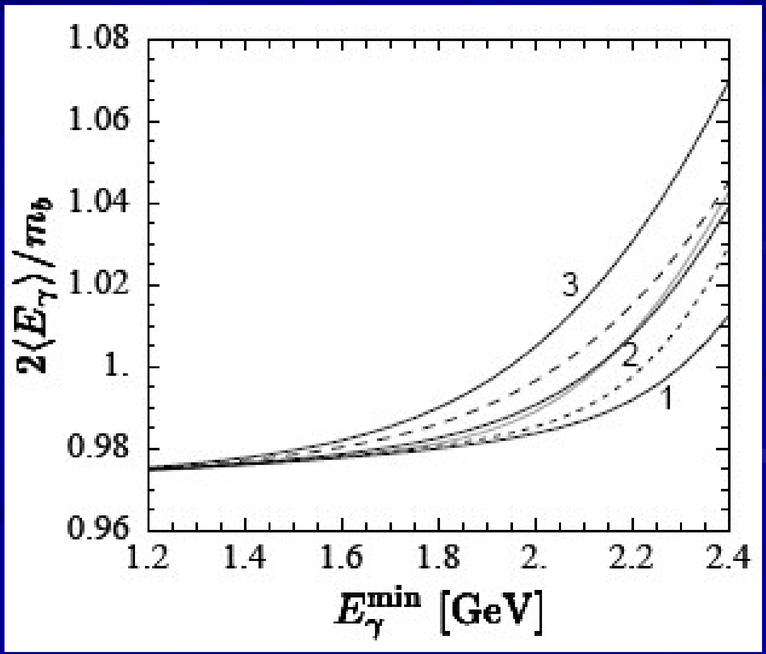
2) Needed for the extraction of the moments of the photon spectrum.

Note: <E>=m_b(1/2+O(a_s)+O(non-pert.)).

Therefore: from a precise measurement one can extract m_b as well as non-perturbative parameters.

However: such extraction depends very strongly on the value of the imposed cut!

Note: very strong sensitivity to m_b for low E_cut.
for large E_cut – low sensitivity to m_b but sensitivity to other shape function parameters.



Kagan and Neubert, hep-ph/9805303

The photon spectrum in B->X_s+gamma.

Current state of the art for theory:

@ The NLO contributions from all operators and their mixing is accounted for. The dominant effect is from O_7 with small contributions from O_2 and O_8.

!!! That is very different from the total rate!

@ Some NNLO contributions (from O_2,7,8) ~a_s^2 β_0 (the so-called BLM terms) are also accounted for:

Therefore, non-O_7 operators are only a few per-cent effect!

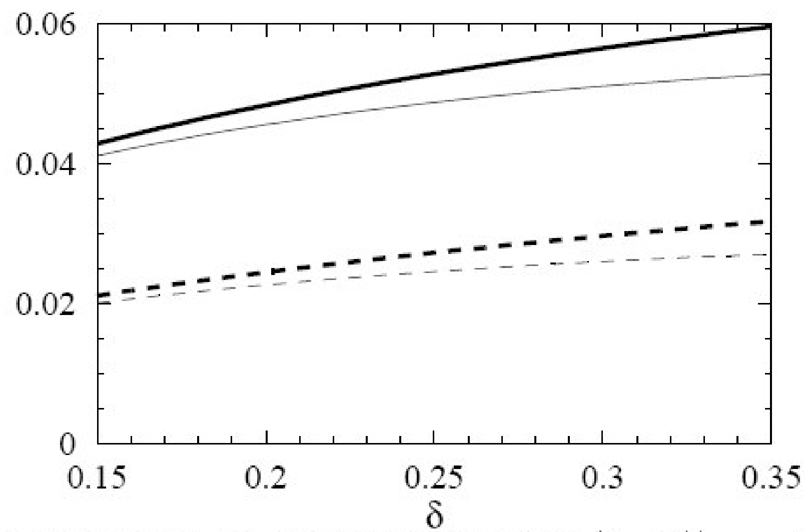


FIG. 3. The sum of the 77, 22, 78, and 27 contributions to $\langle 1-x_b\rangle|_{x_b>1-\delta}$ at order α_s (thick dashed curve) and $\alpha_s^2\beta_0$ (thick solid curve). The thin curves show the 77 contribution only.

Ligeti, Luke, Manohar and Wise, Hep-ph/9903305.

The photon spectrum in B->X_s+gamma.

However:

1) The BLM effects are not 100% genuine NNLO effect; can be absorbed by the choice of scale (cut dependent!)
2) Recently - large differences of the size of the perturbative uncertainty in observables related to the spectrum.

Therefore:

To clarify this we calculate the dominant (genuinely) NNLO effect!

The photon spectrum in B->X_s+gamma.

What we do:

We evaluate analytically the complete O(a_s^2) contribution from the operator O_7 (recall: O_7 gives by far the dominant contribution to the spectrum).

What we achieve with this calculation:

1) We can explicitly check how well the BLM term approximates the spectrum (never checked before at the level of spectrum!).
2) We can calculate (vs. estimate) the true NNLO uncertainty.
3) That will resolve the issue for the size of the theoretical uncertainty.

The photon spectrum in B->X_s+gamma.

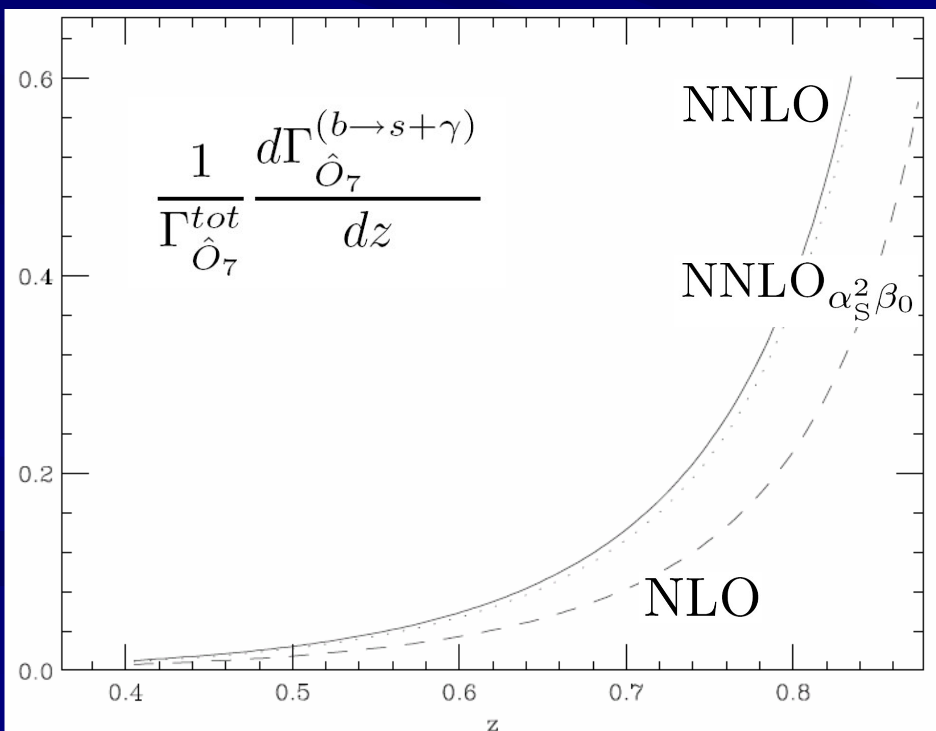
First genuine NNLO results for the photon spectrum

(the analytical expressions are long; will not present them here)

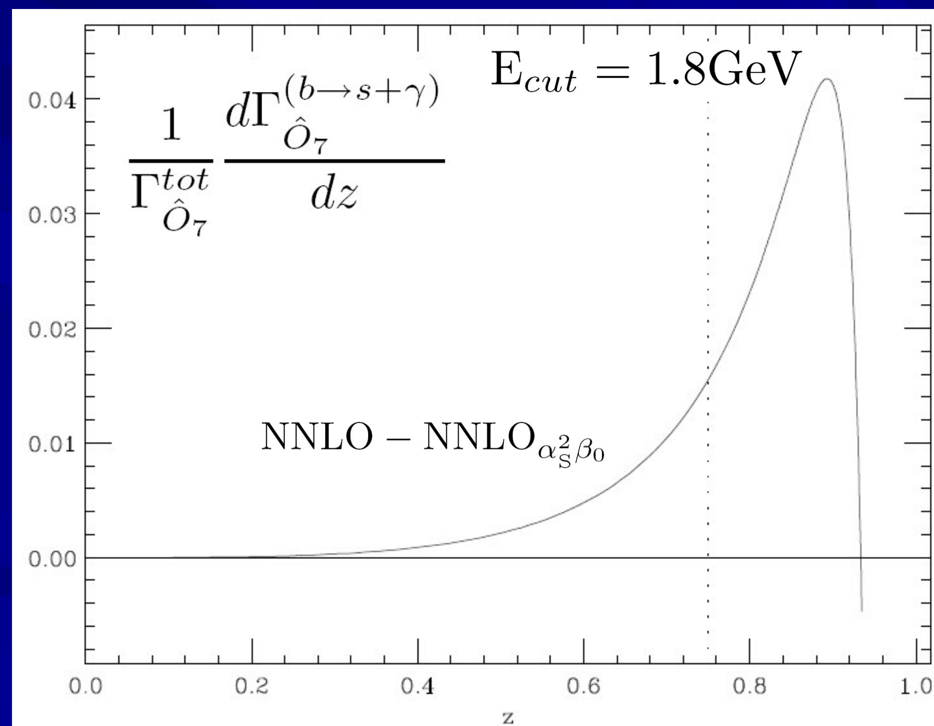
$$z = \frac{2E_\gamma}{m_b}$$

$$\int_0^1 \frac{1}{\Gamma^{tot}_{\hat{O}_7}} \frac{d\Gamma^{(b\to s+\gamma)}_{\hat{O}_7}}{dz} dz = 1$$

$$\Gamma^{tot}_{\hat{O}_7}(\mu_F) = \Gamma^{(0)}_{\hat{O}_7}(\mu_F)\left(1 + \frac{\alpha_S}{\pi} C_F \left(\ln\left(\frac{m_b}{\mu_F}\right) + \frac{4}{3} - \frac{\pi^2}{3}\right) + \mathcal{O}(\alpha_S^2)\right)$$



The genuine NNLO effect



The photon spectrum in B->X_s+gamma.

Contribution of the new genuine NNLO effect to observables:

To the percentage of events above the cut R(1.8 GeV):

ΔR(1.8GeV) = -0.003 (μ=4GeV) … -0.006 (μ=1.6 GeV)

Compare to the perturbative uncertainties from previous results:

R(1.8GeV) = 0.958 +0.013 -0.029 (Kagan, Neubert 1998),

R(1.8GeV) = 0.952 +0.013 -0.029 (Gambino, Misiak 2001),

R(1.8GeV) = 0.95 +0.01 -0.01 (Bigi, Uraltsev 2002).

R(1.8GeV) = 0.89 +0.06 -0.07[pert] ±0.01[param] (Neubert 2004).

To the average energy <E> with a lower cut E>1.8 GeV:

Δ<E>(E>1.8GeV) = 0.01 (μ=4GeV) … 0.02 (μ=1.6 GeV)

Compare to the perturbative uncertainties from previous results:

<E>(1.8GeV) = 2.305 GeV (Benson,Bigi,Uraltsev 2004),

<E>(1.8GeV) =(2.27+0.05-0.07[pert])GeV ±δm ± δλ_1 (Neubert 2004)

<E>(1.8GeV) = 2.292 ±0.026 ±0.034 GeV (Belle 2004).

Our result hints towards the most optimistic estimates for the (perturbative) theoretical error!

Conclusions

1) I have reviewed the current status of the uncertainties of observables related to the photon spectrum in B->X_s+gamma.
2) I have presented new *analytical* results for the dominant NNLO contribution to the photon spectrum in that process.
3) Our result represents first explicit check of the BLM approximation at the level of the spectrum.
4) The effect of the genuinely new NNLO contribution to the observables like the average photon energy and percentage of events above the energy cut is smaller but comparable in size to the most optimistic estimates of the theoretical uncertainties.

Recommendations for the experiment:

1) A decrease in the theoretical uncertainties makes more precise measurements very desirable.
2) Precise measurements of moments as a function of the cut will be extremely valuable in the range below 2.1 GeV
3) One can use such results to extract non-perturbative parameters and to study in detail the relevance of the shape function in that region.