Irreducible theory errors in α and γ extraction

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Motivation

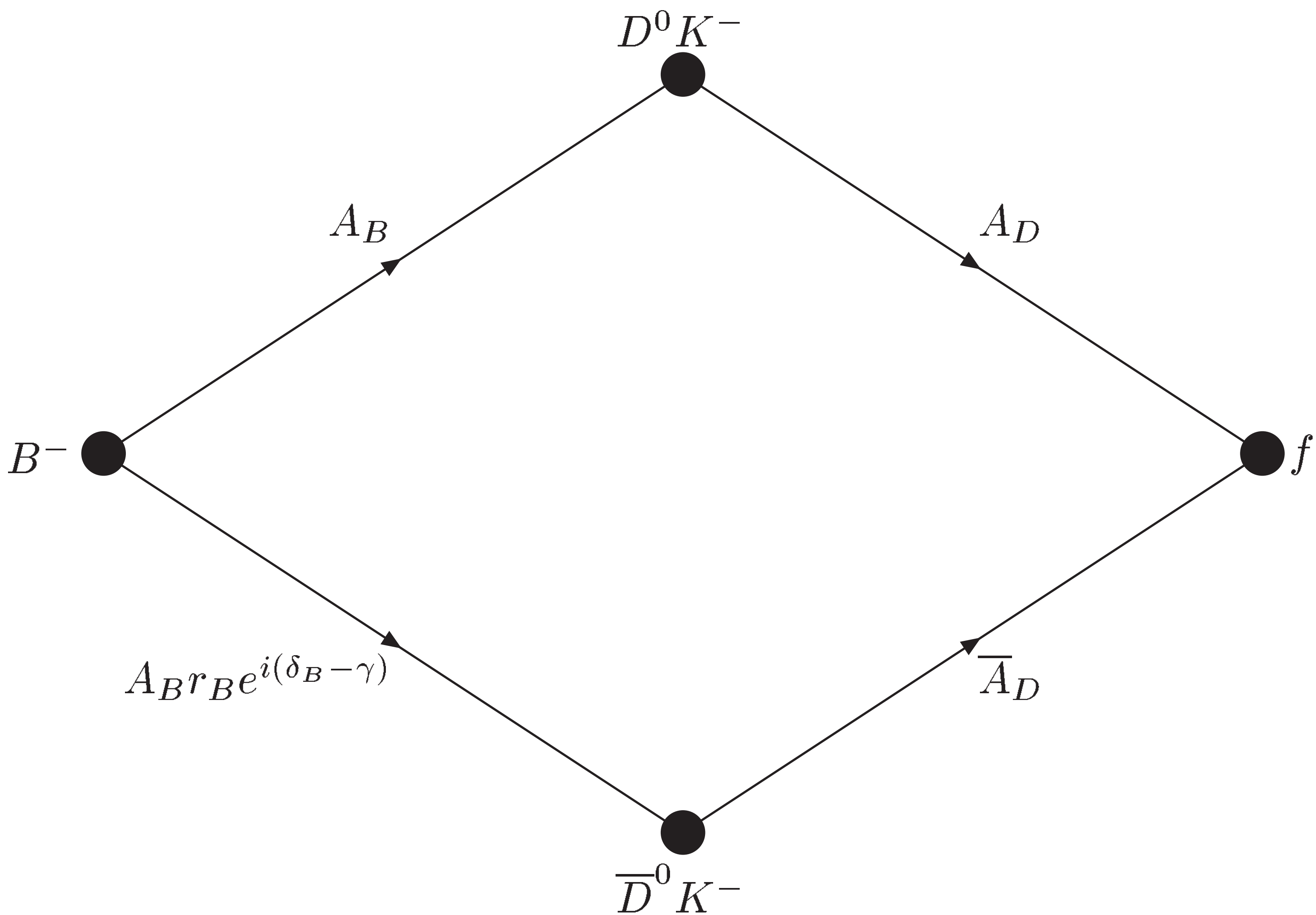
- Assume infinite statistics, what is the ultimate error on γ and α?
- Will discuss only the (theoretically) most precise methods

Outline

- γ from $B \to DK$
- α from $B \to \pi\pi, \rho\rho, \rho\pi$
- conclusions

$$B^{\pm} \to DK^{\pm}$$

graphically...



Different methods

methods can be grouped by the choice of final state f

- CP- eigenstate (e.g. $K_S\pi^0$) Gronau, London, Wyler (1991)
- flavor state (e.g. $K^+\pi^-$) Atwood, Dunietz, Soni (1997)
- singly Cabibbo suppressed (e.g. $K^{*+}K^-$) Grossman, Ligeti, Soffer (2002)
- many-body final state (e.g. $K_S\pi^+\pi^-$) Giri, Grossman, Soffer, JZ (2003)

other extensions:

- many body B final states (e.g. $B^+ \to DK^+\pi^0$) Aleksan, Petersen, Soffer (2002)
- use D^{0*} in addition to D^0
- use self tagging D^{0**} Sinha (2004)
- neutral B decays (time dependent and time-integrated) many refs.

Theory errors

- CP conserving $D - \bar{D}$ mixing does not change the methods
- CP violation in D sector the only uncertainty (!)
  - in SM $\lambda^6 \sim 10^{-4}$ suppressed
  - only relevant if beyond SM CP viol. in D
  - is it present? compare (time integrated) D^0 and $\bar{D}^0$ decays to f, $\bar{f}$

Including CP viol. in D

- enters in two ways
  - direct CP viol. $A(D^0 \to f) \neq A(\bar{D}^0 \to \bar{f})$
  - through $D - \bar{D}$ mixing, $q/p \neq 1$
- can it be included in the analysis?

first focus on 2-body final states with $f \neq \bar{f}$

- most general parametrization of direct CP viol.

$$A(D^0 \to f) = A_f + B_f, \quad A(\bar{D}^0 \to \bar{f}) = A_f - B_f$$

$$A(D^0 \to \bar{f}) = A_{\bar{f}} + B_{\bar{f}}, \quad A(\bar{D}^0 \to f) = A_{\bar{f}} - B_{\bar{f}}$$

Enough info?

- k different channels $(f, \bar{f})$
- $8k$ observables:

$$\Gamma(D^0 \to f), \;\; \Gamma(D^0 \to \bar{f}), \;\; \Gamma(\bar{D}^0 \to f), \;\; \Gamma(\bar{D}^0 \to \bar{f})$$

$$\Gamma(B^\pm \to f_D K^\pm), \;\; \Gamma(B^\pm \to \bar{f}_D K^\pm)$$

- $7k + 6$ unknowns:
  - $7k$ channel specific:
    4 magnitudes and 3 relative phases for each channel

$$A_f, A_{\bar{f}}, B_f, B_{\bar{f}}$$

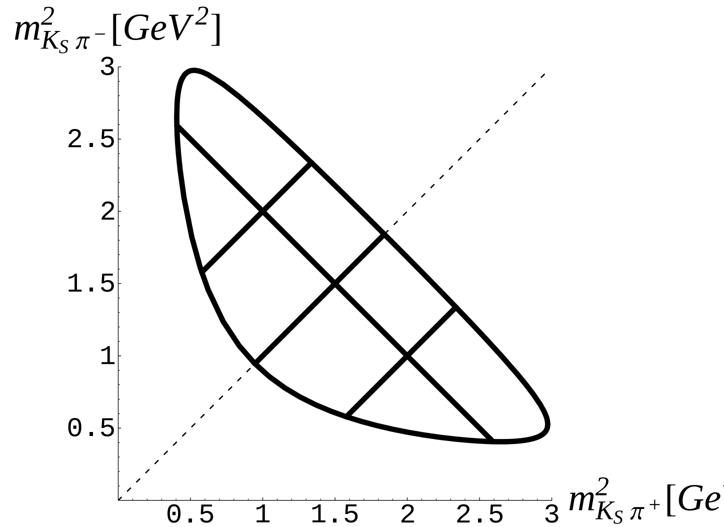
  - 6 common real parameters:

$$\gamma, A_B, r_B, \delta_B, \left(\frac{q}{p} - \frac{p}{q}\right)^* (x + iy)$$

$\boxed{k \geq 6 \text{ general analysis possible}}$

Multibody decay $B^\pm \to (K_S\pi^+\pi^-)_D K^\pm$

- $2k$ bins in the Dalitz plot placed $\pi^+ \leftrightarrow \pi^-$ symmetrically



- too many unknowns

$$T_i, T_{\bar i}, \int_i A_f A^*_{\bar f} = c_i + s_i$$

$$\int_i A_f B^*_f, \int_{\bar i} A_{\bar f} B^*_{\bar f}, \int_i A_f B^*_{\bar f}, \int_{\bar i} A_{\bar f} B^*_f$$

$\boxed{8k \text{ observ.} \Leftrightarrow 12k+6 \text{ unknowns}}$

- model independent method possible if $B_f = 0$, even for $q/p \neq 1$
- B_f can be fit to BW forms

Theory errors in α extraction

- $B(t) \to \pi\pi$
- $B(t) \to \rho\rho$
- $B(t) \to \rho\pi$

Isospin breaking

- the most useful methods for α extraction use isospin relations
- isospin breaking the limiting factor for precision measurements
- typical effect of isospin breaking
$$\sim (m_u - m_d)/\Lambda_{QCD} \sim \alpha_0 \sim 1\%$$
- Questions:
  - Are the isospin breaking effects that we can calculate of this order?
  - Does any of the methods fare better?

Manifestations of isospin breaking

- sources of isospin breaking
  - d and u charges different
  - $m_u \neq m_d$

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- sources of isospin breaking
  - d and u charges different
  - $m_u \neq m_d$
- extends the basis of operators to EWP $Q_{7,\dots,10}$
- mass eigenstates do not coincide with isospin eigenstates: $\pi - \eta - \eta'$ and $\rho - \omega$ mixing
- reduced matrix elements between states in the same isospin multiplet may differ e.g.

$$\langle \pi^+\pi^-|Q_1|B^0\rangle \neq \frac{1}{\sqrt{2}}\langle \pi^+\pi_3|Q_1|B^0\rangle$$

- may induce $\Delta I = 5/2$ operators not present in H_W

Electroweak penguins

- neglecting $Q_{7,8}$

Neubert, Rosner; Gronau, Pirjol, Yan; Buras, Fleischer (1999)

$$H^{\Delta I=3/2}_{\text{eff,EWP}} = -\frac{3}{2}\frac{C_9+C_{10}}{C_1+C_2}\frac{V^*_{tb}V_{td}}{V^*_{ub}V_{ud}}H^{\Delta I=3/2}_{\text{eff,c}-\text{c}}$$

$$\Rightarrow \boxed{\delta\alpha = (1.5 \pm 0.3 \pm 0.3)^\circ}$$

conservatively $\sim 2(|c_7|+|c_8|)/(|c_9|) < 0.2$

- the same shift in $\pi\pi$, $\rho\rho$ and $\rho\pi$ systems

$\pi^0 - \eta - \eta'$ mixing

- π^0 w.f. has η, η' admixtures

$$|\pi^0\rangle = |\pi_3\rangle + \epsilon|\eta\rangle + \epsilon'|\eta'\rangle$$

where $\epsilon = 0.017 \pm 0.003,\ \epsilon' = 0.004 \pm 0.001$ Kroll (2004)

- GL triangle relations in $B \to \pi\pi$ no longer hold

$$A_{+-} + \sqrt{2}A_{00} - \sqrt{2}A_{+0} \neq 0$$
$$\bar{A}_{+-} + \sqrt{2}\bar{A}_{00} - \sqrt{2}\bar{A}_{+0} \neq 0$$

- previous analysis Gardner (1999)
  - estimated using generalized factorization
  - obtained $\Delta\alpha \sim 0.1^\circ - 5^\circ$ (including EWP)

- SU(3) decomposition for $A_{0\eta^{(\prime)}}$, $A_{+\eta^{(\prime)}}$ M. Gronau, J.Z. (2005)

  + exp. information $\boxed{|\Delta\alpha_{\pi-\eta-\eta'}| < 1.4^\circ}$

Snyder-Quinn

Snyder, Quinn (1993), Lipkin et al. (1991), Gronau (1991)

- model the Dalitz plot (similarly for $A(\bar{B}^0 \to 3\pi)$)

$$A(B^0 \to \pi^+\pi^-\pi^0) = \overbrace{A(B^0 \to \rho^+\pi^-)}^{A_+} D_{\rho\rho}(s_+)\cos\theta_+ +$$

$$+ \underbrace{A(B^0 \to \rho^-\pi^+)}_{A_-} D_{\rho\rho}(s_-)\cos\theta_- + \underbrace{A(B^0 \to \rho^0\pi^0)}_{A_0} D_{\rho\rho}(s_0)\cos\theta_0$$

- rotate $A_i(\bar{A}_i) \to e^{i\beta} A_i(e^{-i\beta}\bar{A}_i)$
- tree and penguin defined according to CKM

$$\mathcal{A}_{\pm,0} = e^{-i\alpha} T_{\pm,0} + P_{\pm,0} \,, \qquad \bar{\mathcal{A}}_{\pm,0} = e^{+i\alpha} T_{\pm,0} + P_{\pm,0}$$

- an isospin relation only between penguins

$$P_0 + \tfrac{1}{2}(P_+ + P_-) = 0$$

(EWP and isospin breaking neglected)

Effect of isospin breaking

- isospin breaking affects only the relation between penguins!
- largest shift $\delta\alpha = (1.5 \pm 0.3 \pm 0.3)^\circ$ due to EWP because they are related to tree

$$P_- + P_+ + 2P_0 = P_{EW}$$

- other isospin breaking effects are $P/T \sim 0.2$ suppressed
- using similar approach of SU(3) relations as in $\pi\pi$ to estimate shift due to $\pi^0 - \eta - \eta'$ mixing

$$|\Delta\alpha_{\pi-\eta-\eta'}| \leq 0.1^\circ$$

Conclusions

- the methods based on $B^\pm \to f_D K^\pm$ for measuring γ contain no theory error, even CP violation in D sector can be accomodated
- isospin breaking effect on α extraction from $B \to \rho\pi$ is $P/T \sim 0.2$ suppressed compared to $B \to \pi\pi, \rho\rho$

Backup slides

Effect of CP conserving $D-\bar{D}$ mixing

- in case of no $D-\bar{D}$ mixing

$$|A_f|^2=\int d\tau |A_f^{\rm even}(t)|, \qquad |A_{\bar f}|^2=\int d\tau |A_{\bar f}^{\rm even}(t)|$$

$$A_f A_{\bar f}^*=\int d\tau A_f^{\rm even}(t) A_{\bar f}^{\rm even}(t)^*$$

- CP even $D-\bar{D}$ mixing the same with replacement

$$A_f\to \tilde{A}_f=A_f-\frac{1}{4}(y+ix)\Big(\frac{q}{p}+\frac{p}{q}\Big)A_{\bar f}$$

$$A_{\bar f}\to \tilde{A}_{\bar f}=A_{\bar f}-\frac{1}{4}(y+ix)\Big(\frac{q}{p}+\frac{p}{q}\Big)A_f$$

Using data for $\pi - \eta - \eta'$ mixing

M. Gronau, J.Z. (2005)

- use SU(3) decomposition for $A_{0\eta^{(\prime)}}$, $A_{+\eta^{(\prime)}}$
  + neglect annihilation-like contributions

$$A_{+-} = t + p \overset{SU(2)}{\Longleftrightarrow} A_{33} = \frac{1}{\sqrt{2}}(c - p) \overset{SU(2)}{\Longleftrightarrow} A_{+3} = \frac{1}{\sqrt{2}}(t + c)$$

$$\Uparrow$$

$$SU(3)$$

$$\Downarrow$$

$$A_{3\eta} = \frac{1}{\sqrt{6}}(2p + s) \qquad A_{3\eta'} = \frac{1}{\sqrt{3}}(p + 2s)$$

$$A_{+\eta} = \frac{1}{\sqrt{3}}(t + c + 2p + s) \qquad A_{+\eta'} = \frac{1}{\sqrt{6}}(t + c + 2p + 4s)$$

Using data II

- triangle relation is modified only slightly

$$A_{+-} + \sqrt{2}A_{00} - \sqrt{2}A_{+0}(1 - e_0) = 0$$

where $e_0 = \sqrt{\frac{2}{3}}\epsilon + \sqrt{\frac{1}{3}}\epsilon' = 0.016 \pm 0.003$

- A_{+0} is a sum of pure $\Delta I = 3/2$ amplitude A_{+3} with weak phase γ and isospin-breaking terms

$$A_{+0} = A_{+3}(1 + e_0) + \sqrt{2}\epsilon A_{0\eta} + \sqrt{2}\epsilon' A_{0\eta'} \ .$$

  - while $e^{i\gamma}A_{+3} = e^{-i\gamma}\bar{A}_{+3}$ no longer $e^{i\gamma}A_{+0} = e^{-i\gamma}\bar{A}_{+0}$
  - also $|A_{+0}| \neq |\bar{A}_{+0}| \Leftarrow$ exp. check

Using data III

- varying the phases of $A_{0\eta^{(\prime)}}$, $\bar{A}_{0\eta^{(\prime)}}$ gives bound
$$|\Delta\alpha_{\pi-\eta-\eta}| \leq \sqrt{2\frac{\tau_+}{\tau_0}}\left(\epsilon\sqrt{\frac{\mathcal{B}_{0\eta}}{\mathcal{B}_{+0}}} + \epsilon'\sqrt{\frac{\mathcal{B}_{0\eta'}}{\mathcal{B}_{+0}}}\right)$$
- at 90% CL using WA values
$$|\Delta\alpha_{\pi-\eta-\eta'}| < 1.05\epsilon + 1.28\epsilon' = 1.6^\circ$$
- the bound can be improved using the SU(3) relations
$$A_{+\eta^{(\prime)}} = \frac{\sqrt{2}}{\sqrt{3}}A_{+0} + \sqrt{2}A_{0\eta^{(\prime)}}$$
leading to
$$\boxed{|\Delta\alpha_{\pi-\eta-\eta'}| < 1.4^\circ}$$