Longitudinal Single Bunch Instability by Coherent Synchrotron Radiation

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Motivation

Short bunch length for a high luminosity

Topics

1. Introduction
2. CSR in KEKB and SuperKEKB
3. Longitudinal Instability in SuperKEKB LER
4. Threshold of Instability
5. Summary

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1. Introduction

- Electrons moving in a bending magnet emit **Synchrotron Radiation**.

- In the spectrum of synchrotron radiation, the components such that $\lambda \gtrsim \sigma_z$ produce **Coherent Synchrotron Radiation**. (CSR)

\[ \sigma_z = 3\text{mm} \Rightarrow \lambda \sim 3\text{mm} \]
\[ \nu \sim 100\text{GHz} \]

- Energy change of particles

  - **Short range interaction**

  $\Rightarrow$ Energy spread
  $\Rightarrow$ Single bunch instabilities (Longitudinal, Transverse)
• Shielded CSR by beam chamber
  If shielding is strong:
  \[ a \lesssim (\rho \sigma_z^2)^{1/3} \]
  \( (a = \text{chamber size}) \)
  CSR is suppressed with proper vacuum chambers.

CSR depends on the chamber size.

• At high energy, CSR is determined by
  * bunch distribution \( \lambda(z) (\sigma_z) \)
  * bending radius \( \rho \)
  * size of beam pipe \( a \)
  * magnet length \( \ell_m \)
  * bunch population \( N \)

• LER is affected with CSR in SuperKEKB. (Bends will be used again.)

  \[ \text{LER: } \rho = 16.3\text{m} \quad \text{small bending radius } \Rightarrow \text{Intense CSR} \]
  \[ \text{HER: } \rho = 104.5\text{m} \quad \text{CSR is moderate.} \]
2. CSR in KEKB and SuperKEKB

**KEKB**

- Bunch length: \( \sigma_z = 6\text{mm} \) ⇒ 3mm
- Bunch current: \( I_b = 1.2\text{mA} \) ⇒ 2mA (≈ 20nC)

**SuperKEKB**

Energy change due to CSR in a bending magnet

KEKB ⇒ SuperKEKB
14 times larger \( \Delta E \)

CSR can be suppressed by using chambers of small cross section.
Loss factor due to CSR and Resistive Wall wakefield

\[ \sigma_z = 3\text{mm} \]

**CSR**
- \( k = 12.6 \text{ V/pC} \)
- **CSR + RW**
  - \( k = 18.8 \text{ V/pC} \)

\[ \sigma_z = 6\text{mm} \]

**CSR**
- \( k = 1.0 \text{ V/pC} \)
- **CSR + RW**
  - \( k = 3.2 \text{ V/pC} \)

Loss factor due to CSR+RW is always larger than 12.3 \( \text{V/pC} \).

The minimum value is determined by the dipole magnets \((\rho, \ell_m)\).
3. Longitudinal Instability in SuperKEKB LER

- Field calculation of CSR = Paraxial Approximation in a beam pipe

- Equations of Longitudinal Motion (1 Million macro-particles)
  \[
  \begin{aligned}
  z' &= -\eta \delta \\
  \delta' &= \frac{(2\pi \nu_s)^2}{\eta C^2} z - \frac{2U_0}{E_0} \delta + Q + \text{CSR} + \text{RW}
  \end{aligned}
  \]

- 134 bends in the arc section are considered for CSR, but CSR in wiggler is ignored.
  (It should be considered.)

- Wiggler is taken into account in computing the radiation damping $U_0$.

- Copper pipe of square cross section
  (Actual one is round.)

- RW = Resistive Wall wakefield in the straight section

- Initial condition = Equilibrium without CSR, RW

- parameters
  $E_0 = 3.5 \text{ GeV}$
  $C = 3016.26 \text{ m}$
  $\sigma_z = 3 \text{ mm}$
  $\sigma_\delta = 7.1 \times 10^{-4}$
  $V_{rf} = 15 \text{ MV}$
  $\omega_{rf} = 508.887 \text{ Hz}$
  $h = 5120$
  $\alpha = 2.7 \times 10^{-4}$
  $U_0 = 1.23 \text{ MeV/turn}$
  $\nu_s = 0.031$
**Bunch distribution**

- Initial distribution
- Distribution at 16000 turns

**Energy spread**

- Initial
  - Bunch length $\sigma_z = 3.0\text{mm}$
  - Energy spread $\sigma_\delta = 7.1 \times 10^{-4}$

**Equilibrium**

- Bunch length $\sigma_z \sim 4.3\text{mm}$
- Energy spread $\sigma_\delta \sim 8.8 \times 10^{-4}$
• Bunch distribution

- Initial bunch distribution
- Distribution at 16000 turns

• Energy spread

- Initial energy distribution
- Distribution at 16000 turns

\[ r = 25\text{mm} \]

Initial
Bunch length \( \sigma_z = 3.0\text{mm} \)
Energy spread \( \sigma_\delta = 7.1 \times 10^{-4} \)

Equilibrium
Bunch length \( \sigma_z \sim 3.5\text{mm} \)
Energy spread \( \sigma_\delta \sim 7.1 \times 10^{-4} \)

STABLE
Sawtooth Instability

Resistive wall wakefield reduces the sawtooth amplitude.

But above a certain threshold, the energy spread is increased by CSR, the bunch is not stationary but unstable.

Oscillation: Radiation damping ⇔ CSR burst

Equilibrium/ Initial $\sigma_\delta = 1.24$

Equilibrium/ Initial $\sigma_\delta = 1.35$
- **Bunch distribution**

  ![Bunch distribution graph](image1)

- **Energy spread**

  ![Energy spread graph](image2)

\[ r = 47\text{mm} \]

\[ W_\parallel = \text{CSR} \]

Initial

Bunch length \[ \sigma_z = 3.0\text{mm} \]

Initial

Energy spread \[ \sigma_\delta = 7.1 \times 10^{-4} \]

Average

Bunch length \[ \sigma_z \sim 4.3\text{mm} \]

Average

Energy spread \[ \sigma_\delta \sim 9.6 \times 10^{-4} \]
4. Threshold of longitudinal instability

**Bunch length vs Bunch current**

Initial $\sigma_z = 3\text{mm}$

Energy spread vs Bunch current

Initial $\sigma_\delta = 7.1 \times 10^{-4}$

The length increases fast, and the energy spread starts increasing above a threshold which is determined by the chamber size.

The limit current is $0.8\text{mA}$ ($N_e \sim 8\text{nC}$) in the chamber of $r = 47\text{mm}$.
Threshold for chamber size

- **rms Bunch length and Energy spread**

  ![Graph showing the relationship between chamber half height and equilibrium spread/initial spread for different chamber sizes. The graph includes lines for different chamber sizes and shows how the bunch length and energy spread change with chamber height.](image)

  - \( W_\parallel = \text{CSR} + \text{Resistive Wall} \)
  - \( W_\parallel = \text{CSR} \)

  - **bunch length**
  - **energy spread**

  \( I_b = 2\text{mA} \) \( (N_e \sim 20\text{nC}) \)

- **Longitudinal bunch distribution**

  ![Graph showing the longitudinal distribution of bunches with different chamber sizes.](image)

  - Chamber sizes: 47mm, 40mm, 35mm, 30mm, 25mm, 20mm

  The bunch leans forward because of the energy loss due to the resistive wall.

Threshold for the chamber half height is \( r_{th} \sim 30\text{mm} \), when the bunch current is \( I_b = 2\text{mA} \) \( (N_e \sim 20\text{nC}) \).
SuperKEKB HER

Bunch length, Energy spread vs Bunch current

The limit bunch current is 6.8 mA (∼68 nC).
(Design $I_b = 0.82$ mA: No problem)
Bunch lengthening = 5.6% at $I_b = 0.82$ mA

- Initial Bunch length
  $\sigma_z = 3$ mm
- Bend
  $\rho = 104.5$ m
  $\ell_m = 5.8$ m
- Vacuum chamber
  (rectangular)
  $w \times h = 100 \times 50$ mm
  (full width, height)
- Others
  $\alpha = 1.8 \times 10^{-4}$
  $V_{rf} = 20$ MV
  $U_0 = 3.48$ MeV/turn
  $\nu_s = 0.019$
5. Summary

- SuperKEKB HER has no problem with CSR.
  Design $I_b = 0.82 \text{ mA} \ll \text{Limit } 6.8 \text{ mA (Ne } \sim 68 \text{ nC)}$
  Only 5.6% bunch lengthening at design $I_b$

- LER is affected with CSR because of (1) short bunch length, (2) high bunch charge, (3) small bending radius.
  The bunch of 3mm length and 2mA current is unstable due to CSR in the present chamber $r = 47\text{mm}$.

- Above a bunch current, the longitudinal instability occurs.
  The threshold is $I_b = 0.8\text{mA (} \sim 8\text{nC)}$ in the present chamber.

- Small vacuum chambers suppress CSR.
  The threshold half height is $r = 30\text{mm}$ for $I_b = 2\text{mA (} \sim 20\text{nC)}$.

- Resistive wall wakefield moderates the sawtooth instability.
  However, the instability threshold does not change so much.

- Loss factor by CSR + RW is $k = 18.8 \text{ V/pC}$ for $r = 47\text{mm}$.
  It cannot be smaller than $12.3 \text{ V/pC}$ for any vacuum chamber.

- Small vacuum components may have large impedances.
  Bunch length in the SuperKEKB LER is limited by CSR.