The global flow reconstruction of chaotic stellar pulsation

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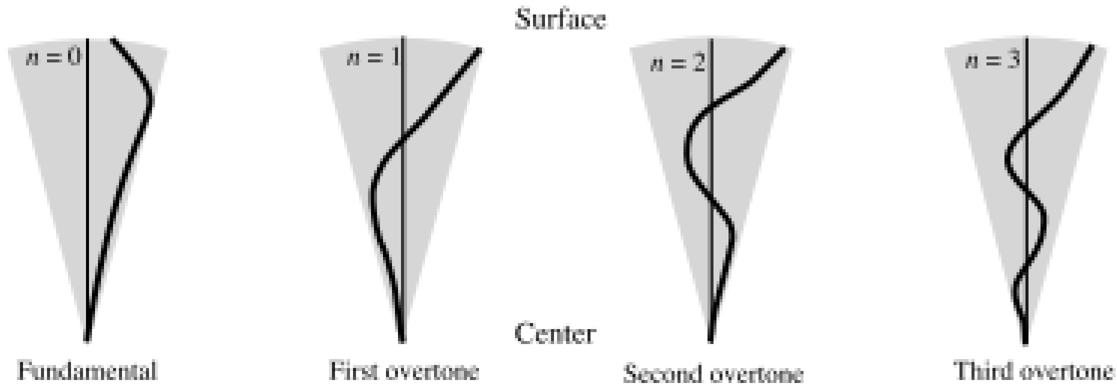
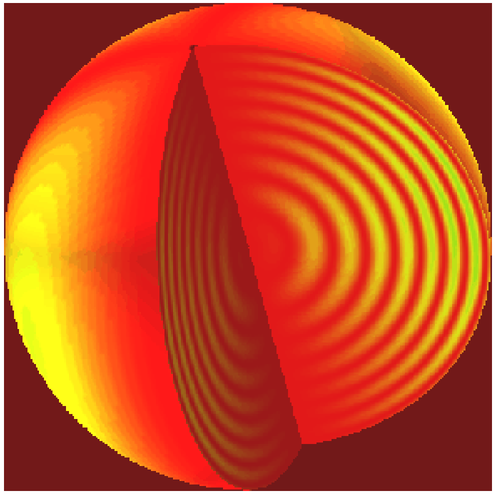
Nonlinear behavior in variable stars

Nonlinear coupling of oscillation modes

- RADIAL

  fundamental mode + overtones



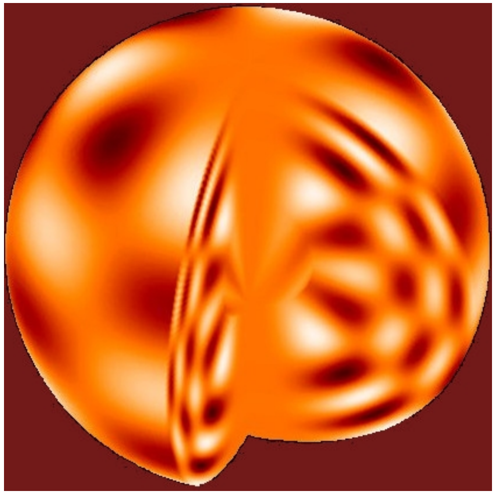


  few are excited -->

  dynamics occur in low dimensional phase space

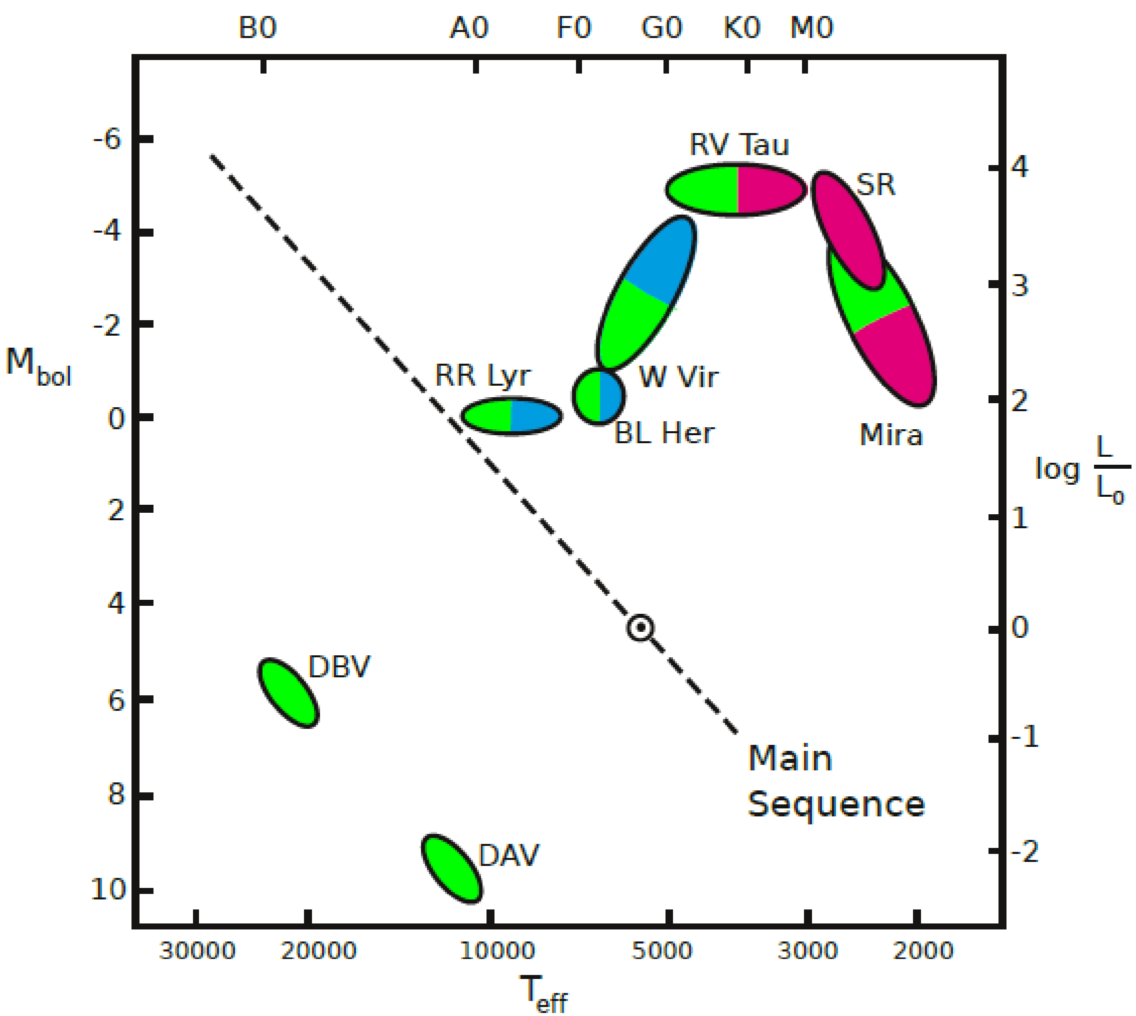
- NONRADIAL

  many modes --> high embedding dimension



Nonlinear behavior in variable stars

Hertzsprung-Russell Diagram



Chaos detected in observations

RV Tauri
Semiregular
Mira

Period doubling in observations

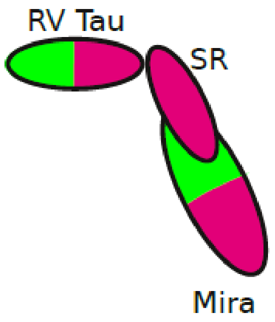
RV Tauri
BL Herculis
RR Lyrae
Pulsating white dwarfs
W Vir (new)

Chaos (and PD) in hidrodynamical models

W Virginis
BL Herculis
RR Lyrae

Nonlinear behavior in variable stars

Red giant and supergiant stars



strongly nonadiabatic (dissipative) stars

pulsation timescale ~ thermal timescale (modulation):

months -> years

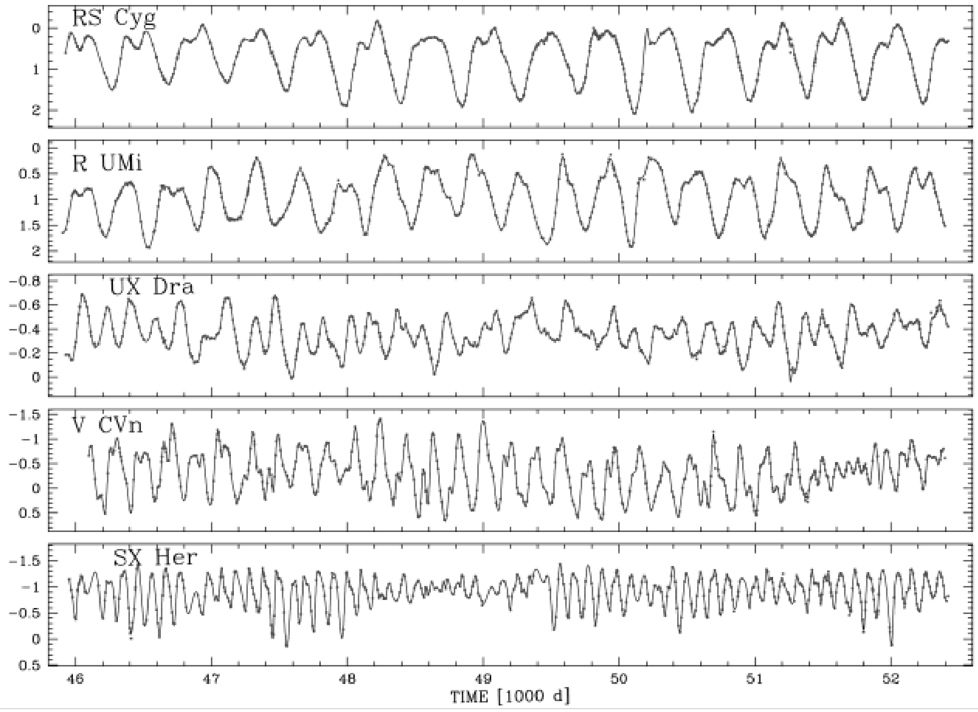
AAVSO data are suitable for nonlinear analysis

Evidence for chaos:

- **RV Tau:** *AC Her* (Kolláth et al. 1998)
  R Scu (Buchler et al. 1996)

- **Semiregular:** *R UMi, RS Cyg, V CVn, UX Dra, SX Her* (Buchler, Kolláth & Cadmus, 2004)

- **Mira:** *R Cyg* (Kiss & Szatmáry 2002)

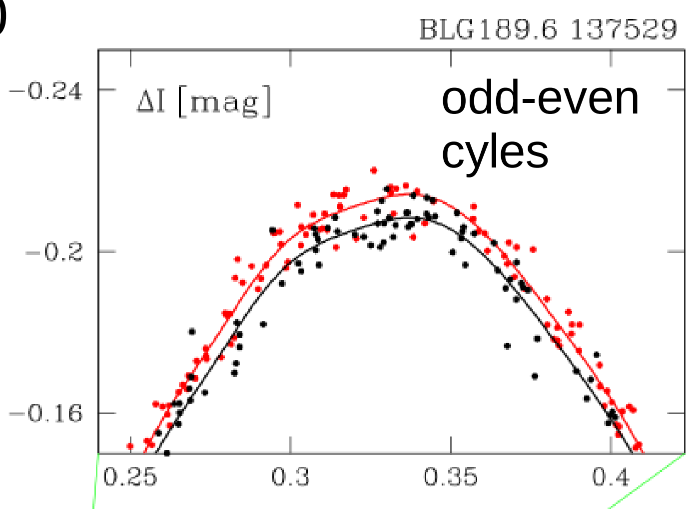


Nonlinear behavior in variable stars

Period doubling

- **RR Lyrae type** (Kepler data) Szabó et al. 2010

  need to observe consecutive cycles



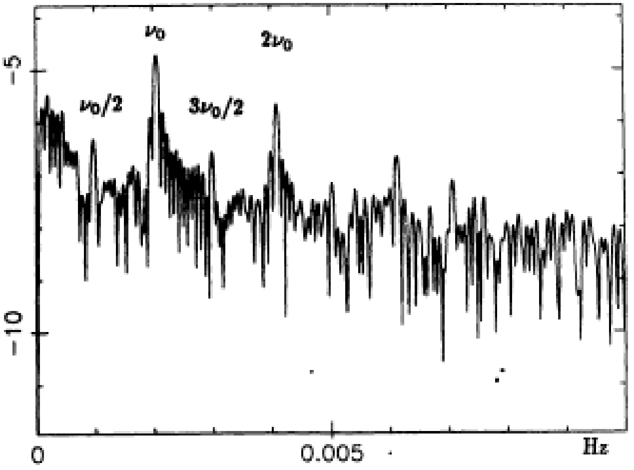
- **BL Her type** (OGLE data) Smolec et al. 2012

- **Pulsating white dwarfs**

  Goupil, Auvergne & Baglin 1988

  some other stars show subharmonics too

  nonradial modes, short lifetime

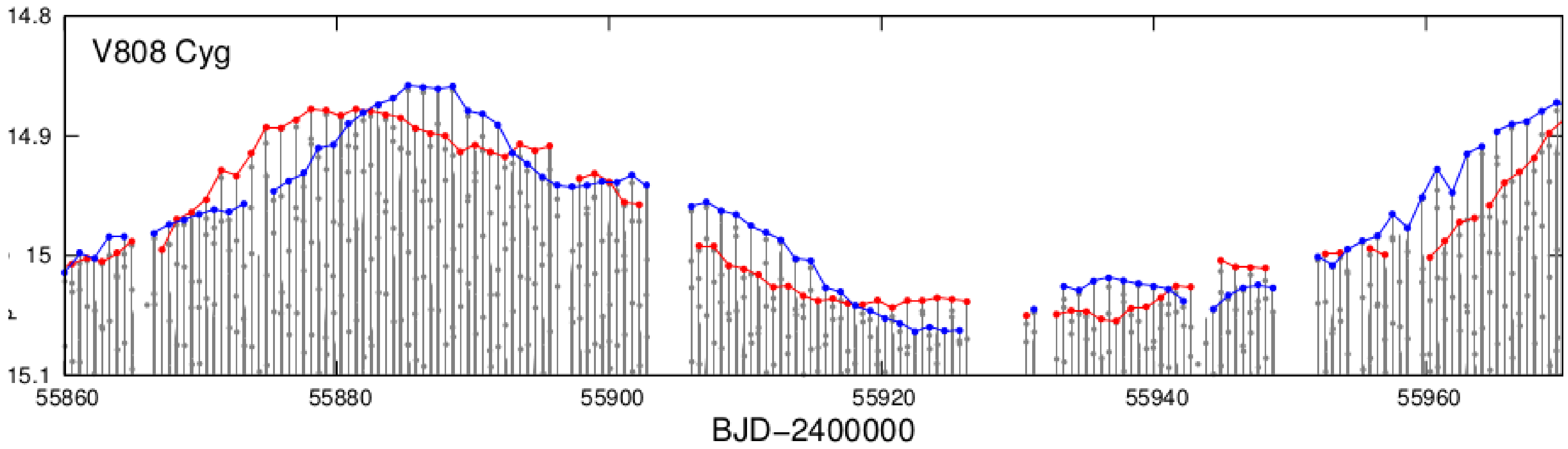


Nonlinear behavior in variable stars

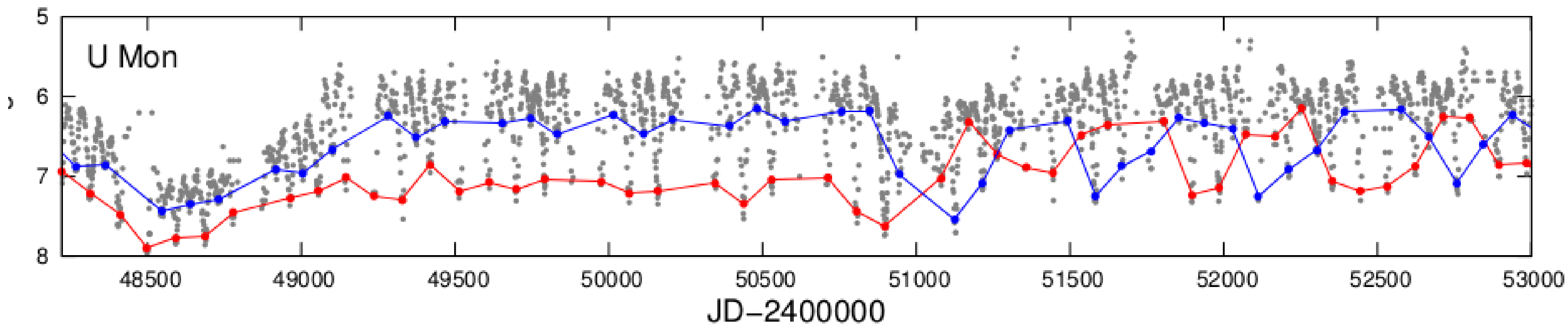
Period doubling

interchanges

RR Lyrae type

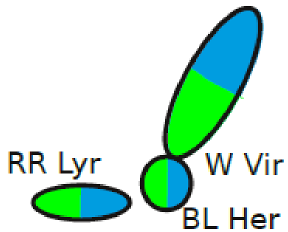


RV Tau type



Nonlinear behavior in variable stars

Hidrodynamical models



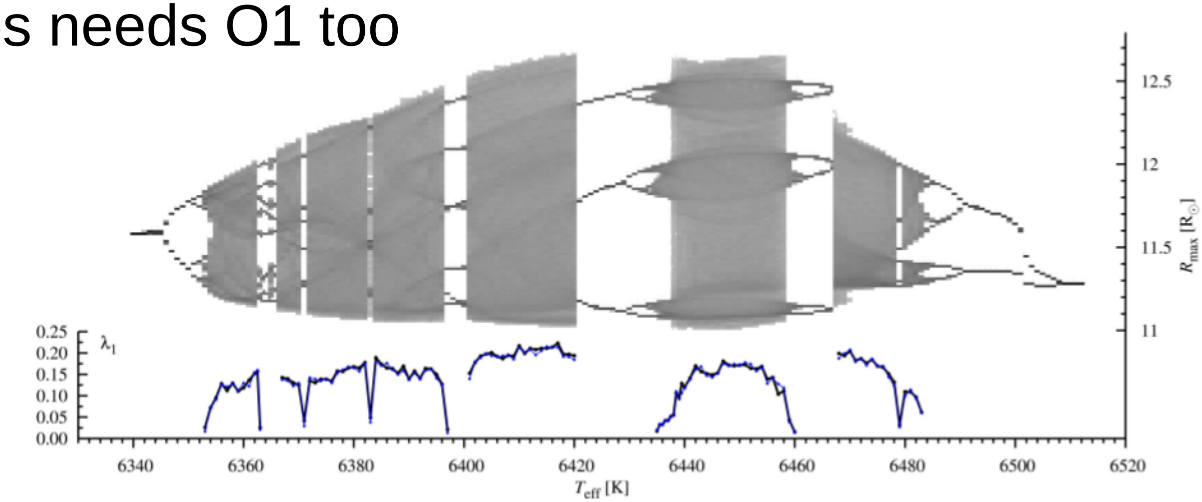
W Vir: strongly dissipative

chaos in purely radiative model (Kovács & Buchler 1988)

chaos is caused by the 5:2 resonance between FM and O1 (Moskalik & Buchler 1990)

RR Lyrae: nonlinear effects were unexpected: weakly dissipative ~ Pop. I. Cepheids, high order resonances were not checked
Kepler –> hydrodynamical calculations
Period doubling due to 9:2 resonance between FM and O9 (Kolláth, Molnár & Szabó, 2011), chaos needs O1 too

BL Her:
PD, intermittency, crisis
(Smolec & Moskalik 2014)



The global flow reconstruction method

Serre, Kolláth, Buchler, 1996

Time series: $g(t_n)$ light curve

Delay vectors:

$\boldsymbol{X}^n = \{ g(t_n), g(t_n - \Delta), \ldots, g(t_n - (d_e - 1)\Delta \}$

Represent the flow in the phase space

d_e (4, 5, 6) embedding dimension, Δ time delay

The neighbouring ponts are linked by the map ***F***

$\boldsymbol{X}^{n+1} = \boldsymbol{F}(\boldsymbol{X}^n)$ where $\boldsymbol{F} = \{F_1, F_2, \ldots, F_{de}\}$

Search $F_1, F_2, \ldots, F_d$ components in polynomial form

--> generate arbitrary long synthetic trajectory

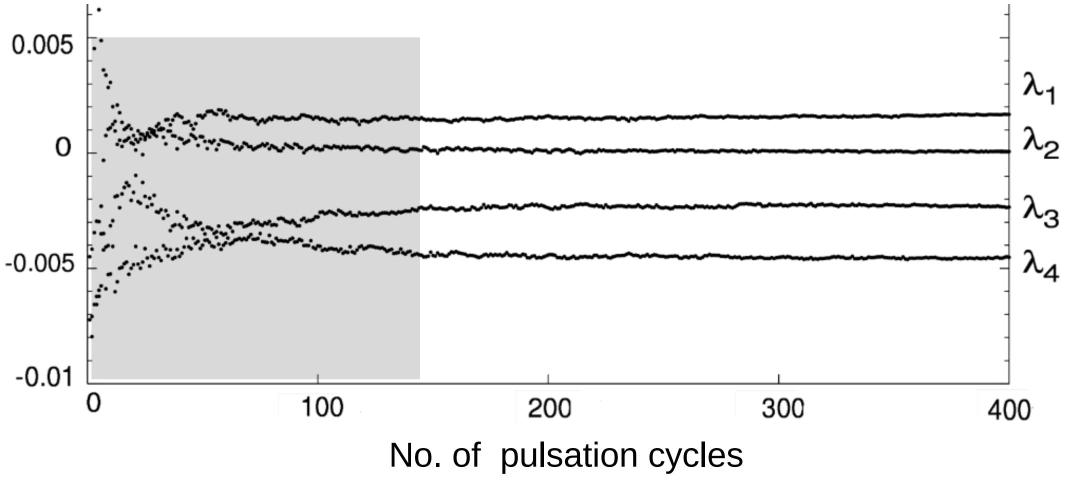
--> Lyapunov exponents

$$d_L = K + \frac{1}{|\lambda_{K+1}|} \sum_{i=1}^{K} \lambda_i,$$

--> Lyapunov dimension

The global flow reconstruction method

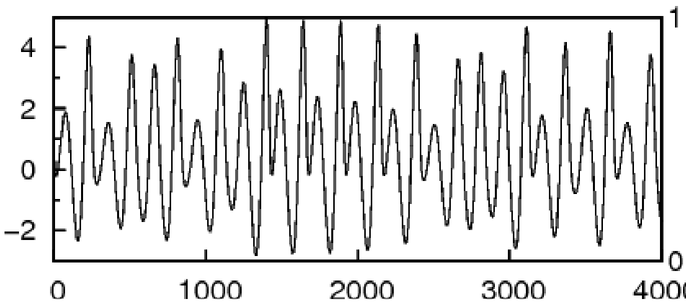
Limited length of continuous observational data is no problem



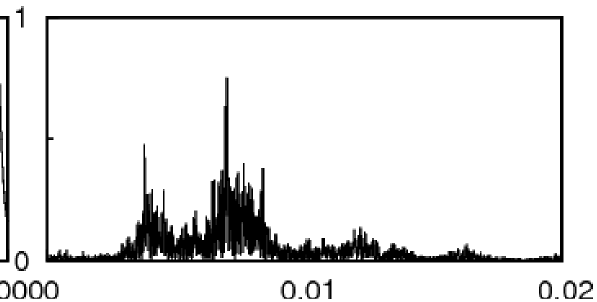
Aim: find a chaotic region in the parameter space: time delay, spline, noise

--> statistical sample of synthetic data --> range for Lyapunov dimension

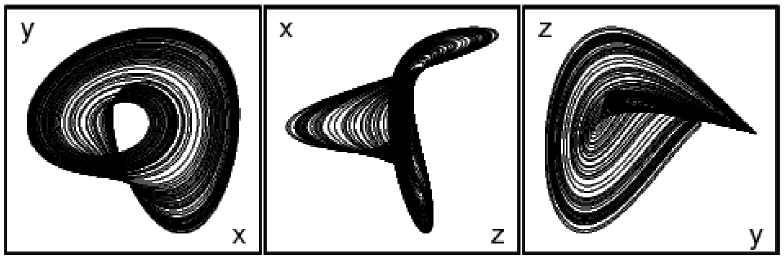
Synthetic data must be similar to the original data!



Time series



Fourier spectrum



B-K projections

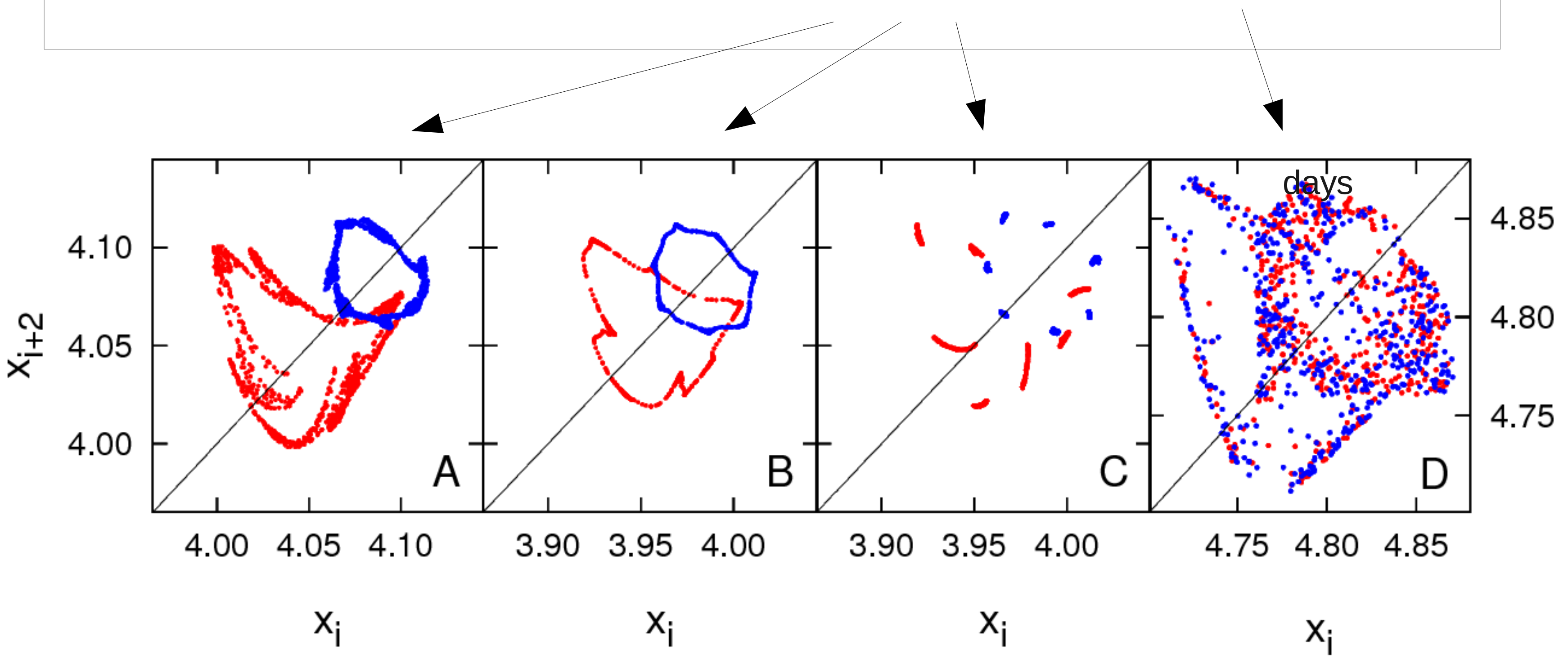
Broomehead & King, 1987: eigenvectors of the correlation matrix

Chaotic RR Lyrae models

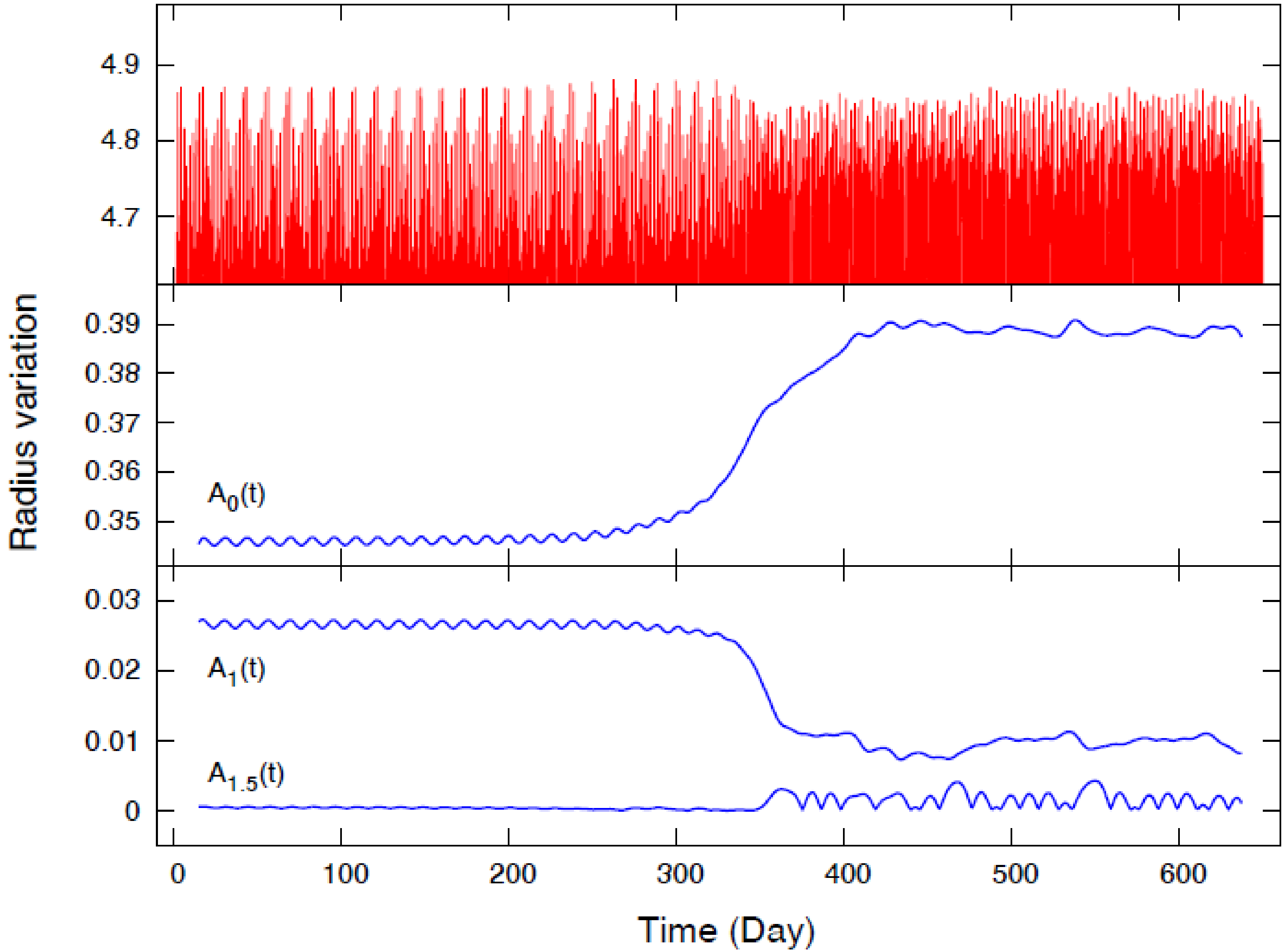
Triple mode models: FM + O1 +O9

Possible resonabces of P_1/P_0 ~ 0.72-0.75 theoretical calculations (Szabó, Kolláth & Buchler 2004), observations (Soszynski, 2011)

6:8	8:11	**14:19**	**20:27**

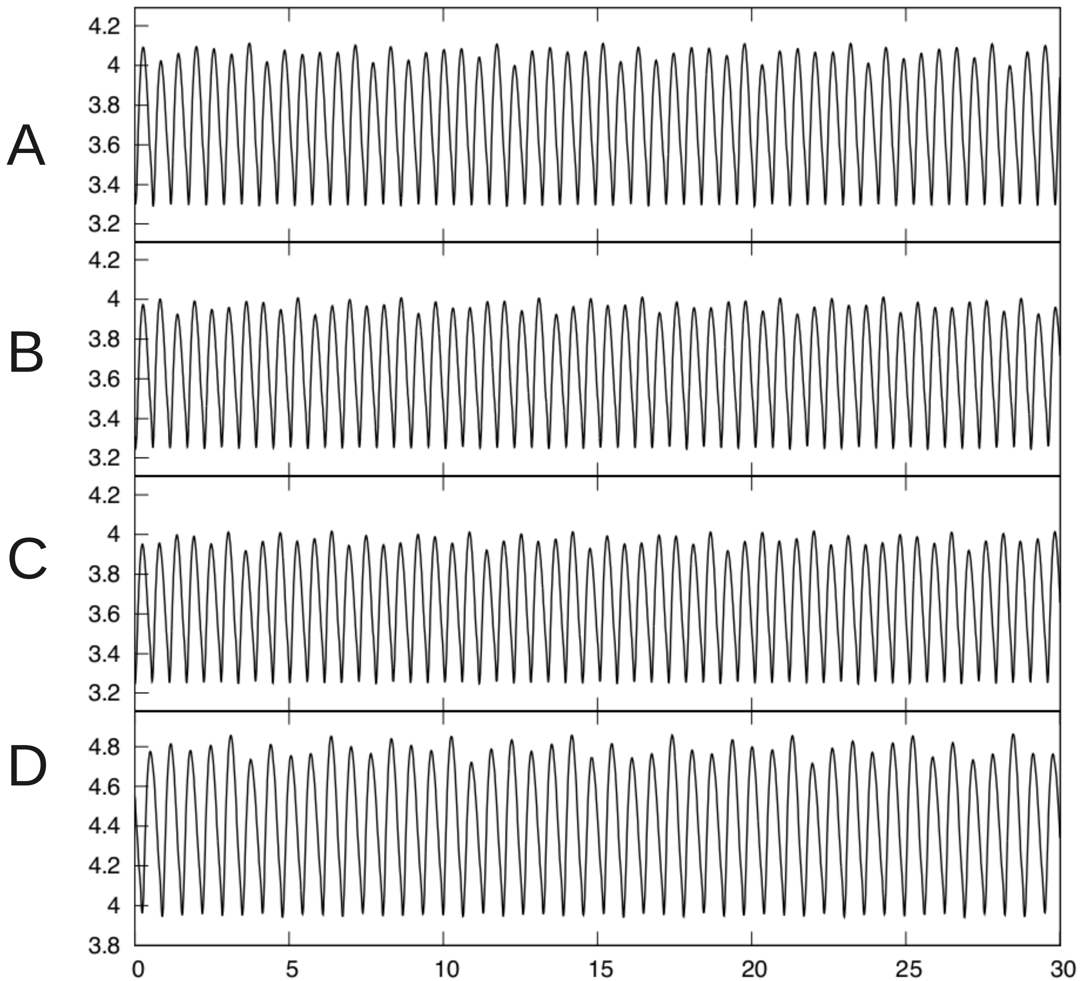


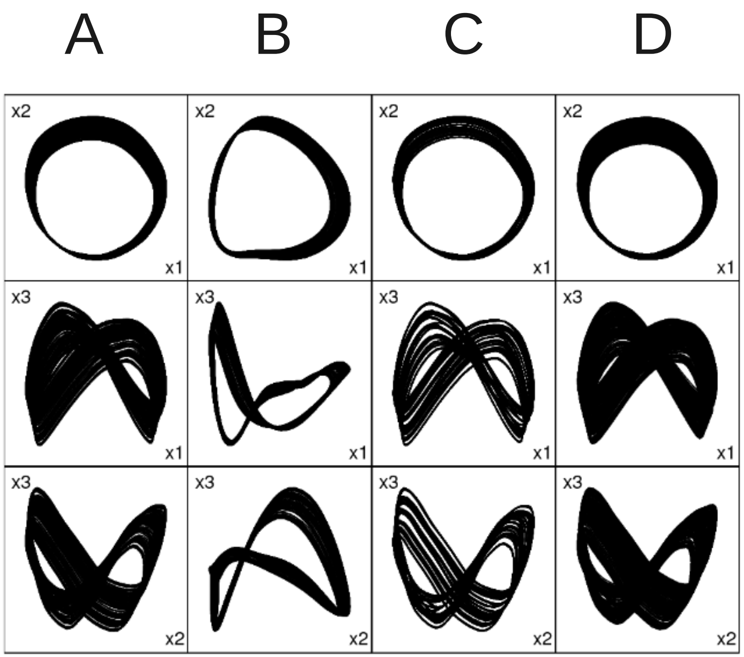
Chaotic RR Lyrae models



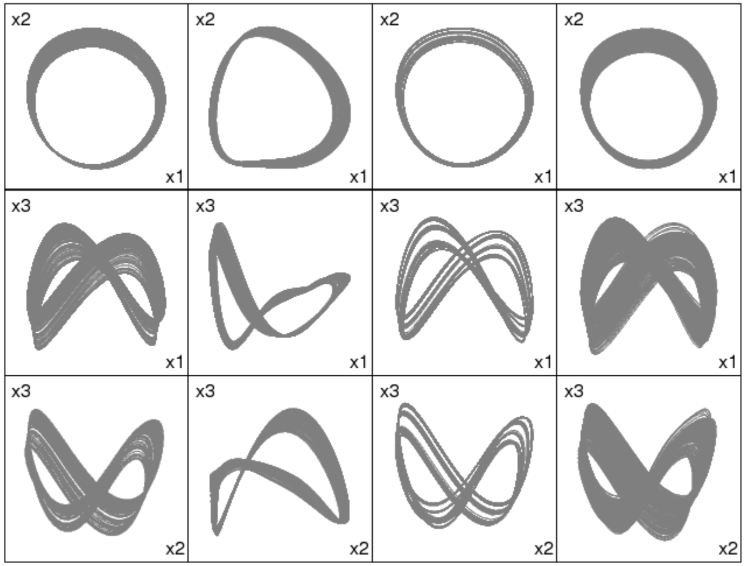
Chaotic RR Lyrae models

The global flow reconstruction of the **radius variation**





Synthetic data

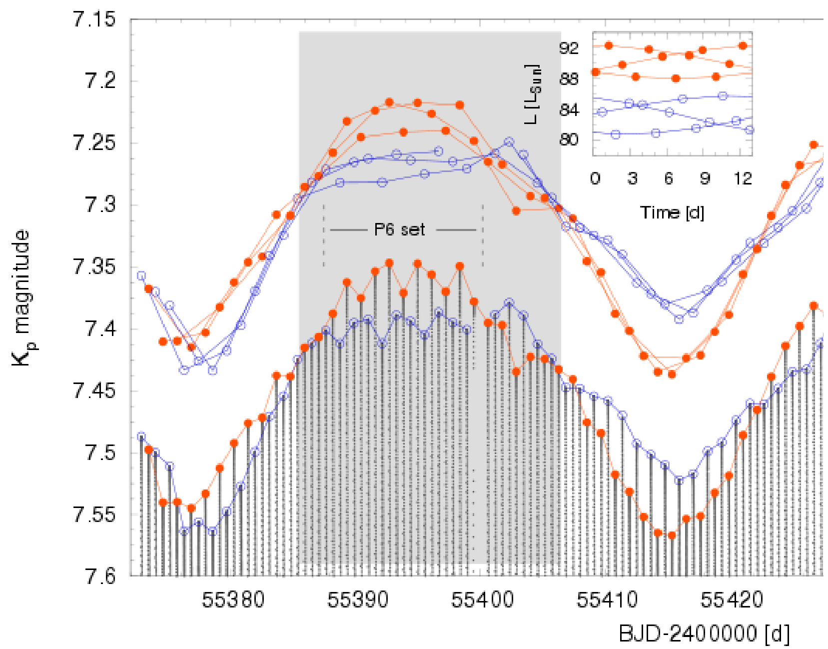


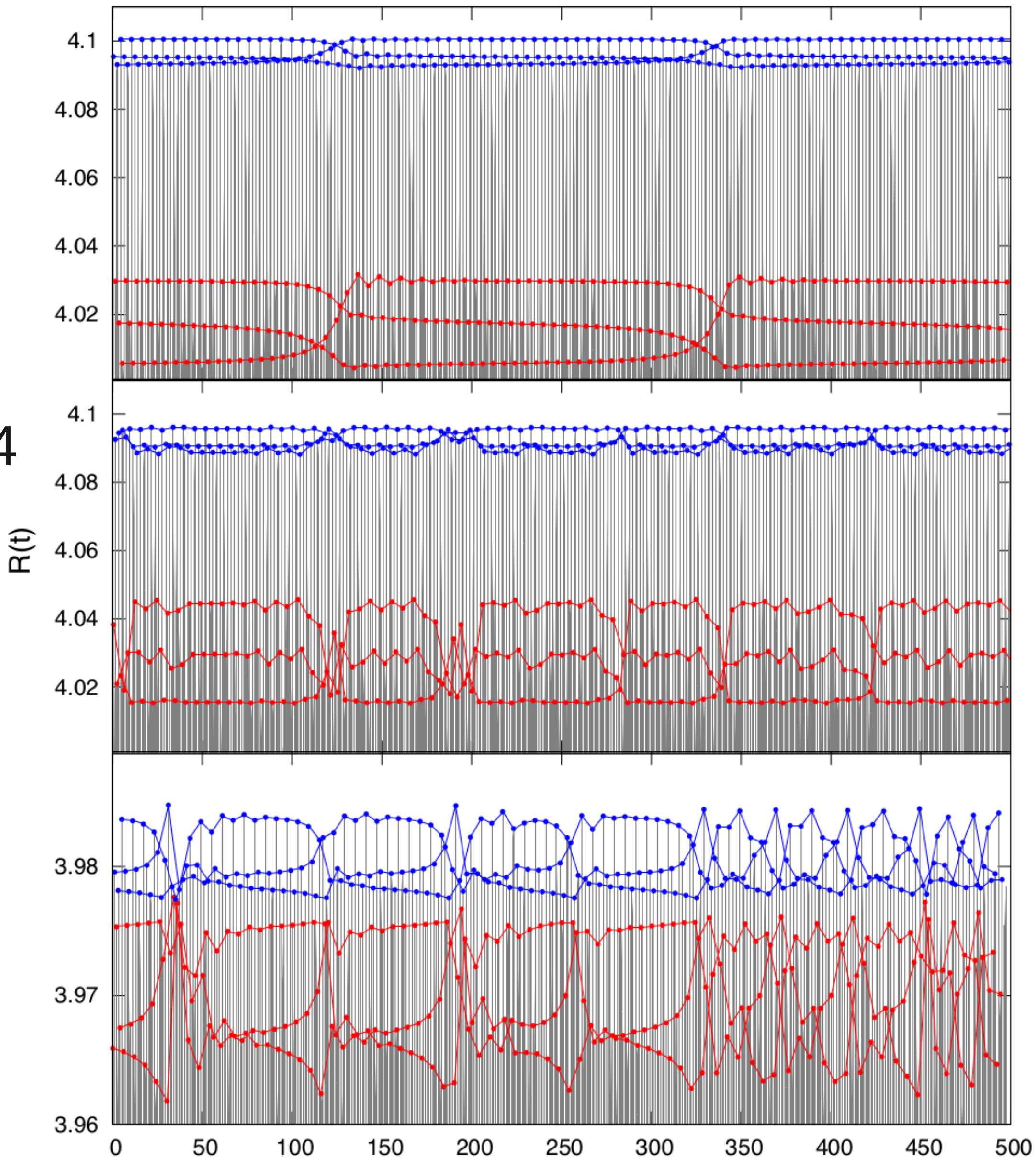
Chaotic RR Lyrae models

Model	Lyapunov dim.	No. of chaotic signals
A	2,24±0,23	452 (∼92%)
B	2,25±0,23	147 (∼80%)
C	2,17±0,23	358 (∼93%)
D	2,21±0,18	173 (∼94%)

Resonances: 6:8, 8:11, 10:13 or 10:14

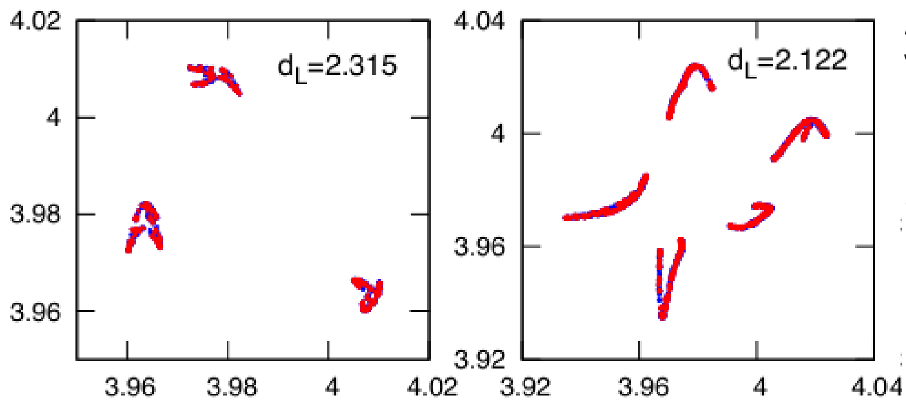
Also seen in RR Lyr (Molnár et al. 2012)





Chaotic RR Lyrae models

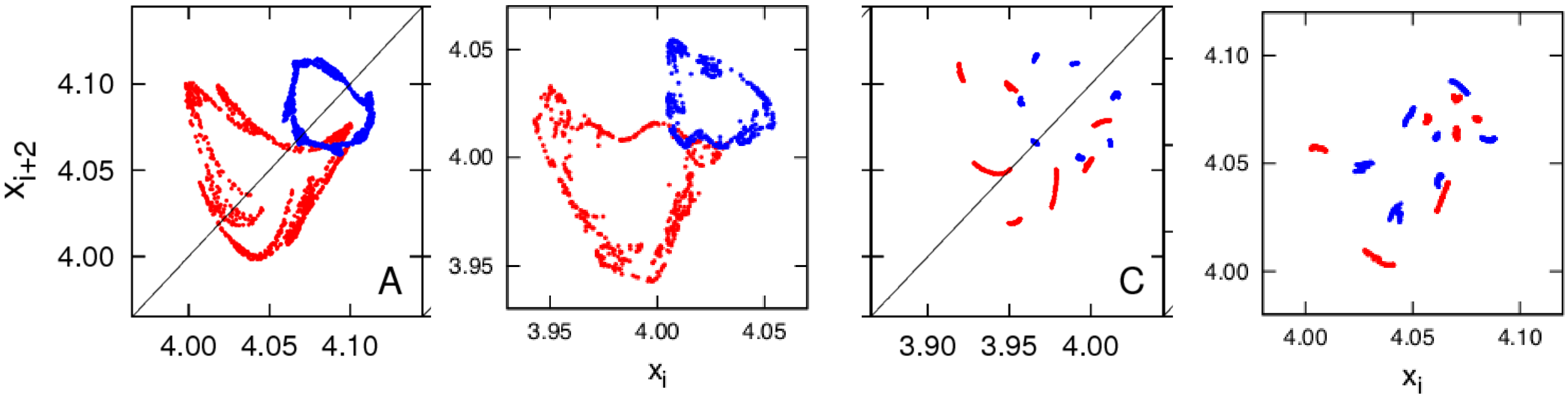
The global flow reconstruction helps mapping resonances in the phase space neighbourhood



3 clumps, 5 clumps
-- > possible odd number resonances

not seen in observations or models

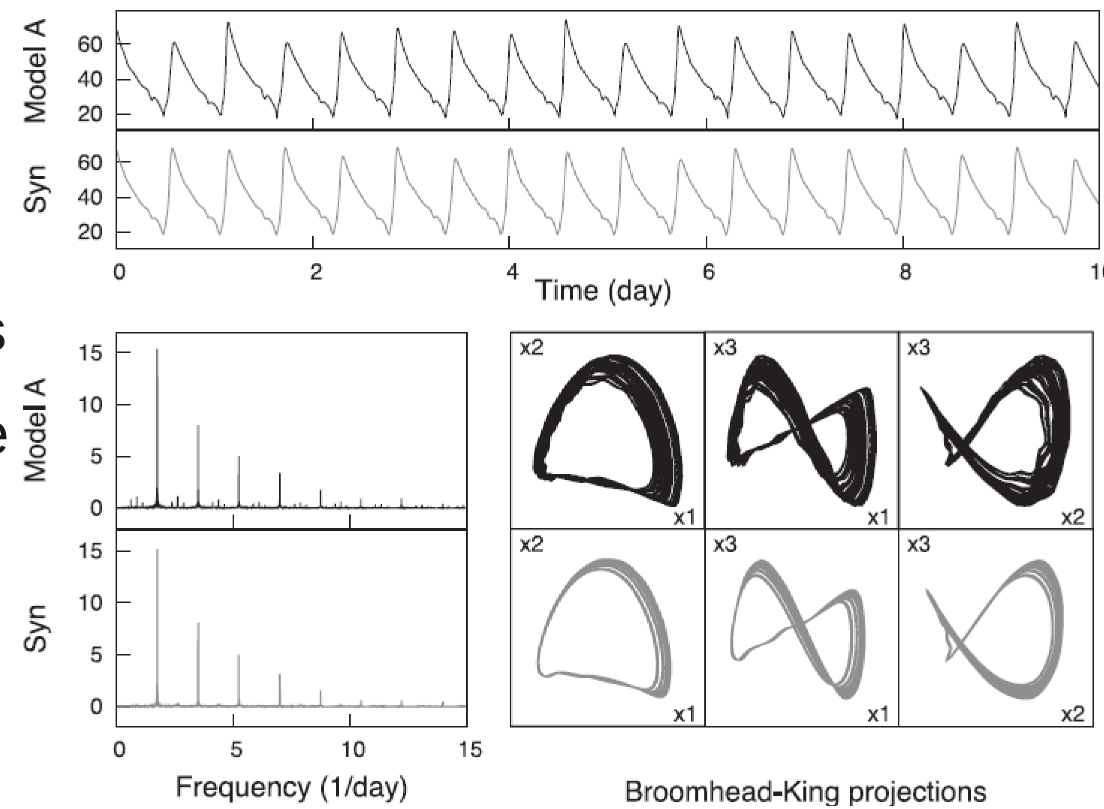
Similarity in return maps



Chaotic RR Lyrea models

Luminosity variation: we can observe that directly

Model	Lyapunov dim.	No. of chaotic signals
A	2,16±0,12	44 (∼92%)
B	2,05±0,02	19 (∼76%)
C	2,15±0,13	47 (∼52%)
D	2,12±0,06	21 (∼88%)



Sharp maxima --> fitting problems

fail to reconstruct the fine structure

Reconstruction results in
much less synthetic signals

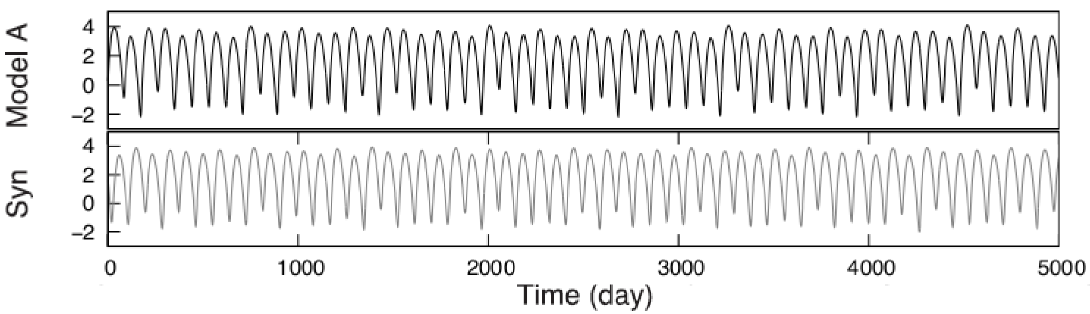
Lyapunov dymensions are good

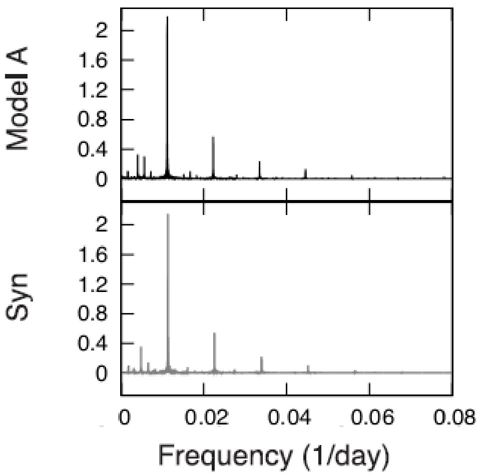
--> it is possibe to reconstruct RR Lyrae light curves

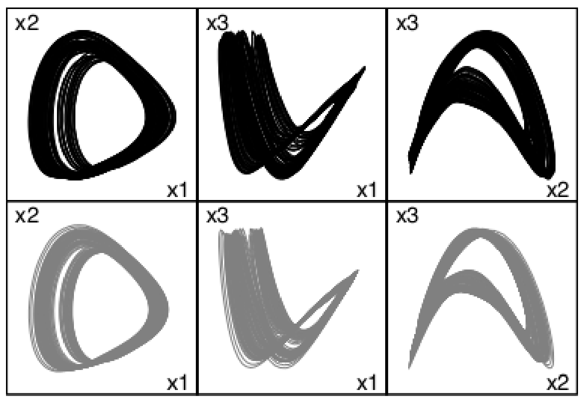
Chaotic RR Lyrae models

Transformation of the luminosity variation into a simpler form

Model	Lyapunov dim.	No. of chaotic signals
A	2,20±0,21	257 (~60%)
B	2,18±0,07	38 (~70%)
C	2,15±0,12	31 (~91%)
D	2,16±0,16	72 (~96%)





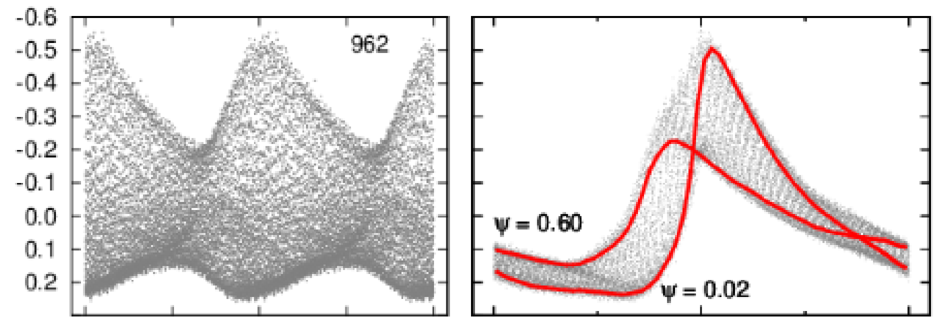


Helped in the case of model A and D

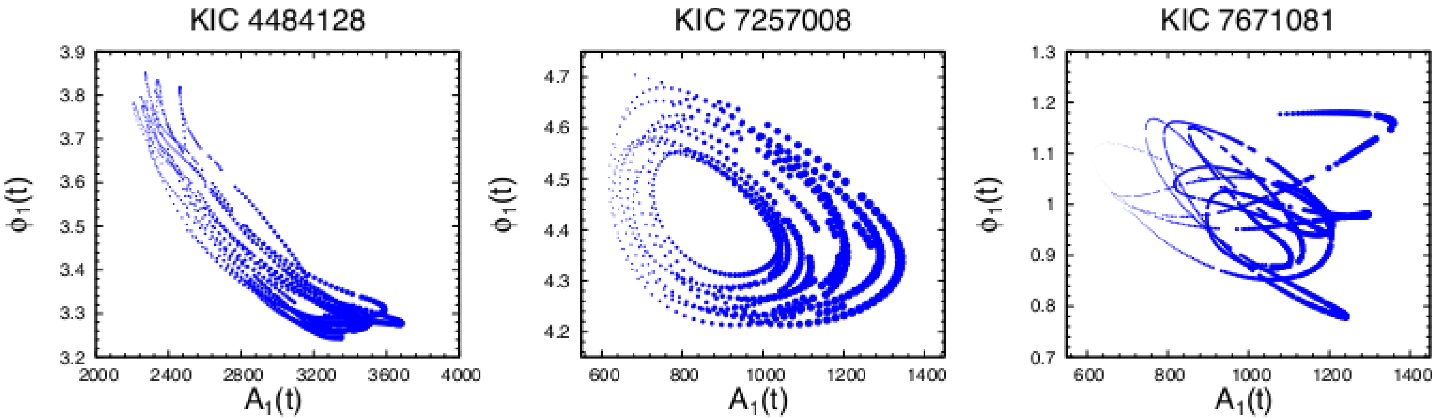
Plachy et al. 2013: Low-dimensional chaos in RR Lyrae models

The dynamical investigation of the Blazhko effect

- discovered in 1907
- amplitude and phase modulation
- half of the RR Lyrae stars show



- amplitude and phase relations vary star by star



- multiperiodic, irregular

--> we can rule out the theoretical explanations that predict strictly periodic modulation

The dynamical investagation of the Blazhko effect

Possible explanations

1. Stothers idea 2006: cyclic change in the turbulent convective properties due to magnetic field build up and decay
--> **stochastic modulation**

2. Gillet idea 2013: shockwave governed compression affects the intensity of the Kappa mechanism --> **stochastic modulation**

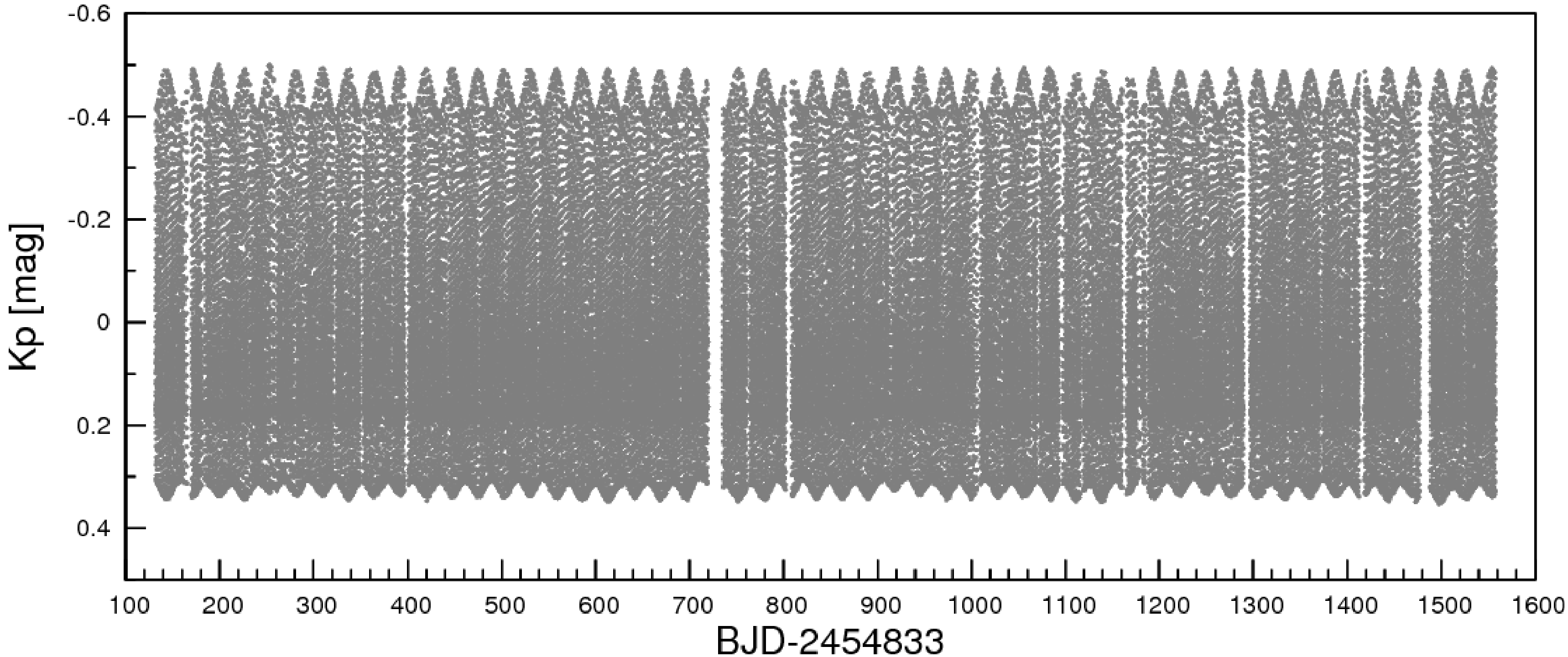
3. Buchler & Kolláth idea (2013): nonlinear coupling of radial modes
--> **chaotic modulation**

amplitude equation formalism

no hydrodynamical confirmation yet

The dynamical investagation of the Blazhko effect

Is the modulation chaotic or stochastic?



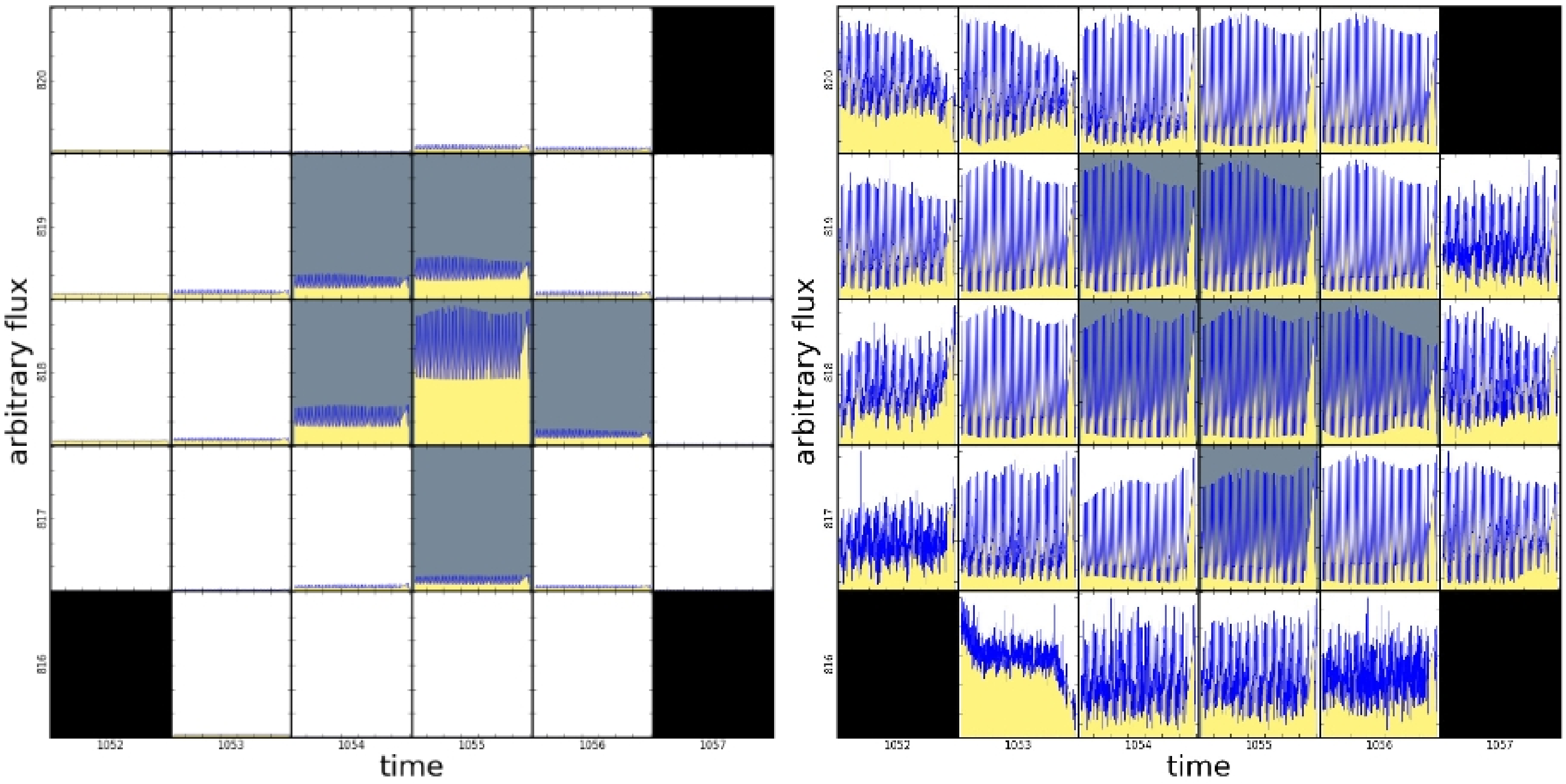
Check Kepler data

V783 Cyg has the shortest modulation period in the Kepler sample

--> 51 modulation cycles

The dynamical investagation of the Blazhko effect

Tailor-made aperture (Benkő et al. 2014)



The dynamical investagation of the Blazhko effect

Extract modulation from light curves

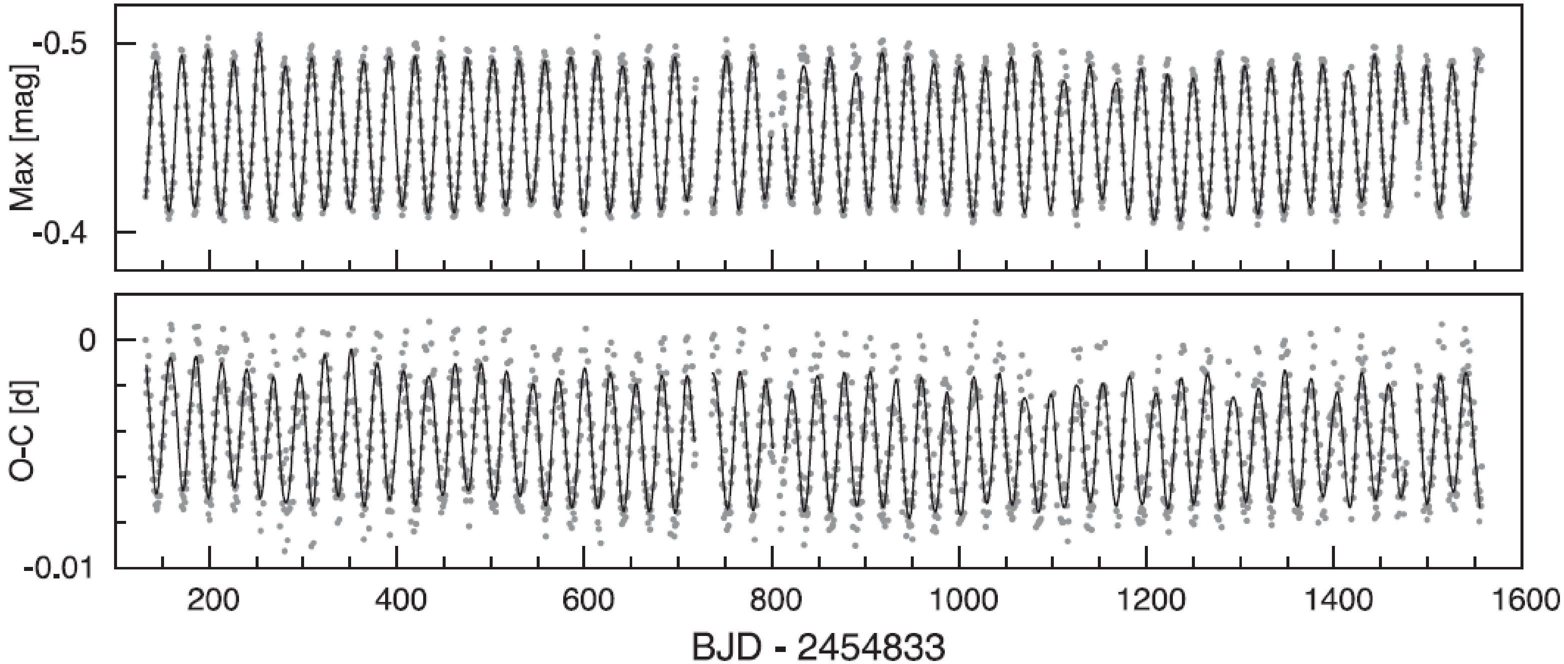
--> different methods:

- maxima determination (by cubic spline)
  - O-C (Observed-Calculated) (1)
  - maximum values (2)
- analythical function method: temporal behavior of Fourier parameters
  - P_1 pulsation period (3)
  - R_{21} A2/A1 amplitude ratio (4)

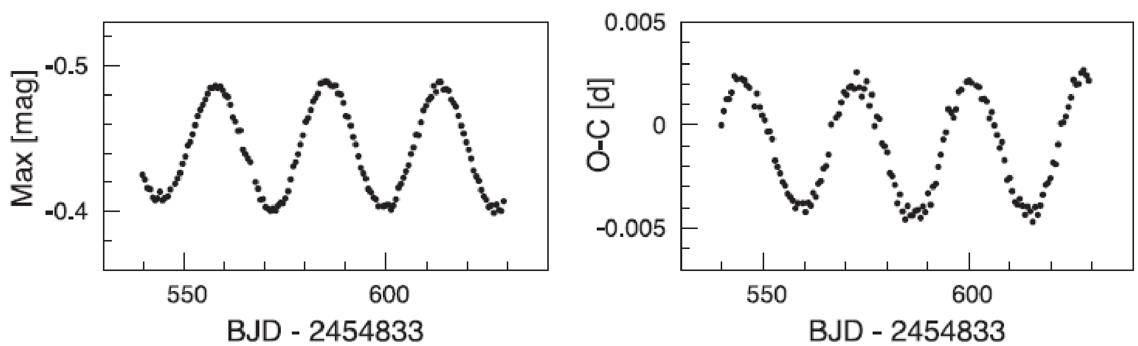
--> 4 modulation curves

The dynamical investagation of the Blazhko effect

Large scatter in maxima values:

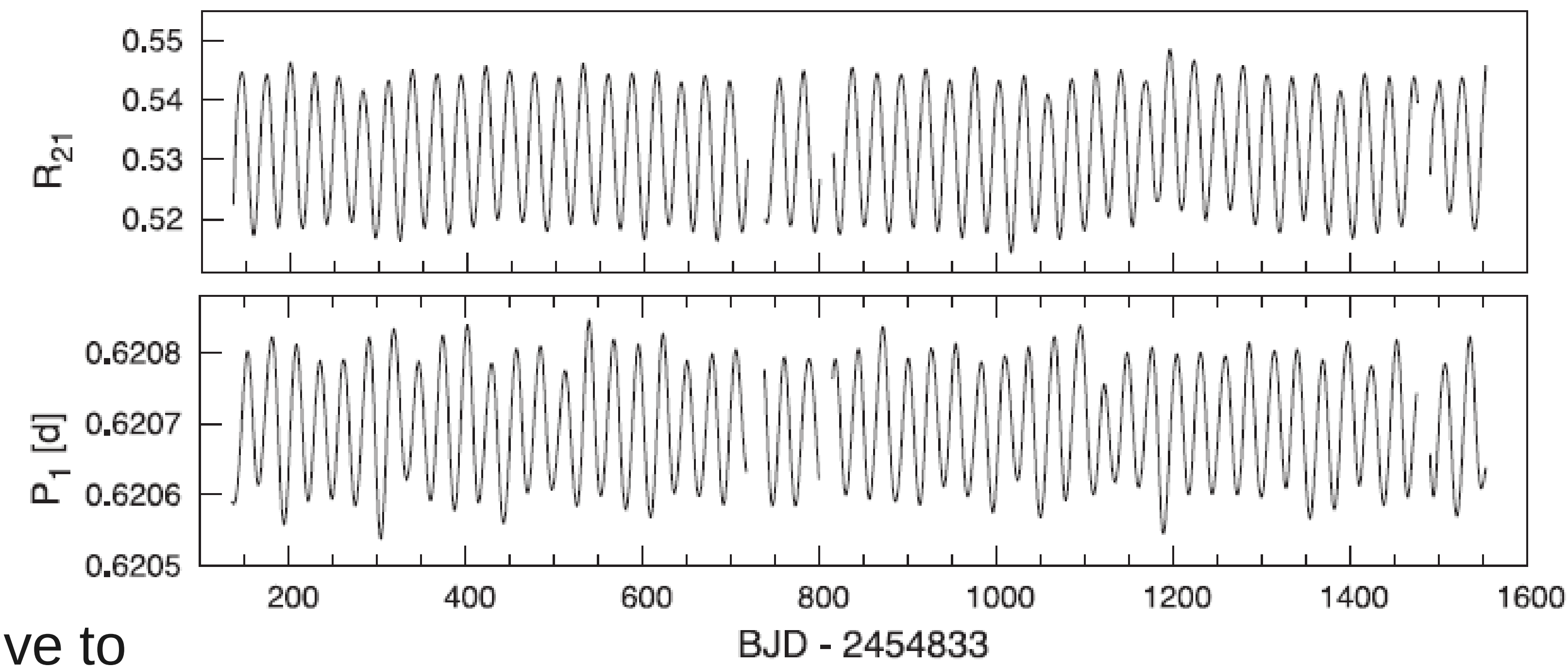


from 1 minute sampling data: one quarter

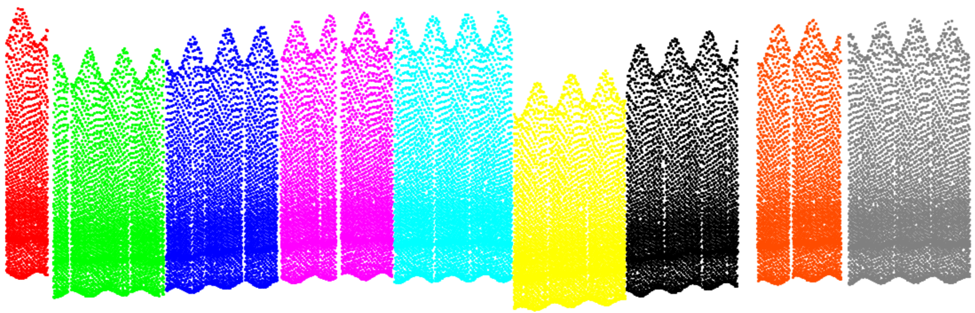


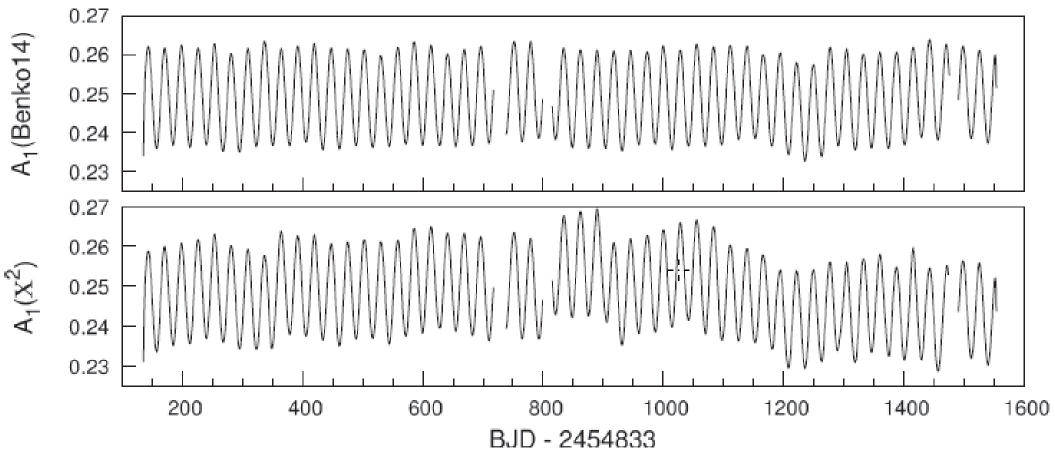
The dynamical investagation of the Blazhko effect

Analytical functions



R_{21} is less sensitive to instrumental effects than A_1





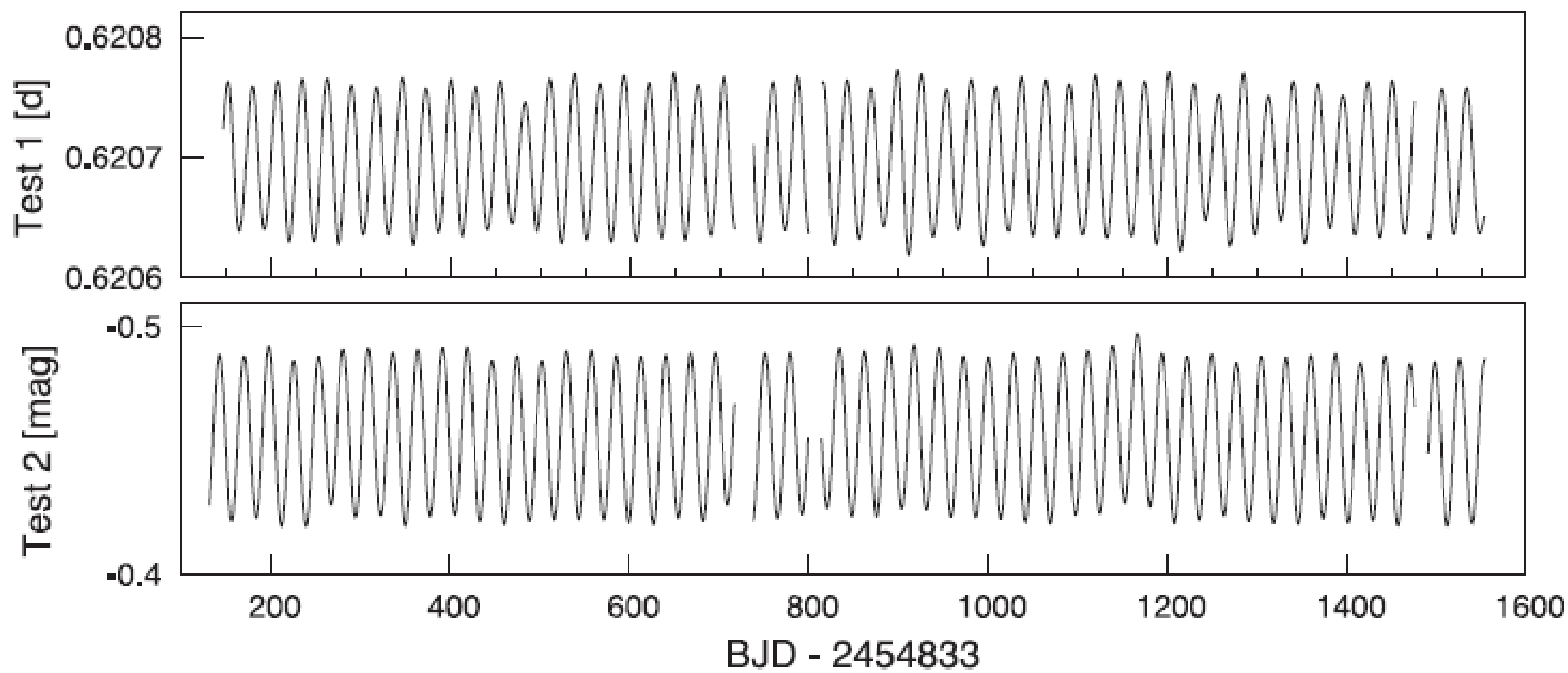
The dynamical investigation of the Blazhko effect

Problems:

- rare sampling
- quarter stitching uncertainty
- detrending distortions

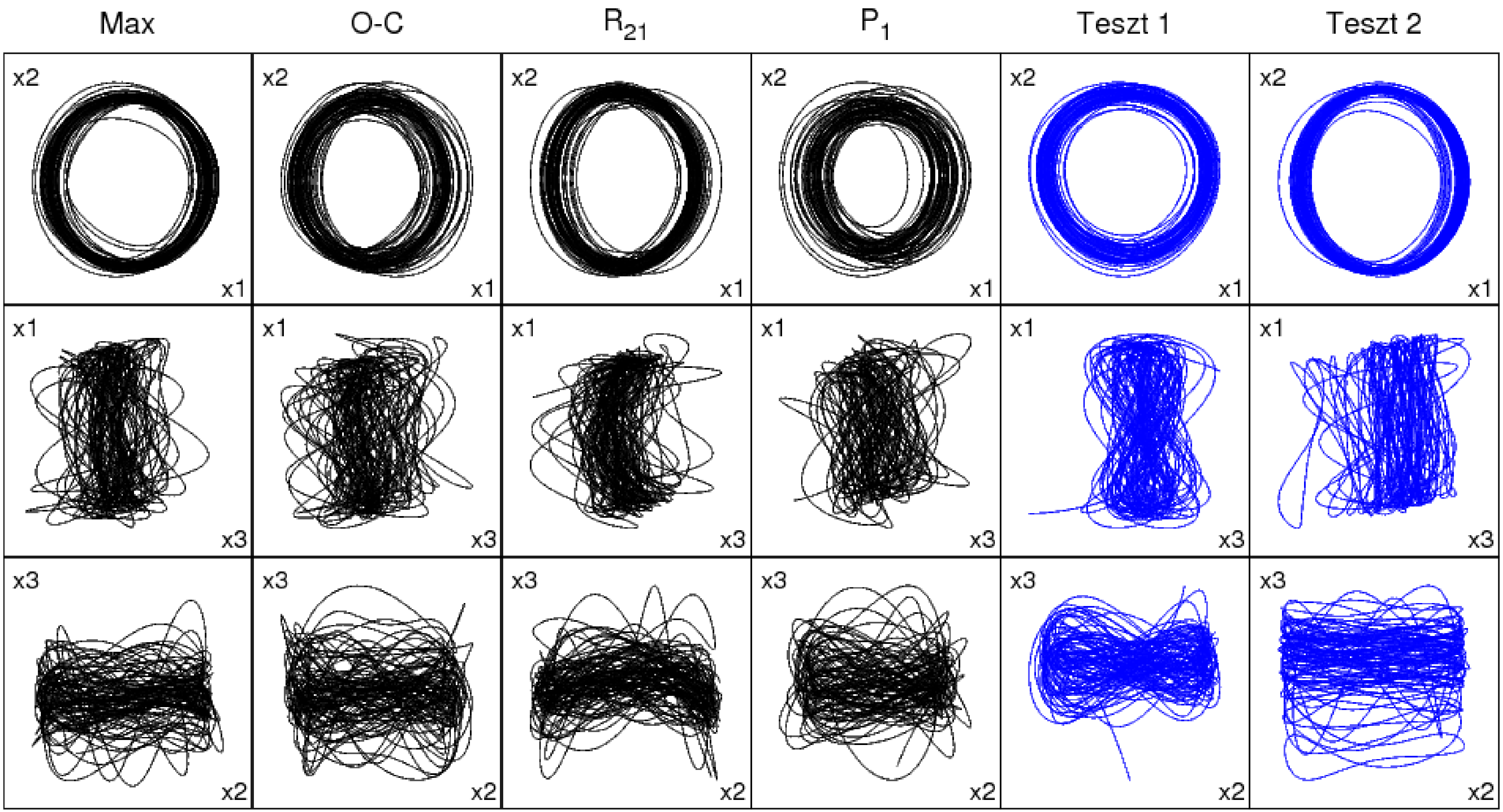
--> test modulation curves

- effect of noise
  (on P1 modulation curve)
- effect of data processing
  (on pulsation maxima)



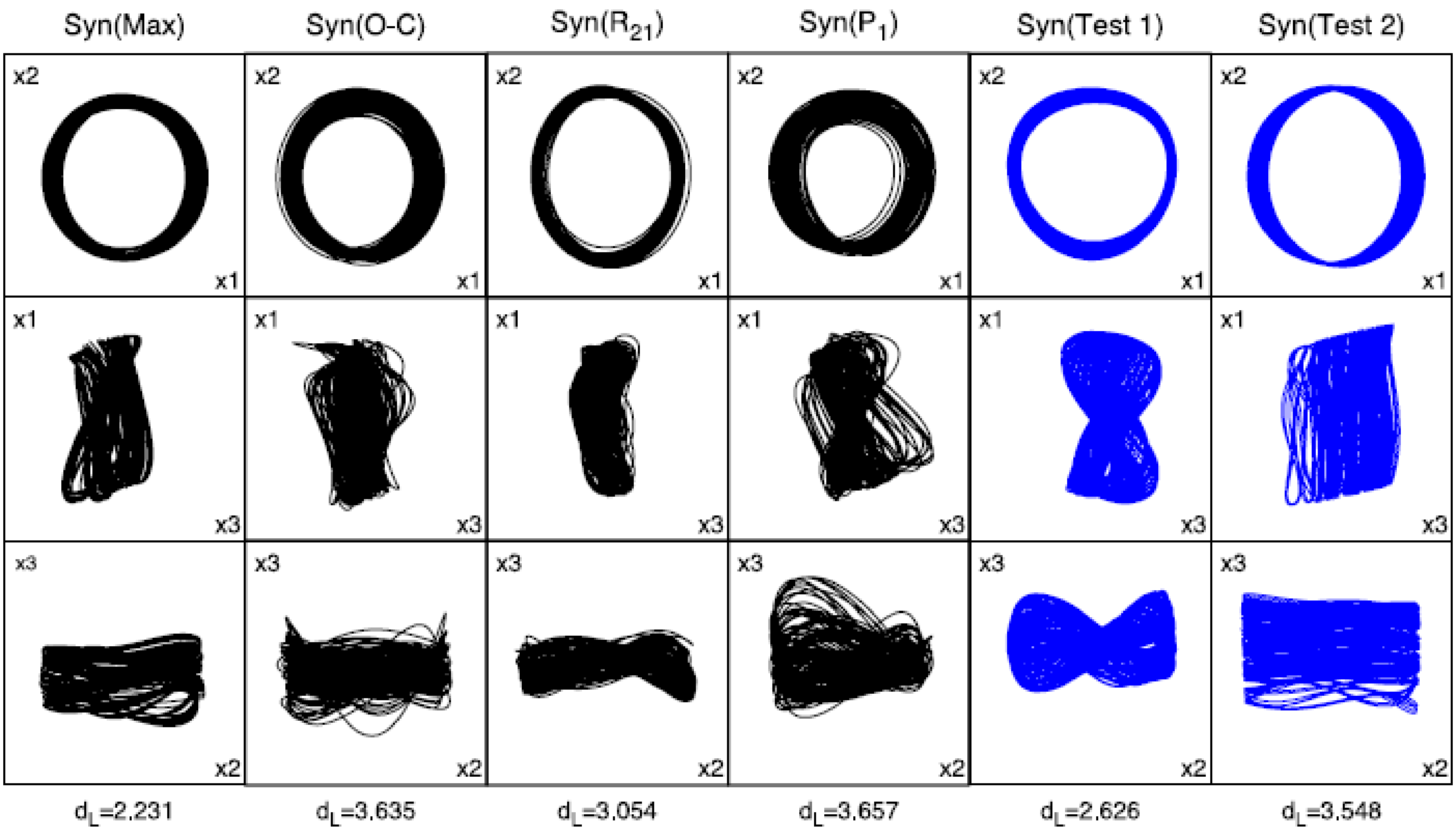
The dynamical investagation of the Blazhko effect

Broomhead – King projections of the modulation curves and the test data



The dynamical investagation of the Blazhko effect

Broomhead – King projections of
the synthetic signals



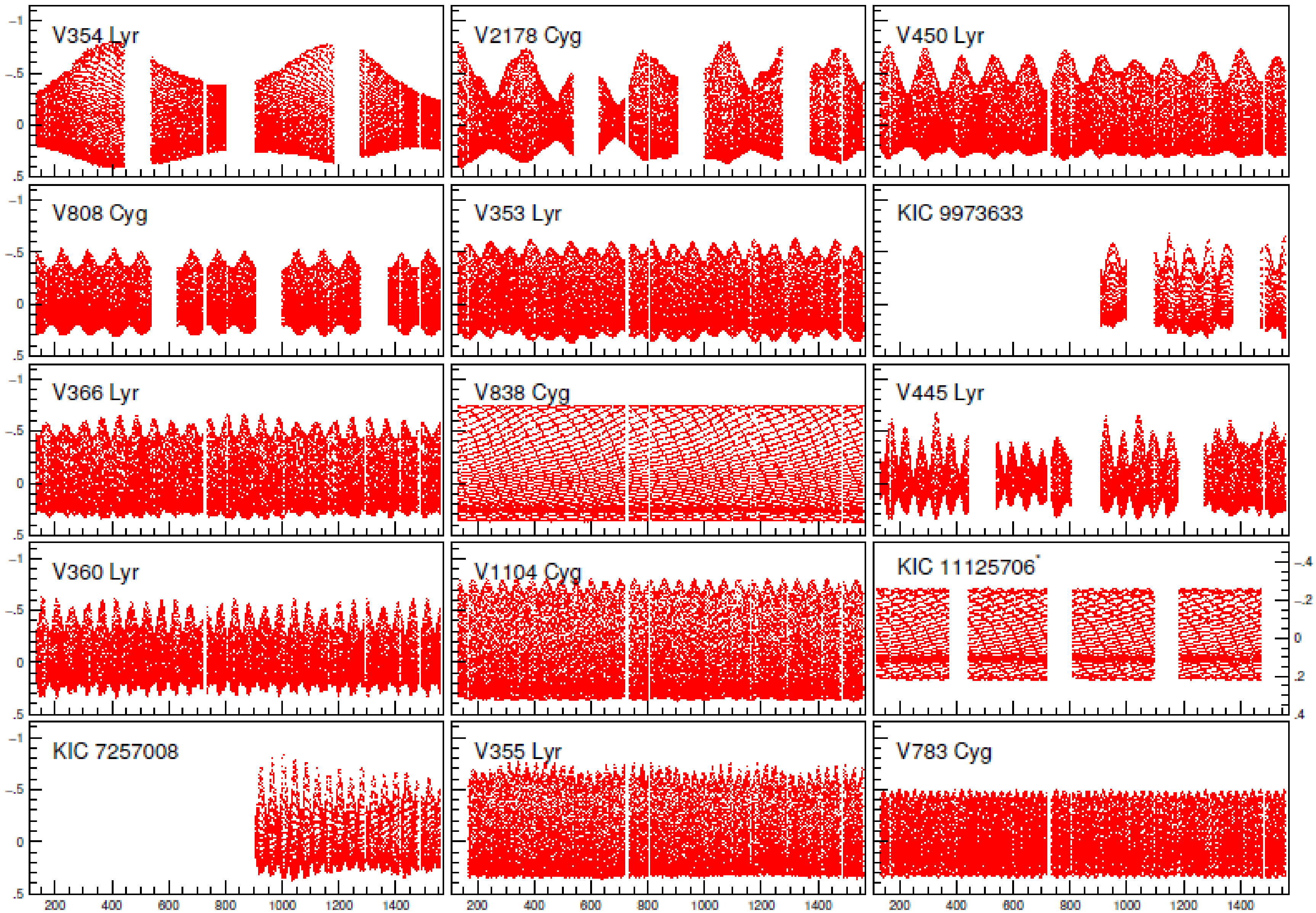
The dynamical investagation of the Blazhko effect

	Chaotic maps	Acceptable	Fraction	D_L
Max	154	143	93 per cent	2.48 ± 0.47
$O-C$	207	166	80 per cent	2.63 ± 0.54
R_{21}	647	275	43 per cent	2.43 ± 0.37
P_1	500	411	82 per cent	2.46 ± 0.44
Test 1	301	227	75 per cent	2.85 ± 0.43
Test 2	166	139	84 per cent	2.39 ± 0.37

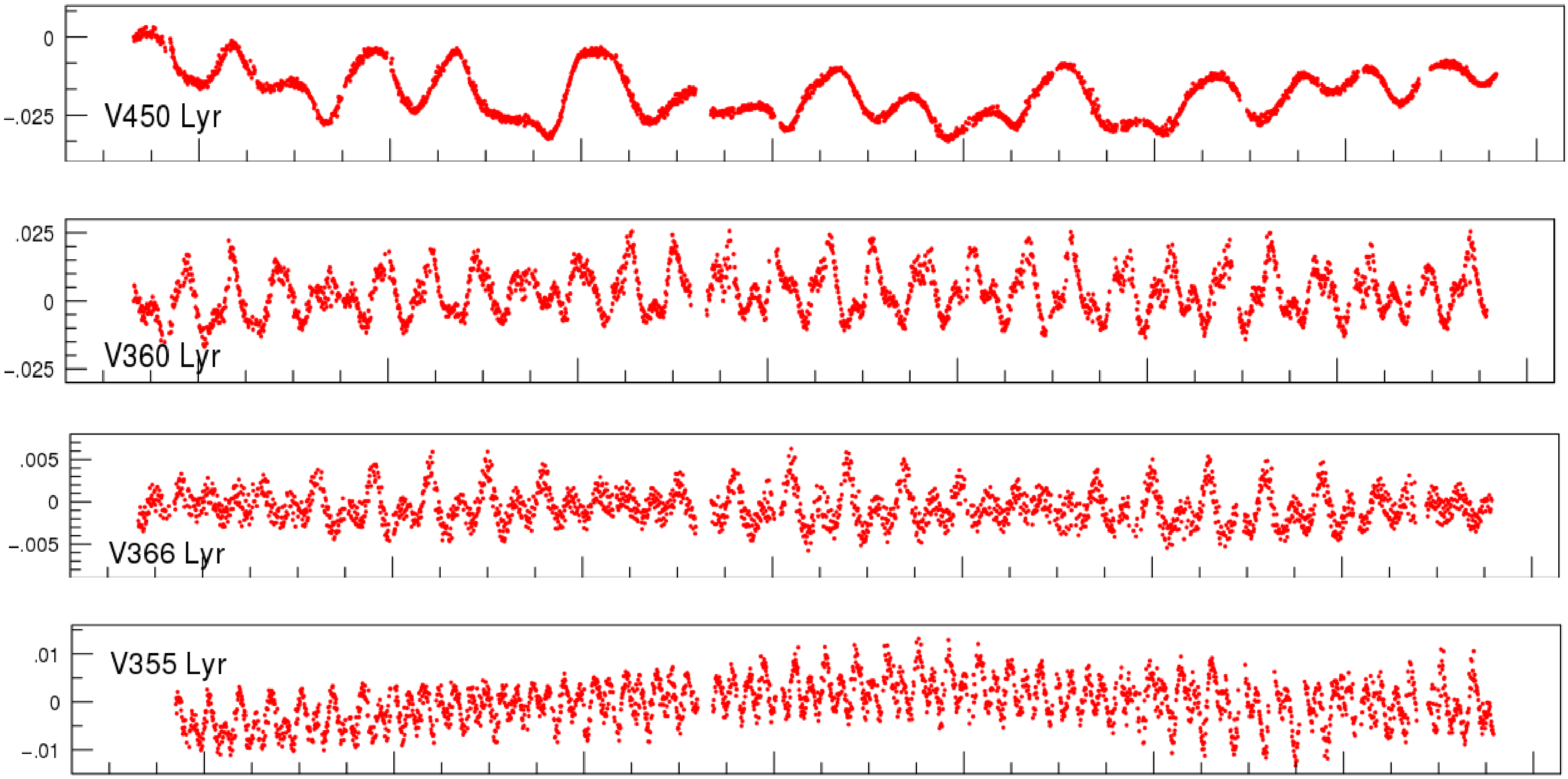
False positive results
--> the global flow reconstruction of themodulation is uncertain

Plachy et al. 2014: Non-linear dynamical analysis of the Blazhko effect with the Kepler space telescope: the case of V783 Cyg

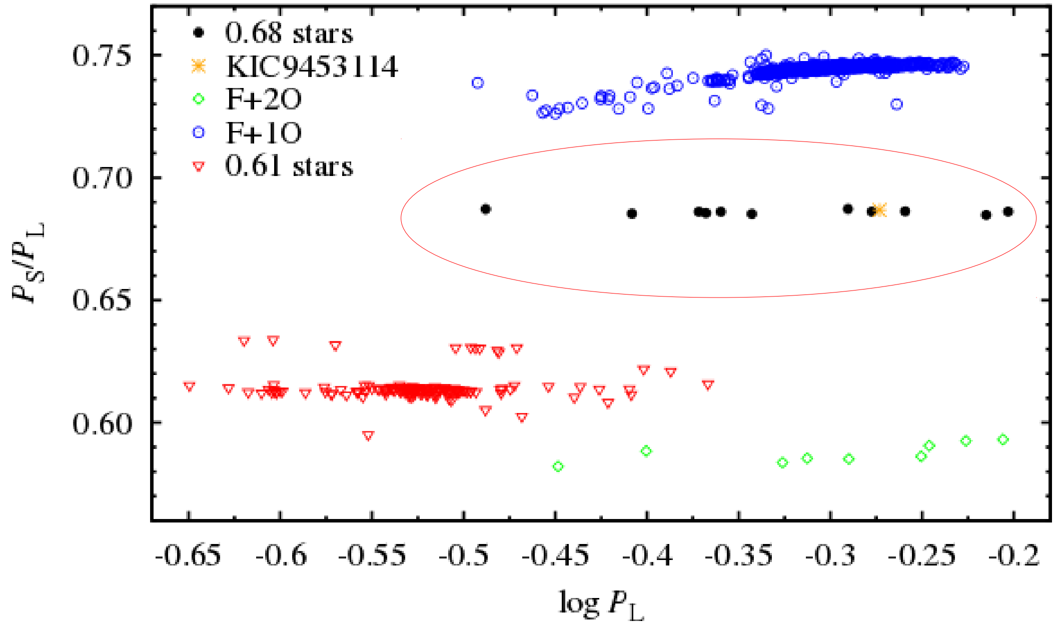
The dynamical investagation of the Blazhko effect



The dynamical investagation of the Blazhko effect



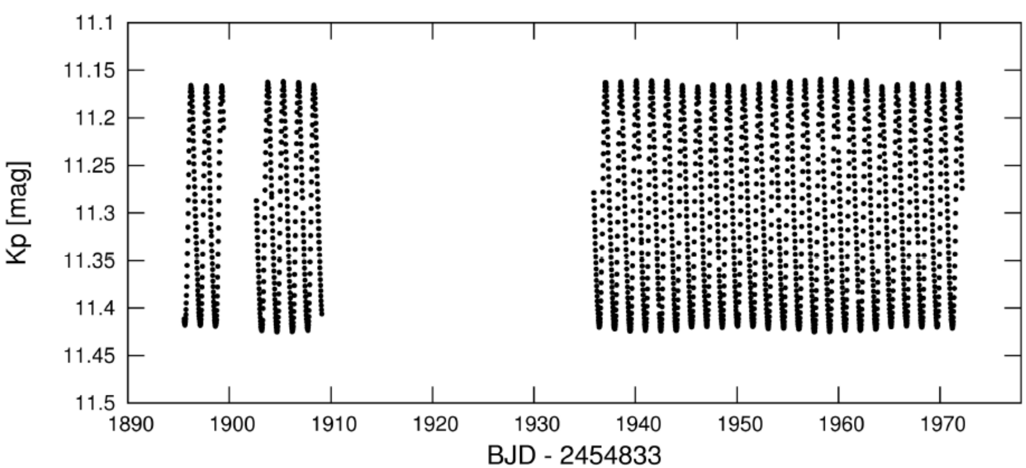
New discoveries



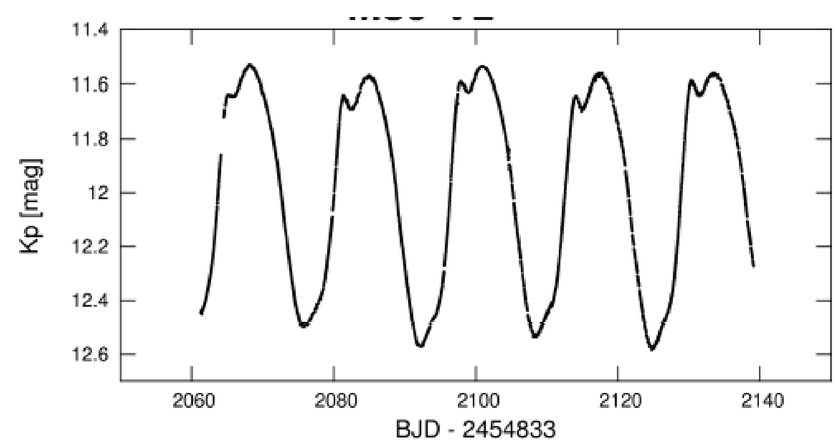
Netzel et al. 2015

K2 cepheids (Plachy et al. In prep)

~ 0.69 period ratio



period doubling



rrl2015.hu


RRL 2015
High-precision studies of RR Lyrae stars
from dynamical phenomena to mapping the galactic structure
19-22 October 2015 Hotel Visegrád, Visegrád, Hungary
Time until the conference 61 days
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Committees
Preliminary program
Invited speakers (confirmed)
Registration
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Invitation to the RR Lyrae 2015 Conference
The field of RR Lyrae studies is vigorous and has recently gained new momentum. These pulsating variable stars are part of the space photometric revolution: continuous, high-precision observations shed new light on the still mysterious Blazhko-modulation, unexpected dynamical phenomena have been discovered, novel ideas and hypotheses have emerged. On the theoretical front multidimensional hydrocodes are about to mature, allowing for an improved description of the interaction between convection and pulsation,
Deadlines
Abstract submission ~~July 31, 2015~~
Extended to August 31
Payment of early registration fees ~~July 31, 2015~~
Extended to August 31